Transition to turbulence in forced vibrations of perfect plates

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Summary. The transition from periodic to chaotic vibrations for perfect plates harmonically forced with an increasing amplitude, is numerically studied. Von Kármán model for a rectangular plate with simply supported boundary conditions is considered. Second order finite differences in space are combined with an energy-conserving scheme to provide numerically robust and accurate solutions handling complex dynamics, damped and undamped cases as well as pointwise external forcing. The numerical results are in agreement with experimental observations, recovering a generic scheme for the transition to turbulence. The chaotic regime is described in the framework of wave turbulence, so that the dynamics exhibited by plates can be interpreted as a turbulent state in a solid. For undamped plates, the power spectra of turbulence are in agreement with those theoretically predicted. Adding damping to the plate has a significant effect on the turbulent spectra, hence giving a clue for explaining the discrepancy with measurements realized on plates with large dimensions.

Introduction

When subjected to large-amplitude external loads, thin plates and shells can experience a complex vibratory regime that can be interpreted as turbulence in a solid medium [1, 2, 3]. The transition from periodic response to the wave turbulent state has been extensively studied experimentally on a number of different plates, shells, cymbals and gongs [4, 5, 6]. A generic transition scenario has been proposed, involving two bifurcations separating three regimes. For low amplitudes of excitation, a periodic (quasilinear, unimodal) regime is at hand. The first bifurcation is characterized by energy exchange between internally resonant modes, resulting in a quasiperiodic regime. This second regime is obtained only when internal resonance relationships exists, otherwise a direct transition to the third -turbulent- regime is observed. The wave turbulence motion is characterized by an energy transfer from large to small wavelengths. A discrepancy has been found between the power spectra of the transverse velocity theoretically predicted in the framework of wave turbulence [1] with those experimentally measured [2, 3]. The aim of this work is to study numerically the forced vibrations of plates in order to confirm the transition scenario, and to gain insight in understanding the turbulent regime.

Model

A simply-supported, perfect rectangular plate is considered in the framework of von Kármán equations for thin, geometrically nonlinear plates. The equations of motion reads [7, 8, 6]:

$$\rho h \ddot{w} + D\Delta\Delta w + \sigma_0 \dot{w} + \sigma_1 \Delta \dot{w} = L(w, F) + p, \tag{1a}$$

$$\Delta\Delta F = -\frac{Eh}{2}[L(w, w)], \tag{1b}$$

where w is the transverse displacement, F the Airy stress function, $p = F_0 \cos 2\pi f_{exc} t$ the external forcing, $D = Eh^3/12(1-\nu^2)$ the flexural rigidity and L the usual bilinear operator of the von Kármán equations. Two damping laws has been considered, the first one, controlled by σ_0, is independent of the frequency, the second one, controlled by σ_1 is linear with the frequency. The second-order finite difference scheme with an energy-conserving time integration defined in [8], is applied to (1).

Simulation results

The plate selected for simulations has dimensions $L_x = 0.4$m $\times$ $L_y = 0.6$m, and thickness h=1 mm. Material parameters have been set so as to modelize a metallic plate, so that E= 200 GPa, ν=0.3 and ρ=7860 kg.m^{-3}. Fig. 1 shows the result obtained for a damped plate with σ_0=0.75 and σ_1=0, excited at f_{exc}= 75 Hz, near the fourth eigenfrequency. The simulation lasts 12 seconds, where the amplitude of the forcing F_0 is linearly increased from 0 to 24 N. The sampling rate of the simulation is 50000 Hz. The spectrogram of the displacement of one selected point of the plate is shown, as well as the Fourier transform of a window of 1 second (between the sixth and the seventh), in order to get a better visualization of the spectral content. A three-stage scenario is obtained, where a 1:3 superharmonic resonance is excited before the turbulent state. The first mode of the plate has for eigenfrequency 22 Hz and is excited through mode coupling, resulting in a two-mode regime. Then the turbulent regime sets in, it is characterized by a broadband Fourier spectrum with energy up to 12000 Hz.

The effect of the damping on the spectra of turbulence is shown in Fig. 2. The plate is excited at 75 Hz with a forcing amplitude of 24 N, ensuring the turbulent behaviour is at hand. Sampling rate is now sets at 100 000 Hz. The damping

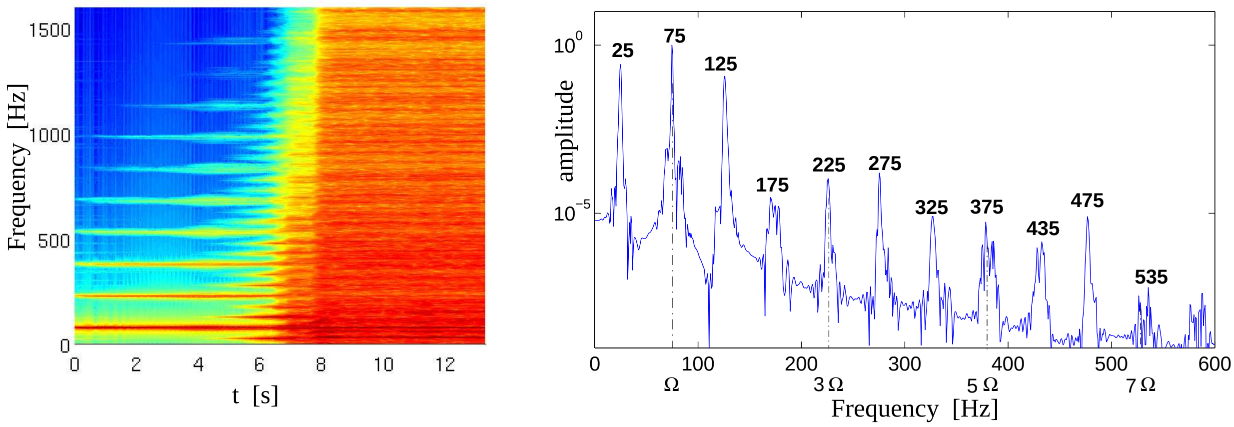

Figure 1: Spectrogram of the displacement of a selected point on the plate (left), and FFT of the signal between 6^{th} and 7^{th} seconds (left). The plate is harmonically forced with f_{exc}=75 Hz and an increasing amplitude F_0 from 0 to 24 N. The spectra reveals the superharmonic 1:3 coupling between the directly excited mode at 75 Hz and the fundamental mode of the plate.

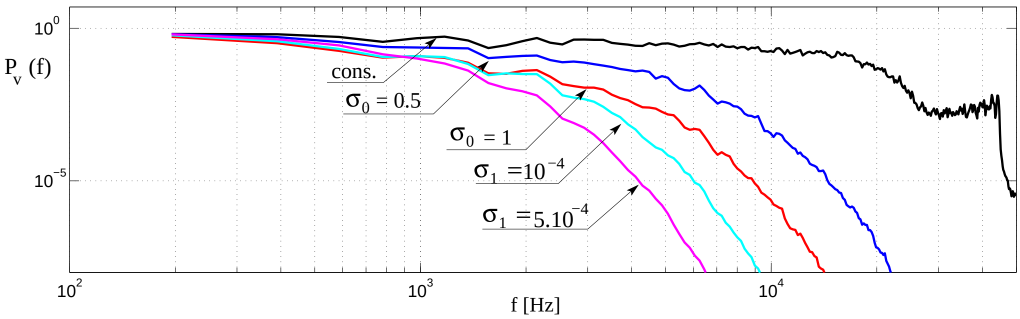

Figure 2: power spectra $P_v(f)$ of the transverse velocity in the turbulent regime, plate excited at 75 Hz and 24 N. The conservative case (cons.) is compared to damped plates with σ_0=0.5 and 1 (and σ_1=0), then σ_1=10^{-4} and 5.10^{-4} (and σ_0=0).

factors are then increased: four cases are tested and compared with the undamped case. In the first two, σ_1=0 and σ_0=0.5 and 1, then σ_0=0 and σ_1=10^{-4} and 5.10^{-4}. The power spectra $P_v(f)$ of the transverse velocity is shown. For undamped perfect plates, wave turbulence theory predicts that $P_v(f)$ should be independent on f [1]. This result is recovered with a fairly good accuracy with our simulation, where a flat spectrum is observed on two decades. As reported in [2, 3], experimental measurements on a real plate are markedly different from this prediction, with the appearance of a clear cut-off frequency that can be scaled to the injected power. Fig. 2 highlights the fact that taking the damping into account leads in the appearance of this cut-off frequency. More thorough analysis will be presented at the conference, showing different cases for the transition scenario as well as further insight into the turbulent regime.

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