EXPERIMENTAL EVIDENCE OF MECHANICAL FREQUENCY COMB IN A QUAD-MASS MEMS STRUCTURE

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ABSTRACT

Challenging applications and the increasing request of higher performances of MEMS devices are focusing the attention of the MEMS community on the understanding and modelling of complex dynamic phenomena. In this work, we experimentally detect a mechanical frequency comb arising in a quad-mass MEMS structure electrostatically actuated through comb fingers electrodes and we propose a simplified model able to predict such nonlinear dynamic behaviour. In particular, we hypothesize a 1:3 internal resonance between two in-plane modes of the structure and exhibit the presence of a Neimark-Sacker bifurcation responsible of frequency comb phenomena.

KEYWORDS

Modelling, Nonlinearities, Frequency Comb, Bifurcation

INTRODUCTION

Micro-Electro-Mechanical-Systems (MEMS) are experiencing exponential growth in terms of popularity and technical challenges. They are used in countless applications ranging from sensing (e.g. Gyroscopes, Accelerometers, Magnetometers, etc.) to timing (e.g. resonators) and actuation (e.g. microactuators, micro-mirrors).

Moreover, emerging cutting-edge applications are requiring the development of complex nonlinear models to predict the in-operation behaviour of MEMS devices. Due to the typical low damping conditions and the large displacements regime employed in many MEMS to obtain high performances in terms of noise and sensitivity, nonlinear phenomena can indeed arise mainly from geometric and electrostatic sources.

Several numerical and analytical electro-mechanical nonlinear models have been proposed so far to describe the dynamic behaviour of MEMS devices [1]–[4]. Nonlinear phenomena have been also studied for their potential benefits on the MEMS performances [5]–[7], but the topic still remains very attractive and challenging for the MEMS community.

In particular, the study of bifurcations and chaotic phenomena are recently attracting interest due to their increasing popularity in MEMS devices working in the nonlinear regime [8]–[11]. Different types of bifurcations can indeed arise in MEMS as soon as the nonlinear level overcomes a certain threshold and more than one stable configuration is possible. By analysing the bifurcation diagrams, it is possible to classify the bifurcation nature and consequently identify the dynamic state of the MEMS structure, e.g. period-doubling, quasiperiodic, chaotic motion.

Among the possible alternative states, the quasiperiodic regime, that arises after a Neimark-Sacker (NS) bifurcation, is of special interest in MEMS devices featuring internal resonance phenomena [1], [2] . Internal resonance arises when two or more modes are nonlinearly coupled and then able to exchange energy [12]. The internal resonance arising from the coupling of two modes is defined as 1:1, 1:2, and 1:3 when the frequency ratio between the higher and the lower coupled modes is close to 1, 2 or 3, respectively. In the quasiperiodic regime, the main signal at the driven frequency, i.e. lower mode, is modulated in amplitude with a period longer than the main one, thus resulting in a Power Spectral Density (PSD) made by peaks equally spaced around the main resonant frequency that is known as frequency comb.

Frequency combs can be found in mechanical and nonmechanical systems like e.g. optical devices [13], [14]. In a mechanical system, the frequency comb phenomenon can be achieved through nonlinear mode coupling [15] as mentioned before but also through different mechanisms like for example contact between elastic components [16].

In this paper we present a mechanical frequency comb detected experimentally in a quad-mass MEMS structure similar to the one studied in [17]. We identify the 1:3 internal resonance between the drive mode and a spurious in-plane mode as the possible mechanism triggering the NS bifurcation responsible for the frequency comb and we provide a simplified model able to reproduce it. Being the simplified model based on a-priori numerical simulations and not on an experimental calibration, this work represents an important step towards the numerical and analytical prediction of very complex dynamic behaviours arising in MEMS.

QUAD-MASS STRUCTURE

The MEMS quad-mass structure here considered is inspired by the one proposed in [17]. It consists of four shuttle masses suspended through folded springs and connected one to the others through angular springs as shown in Fig. 1a.

Four sets of comb fingers allow the actuation and the readout of in-plane modes. In Figs. 1b-c, two in-plane modes of the quad-mass structure are reported. The first one, that will be referred to as the beating heart drive mode in the following, consists in an in-plane expansion (contraction) of the structure (see yellow arrows in Fig. 1b), while the second one will be referred in the following as spurious drive mode and consists in an expansion (contraction) in the horizontal direction and a simultaneous contraction (expansion) in the vertical direction (see yellow arrows in Fig. 1c). The natural frequencies of the two in-plane modes under study are computed through modal analysis and read 19.4 kHz and approximately 60 kHz, respectively.

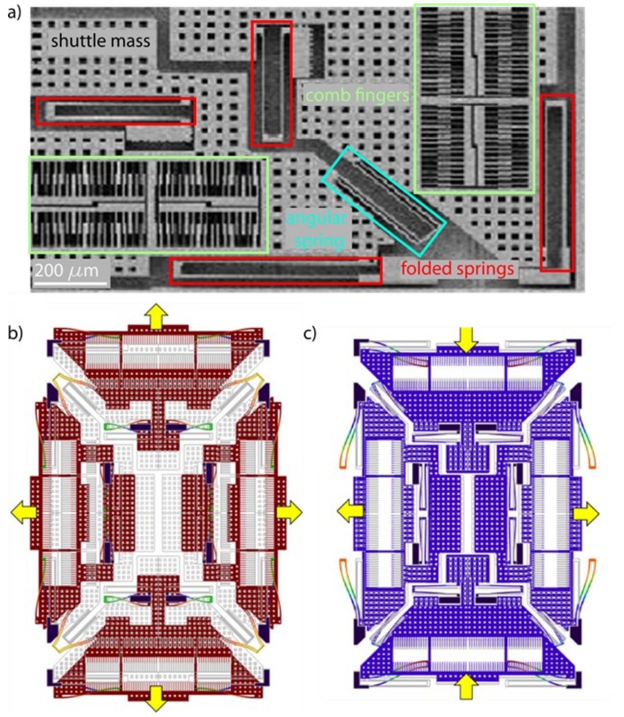

Figure 1: a) Scanning Electron Microscope (SEM) image of the quad-mass structure. Only a quarter of the structure is reported for symmetry reason. b) Beating heart drive mode (f = 19.4 kHz). c) Spurious drive mode (f ≈ 60 kHz). The contour of the displacement magnitude is shown in colour.

It is worth mentioning that the two modes reported in Figs. 1b-c have similar nature, i.e. in-plane translation of the shuttle masses and that the ratio between their natural frequencies is around three. Moreover, in [17], it is shown that the angular springs can exhibit mechanical softening when the structure is excited according to the beating heart mode in the large displacement regime. All these ingredients open the way to a possible 1:3 internal resonance mechanism and a consequent PSD in the form of a frequency comb.

EXPERIMENTAL RESULTS

The nonlinear dynamics of the proposed structure is firstly experimentally investigated. A bias voltage $V_{DC} = 10.43$ V is applied to the MEMS quad-mass structure through a DC power supply Agilent E3631A, while a sinusoidal voltage of amplitude $|V_{AC}| = 0.75$ V generated through the Waveform generator Agilent 81150A, is applied on two sets of comb fingers to provide the push-pull actuation of the beating heart mode. The readout is performed in a differential way from the other two sets of comb fingers. A Signal amplifier SRS model SR570 is used to amplify the output signal, while a Spectrum Analyser Agilent 35670A is employed to collect PSD diagram reported in Fig. 2.

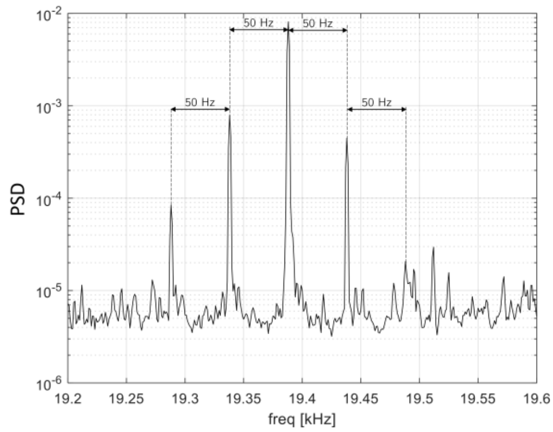

Figure 2: Experimental PSD measured on the quad-mass structure for a V_{DC} = 10.43 V when a V_{AC} of amplitude 0.75V and frequency around the natural frequency of the beating heart mode is applied.

From Fig. 2, it is evident that the frequency comb phenomenon is activated in the quad-mass structure under study: equally spaced multiple resonance peaks are indeed present around the driven frequency, i.e. 19.388 kHz. To better characterize the nonlinear behaviour of the driven beating heart mode, a second set of measurements is also performed. The nonlinear frequency response of the MEMS structure for a $V_{DC} = 10$ V and a $|V_{AC}| = 2$ V at the natural frequency of the beating heart mode is measured through the ITMEMS and is reported in Fig. 3. The experimental curve bends towards the left, thus showing an overall softening behaviour as expected from [17].

FREQUENCY COMB IN COUPLED OSCILLATORS: A SIMPLIFIED MODEL

To provide a valuable theoretical explanation of the experimental results shown in the previous section, we present a simplified mechanical model able to reproduce the frequency comb starting from the 1:3 internal resonance between the driven and the spurious modes represented in Fig. 1. In the following, u and v are the degrees of freedom representing the beating heart mode and the spurious mode, respectively.

The Hamiltonian form usually employed to describe the nonlinear dynamics of systems experiencing the 1:3 internal resonance is here considered as the starting point of our simplified model [7], [18]:

$$H = \frac{1}{2}(\dot{u}^2 + \dot{v}^2) + \frac{1}{2}(\omega_1^2 u^2 + \omega_2^2 v^2) + \frac{1}{4}\beta u^4 + + \frac{1}{3}\gamma u^3 v \tag{1}$$

where ω_i is the angular frequency of the $i-th$ mode and β and γ are the nonlinear coupling coefficients. For the sake of clarity, let us define $\omega_2 = \omega_1(3+\sigma_1)$, where σ_1 is a detuning parameter allowing accurate quantification of the fulfilment of the 1:3 internal resonance.

Starting from the Hamiltonian of eq. (1) and adding the damping and electrostatic contributions, we get the equations of motion describing the nonlinear dynamics of the MEMS quad-mass structure:

$$\ddot{u} + \frac{\omega_1}{Q_1}\dot{u} + \omega_1^2 u + \beta u^3 + \gamma u^2 v = F_1 \cos \Omega t \qquad (2a)$$

$$\ddot{v} + \frac{\omega_2}{Q_2}\dot{v} + \omega_2^2 v + \frac{\gamma}{3} u^3 = 0 \qquad (2b)$$

where Q_i is the quality factor of the *i−th* mode, F_1 is the load multiplier representing the electrostatic actuation performed through comb fingers, and Ω is the external forcing pulsation. As expected from the bifurcation theory, only cubic coupling coefficients are present in the two-degrees of freedom model. From the modal coordinates u and v the gyroscope shuttle mass displacement d is reconstructed. This is done by means of the modal expansion d= $u\,\varphi_1 + v\,\varphi_2$, with φ_1 and φ_2 denoting the modal displacement field of the linear main modes mass normalized. In our case $\varphi_1 = 0.00413\,\mu$m and $\varphi_2 = 0.0037\,\mu$m.

To estimate the unknown coefficients of eqs. (2), we employed the normal form theory [19]. In particular the static condensation approach proposed in [20] is here applied for the a-priori numerical estimation of the mechanical coupling coefficients. The electrostatic forcing term is instead estimated on the basis of analytical formula available for push-pull actuation system based on comb fingers electrodes. A fully-coupled electro-mechanical simulation as the one shown in [21] is here avoided being the electrostatic nonlinearities induced by comb fingers actuation/readout schemes negligible with respect to mechanical ones. Finally, quality factors are estimated starting from the data available in [17] for a similar structure. Uncertainties on such values must be then considered in the comparison with experimental data.

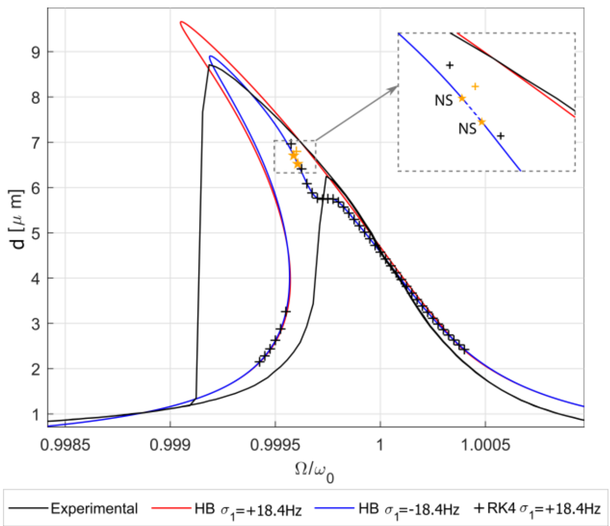

Figure 3: Comparison between experimental frequency response function and theoretical model

In Table I, the coefficients of eqs. (2) are reported. It is worth mentioning that the cubic coefficient β that governs the nonlinear behaviour of the beating heart mode has a negative value as expected from the experimental measurements presented in the previous Section.

The solution branches of eqs. (2) are found by numerical continuation, using the package MANLAB [22] that implements a combination of Harmonic Balance (HB) with an asymptotic numerical method (ANM) for path-following procedure. Fig. 3 compares the numerical outcome to the experimental measurements. Two different values of the detuning parameter σ_1 are considered: when σ_1 is positive the two modes have minor interactions (red curve of Fig. 3), while when σ_1 is negative (blue curve of Fig. 3) the two modes have strong interaction and the NS bifurcation responsible of the frequency comb phenomenon is achieved.

Table 1: Simplified model parameters.

Parameters	Value
ω_1	0.1219 rad/µs (19.4 kHz)
β	$-6.86 \cdot 10^{-12}$
γ	$-3.43 \cdot 10^{-11}$
σ_1	$\pm 1.15 \cdot 10^{-4}$ rad/µs (± 18.4 Hz)
Q_1	4200
Q_2	5100
F_1	$46.89\ \epsilon_0 V_{DC} V_{AC}$

The NS region is enclosed between the orange star markers labelled with NS (computed numerically with MANLAB), and the corresponding frequency response function periodic solution highlighted with a dashed line. The close-up view in the grey box shows a magnification of the bifurcation region. In order to inspect the temporal solution and investigate the frequency comb, the quasiperiodic regime is computed through a fourth-order Runge-Kutta algorithm (RK4). From Fig. 3, it is clear that the RK4 solution is in perfect agreement with the one obtained through the HB method except for the values near the bifurcation region. In this region, indeed, the quasiperiodic regime gives rise to an amplitude modulation typical in frequency comb phenomena, not caught by the periodic solution of the HB method.

The RK4 solution inside the NS region, marked with the orange cross in Fig.3, is inspected to put in evidence the frequency comb phenomenon reproduced through the simplified model proposed in this Section. The results are reported in Fig.4. The PSD of the time dependent solution is reported in Fig. 4a, while from Fig. 4b it is clearly visible the amplitude modulation of the shuttle displacement time history expected in presence of frequency comb.

CONCLUSIONS

A frequency comb is experimentally measured in a quad-mass MEMS structure driven according to the beating heart in-plane mode. A simplified two-degrees of freedom theoretical model based on the nonlinear coupling of two in-plane modes of the structure is proposed to explain such complex nonlinear dynamics. Despite the excellent agreement between the theoretical and experimental frequency response curves, the frequency comb obtained through the simplified model is only in a qualitative agreement with the experimental one. This discrepancy can be explained by the uncertainties we have on the quality factors of the two modes and in general on the effects of fabrication imperfections on the geometry of the structure. Moreover, the extremely simplified model employed in this work can also play a role in this direction.

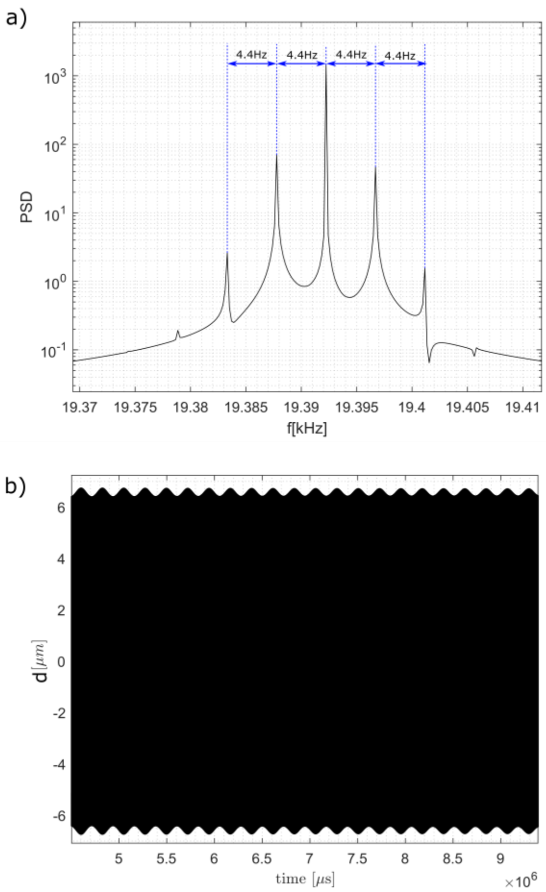

Figure 4: a) PSD and b) time history of the shuttle displacement obtained through the proposed simplified model.

We are currently working towards more complex and complete models able to explain also quantitatively the arising of frequency combs in quad-mass MEMS structures. A wider experimental campaign will be also performed in the near future to identify the quality factors and other unknown geometric and electrostatic parameters.

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