Quelques exemples de dynamique non-linéaire dans la mécanique du contact/frottement

Vladislav A. Yastrebov

MINES ParisTech, PSL Research University, Centre des Matériaux
CNRS UMR 7633, Evry, France

Manifestation du GDR DYNOLIN
DYnamique **NO**n **LIN**éaire
ENSTA ParisTech, Palaiseau, France
11 octobre 2016

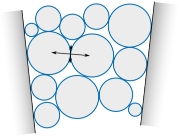
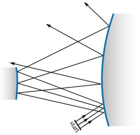
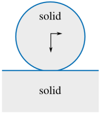
• Outline

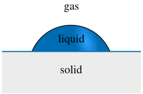
- Introduction to contact mechanics
  - Industrial and natural problems
  - Complex interface physics
  - Several important examples
- Elastodynamic frictional sliding on bi-material interface

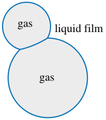
\- *Vibration of asymmetric materials with internal contacts*

Introduction

- **Dynamical systems are often defined over a single independent variable: scalar time** t
- **Multi-/infinite-dimensional extension: loading conditions (tractions, displacements):** $\underline{p}(x,t)$ for $x \in \Gamma_f(t), t \in \mathbb{R}$
- **Examples of dynamical systems with contacts**
  - Granular matter
  - Dynamical billiards
  - Mechanical contact between solids (quasi-static/dynamic)
  - Liquid/solid contact (*e.g., liquid drop on a substrate*)
  - Liquid/liquid contact (*e.g., merger of liquid drops*)





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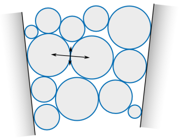
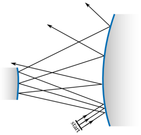
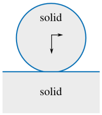
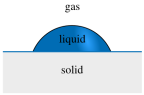
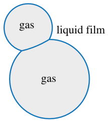








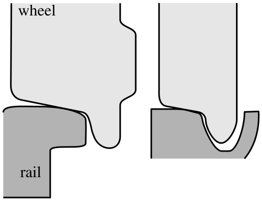
- Tyre/road contact

www.motortrend.com

• Industrial and natural contact problems

- Tyre/road contact
- Wheel/rail contact

Railway wheel www.railway-wheel-axle.com

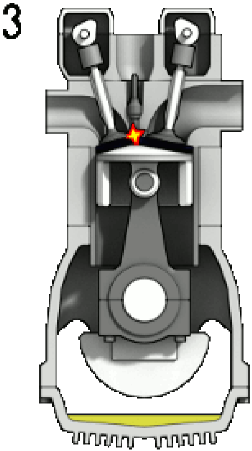


Railway Tram

Wheel flange
adapted from Wikipedia

• Industrial and natural contact problems

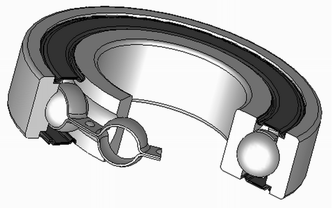
- Tyre/road contact
- Wheel/rail contact
- Piston/cylinder



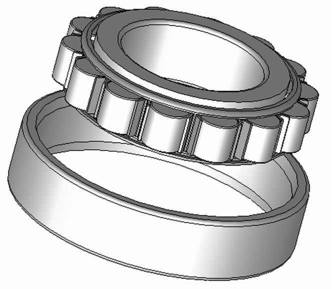
Four-stroke cycle in cylinder
from Wikipedia

• Industrial and natural contact problems

- Tyre/road contact
- Wheel/rail contact
- Piston/cylinder
- Bearings



A sealed deep groove ball bearing
from Wikipedia



A cylindrical roller bearing
from Wikipedia

• Industrial and natural contact problems

- Tyre/road contact
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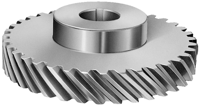
- Piston/cylinder
- Bearings
- Gears

Change gear



Helical gear

Bevel gear

www.linngear.com

• Industrial and natural contact problems

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Interior of Rolex watches
www.rolex.com

Ship reduction gearbox 14 MW
(e.g. Renault Mégane 1.4 $\approx$ 60 KW)
www.renk.eu

• Industrial and natural contact problems

- Tyre/road contact
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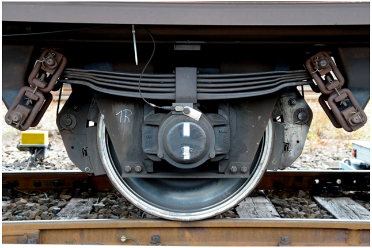
- Gears
- Brake systems

Reinforced carbon brake disc on a Ferrari F430
Wikipedia



New LL brake blocks aimed to reduce noise from rail sector photo: UIC/EuropeTrain

• Industrial and natural contact problems

- Tyre/road contact
- Wheel/rail contact
- Piston/cylinder
- Bearings
- Gears
- Brake systems
- Assembled pieces

Modern steam turbine Wikipedia

Fuselage of modern aircrafts contains ≈100 000 rivets
www.news.cn

• Industrial and natural contact problems

- Tyre/road contact
- Wheel/rail contact
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- Bearings
- Gears
- Brake systems
- Assembled pieces
- Impact and crash

Bird impact traces on aircraft's nose

Mercedes crash test
Insurance Institute for Highway Safety

• Industrial and natural contact problems

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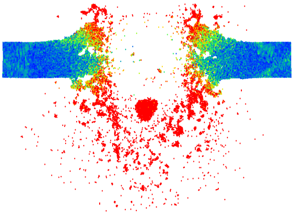
- Impact and crash
- Penetration and perforation

Sherman Firefly armor piercing shell on Tiger tank armor, Bovington Tank Museum
Andy's photo www.flickr.com



Hyper velocity perforation
Molecular Dynamics simulation

• Industrial and natural contact problems

- Tyre/road contact
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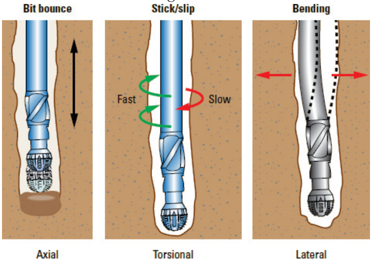
- Brake systems
- Assembled pieces
- Impact and crash
- Penetration and perforation
- Drilling and cutting

Drill crown and a single drill-bit button WC-Co



Drilling column vibrations

• Industrial and natural contact problems

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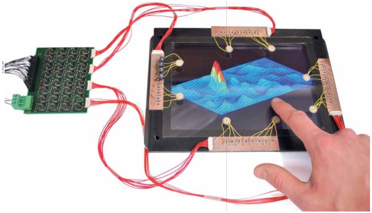
- Haptic perception



Sensory interacting system
V. Hayward, ISIR UPMC
CNRS International Magazine 34 (2014)

Braille page www.todayifoundout.com

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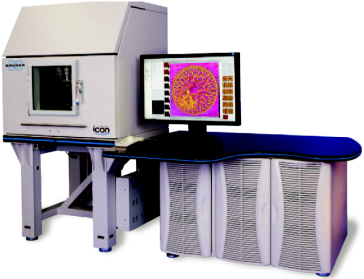
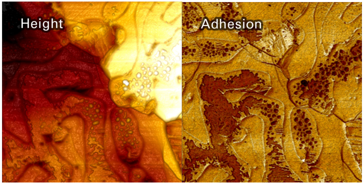
- Haptic perception
- Atomic force microscopy



Atomic force microscope (www.brucker.com)



Height and adhesion measurements of Sn-Pb alloy surface (AFM)
www.brucker.com

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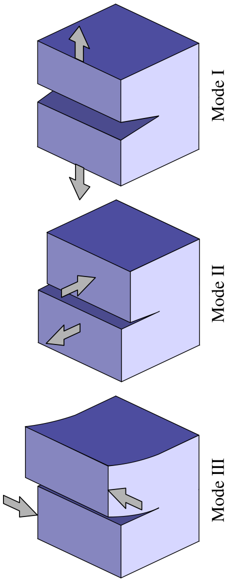
- Interface cracks



Crack types

• Industrial and natural contact problems

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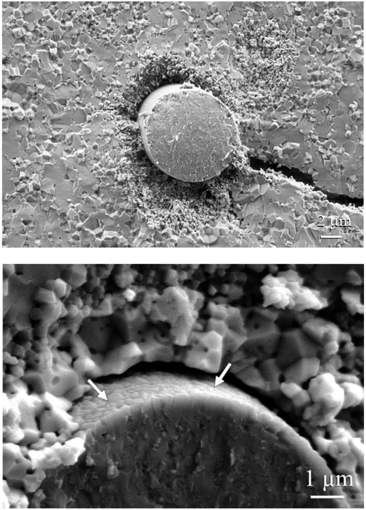

Fiber-matrix interface

[1] D. Blaese et al. ZrO_2 fiber-matrix interfaces in alumina fiber-reinforced model composites, J Eur Ceramic Soc 35 (2015)

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- Interface cracks
- Electrical contact

Siemens Switch www.siemens.com

Rouen's tram brush
Wikipedia

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- Atomic force microscopy
- Interface cracks
- Electrical contact
- Music instruments & sound

Violin and bow
www.walmart.com

Grasshopper's leg
by Nico Angleys on www.flickr.com

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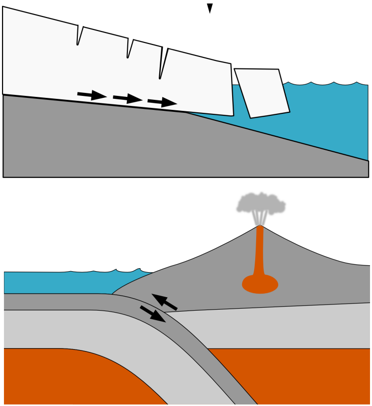
- Geophysical contacts



0.1-100 km

Geophysical scale slip
- basal glacial slip on the bedrock
- rock-rock slip in faults

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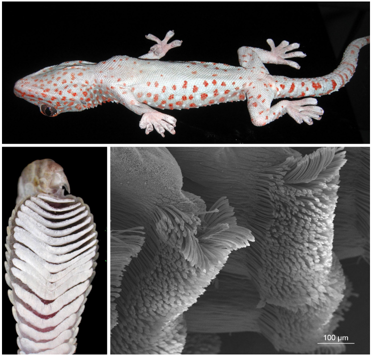


Gecko adhesive toes

- Adhesion
- etc . . .

• Frictionless or normal contact conditions

- **No penetration**
  Always non-negative gap

  $$g \ge 0$$

- **No adhesion**
  Always non-positive contact pressure

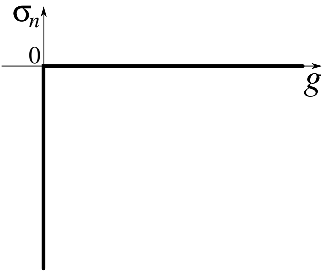
  $$\sigma_n^* \le 0$$

- **Complementary condition**
  Either zero gap and non-zero pressure, or non-zero gap and zero pressure

  $$g\,\sigma_n = 0$$

- **No shear transfer** *(automatically)*

  $$\underline{\sigma}_t^{**} = 0$$



Scheme explaining normal contact conditions

$$\sigma_n^* = (\underline{\underline{\sigma}} \cdot \underline{n}) \cdot \underline{n} = \underline{\underline{\sigma}} : (\underline{n} \otimes \underline{n})$$

$$\underline{\sigma}_t^{**} = \underline{\underline{\sigma}} \cdot \underline{n} - \sigma_n \underline{n} = \underline{n} \cdot \underline{\underline{\sigma}} \cdot \left(\underline{\underline{I}} - \underline{n} \otimes \underline{n}\right)$$

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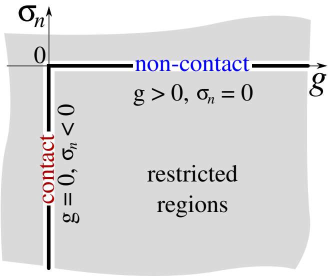

Improved scheme explaining normal contact conditions

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• Frictionless or normal contact conditions

In **mechanics:**

Normal contact conditions

≡

Frictionless contact conditions

≡

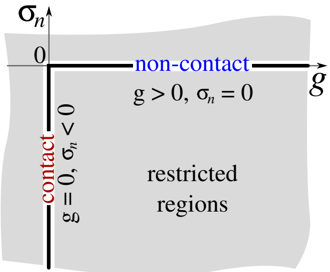
Hertz[1]*-Signorini*[2] *conditions*

≡

Hertz[1]*-Signorini*[2]*-Moreau*[3] *conditions*

also known in **optimization theory** as

Karush[4]*-Kuhn*[5]*-Tucker*[6] *conditions*



Improved scheme explaining normal contact conditions

$$\boxed{g \ge 0, \qquad \sigma_n \le 0, \qquad g\sigma_n = 0}$$

[1] **Heinrich Rudolf Hertz** (1857–1894) a German physicist who first formulated and solved the frictionless contact problem between elastic ellipsoidal bodies.
[2] **Antonio Signorini** (1888–1963) an Italian mathematical physicist who gave a general and rigorous mathematical formulation of contact constraints.
[3] **Jean Jacques Moreau** (1923–2014) a French mathematician who formulated a non-convex optimization problem based on these conditions and introduced pseudo-potentials in contact mechanics.
[4] **William Karush** (1917–1997), [5] **Harold William Kuhn** (1925) American mathematicians,
[6] **Albert William Tucker** (1905–1995) a Canadian mathematician.

• Amontons-Coulomb's friction

- **No contact** $g > 0, \ \sigma_n = 0$
- **Stick** $|\underline{v}_t| = 0$

  Inside slip surface/Coulomb's cone

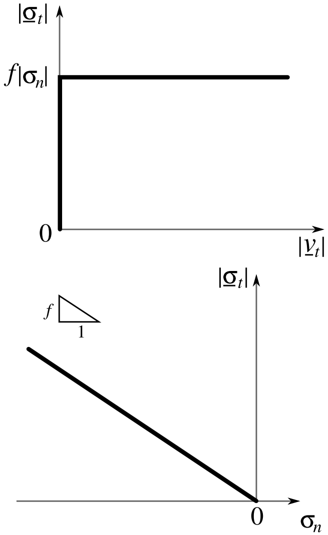
  $$F = |\underline{\sigma}_t| - f|\sigma_n| < 0$$
- **Slip** $|\underline{v}_t| > 0$

  On slip surface/Coulomb's cone

  $$F = |\underline{\sigma}_t| - f|\sigma_n| = 0$$
- **Complementary condition**

  Either zero velocity and negative slip criterion, or non-zero velocity and zero slip criterion

  $$|\underline{v}_t|\Big(|\underline{\sigma}_t| - f|\sigma_n|\Big) = 0$$



Scheme explaining frictional contact conditions

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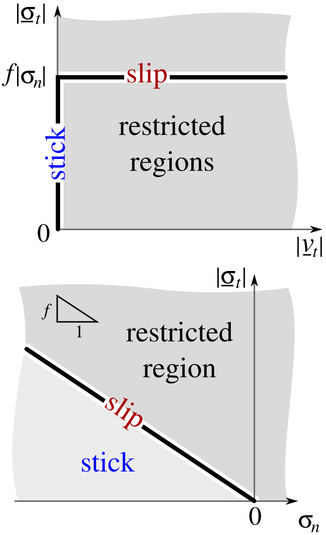

Improved scheme explaining frictional contact conditions

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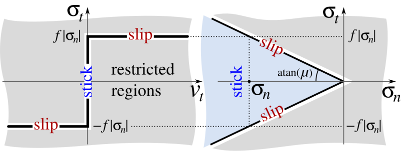
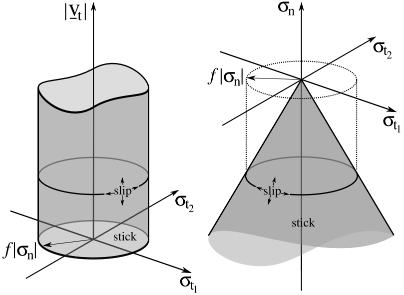


Scheme of 2D frictional contact

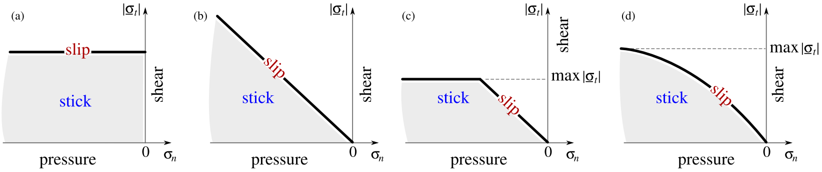


Scheme of 3D frictional contact

$$|\underline{v}_t| \geq 0, \qquad |\underline{\sigma}_t| - f|\sigma_n| \leq 0, \qquad |\underline{v}_t| \Big(|\underline{\sigma}_t| - f|\sigma_n| \Big) = 0$$

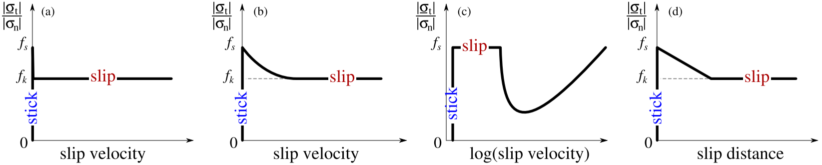
• Other friction laws I

• Static criteria



(a) Tresca (b) Amontons-Coulomb (c) Coulomb-Orowan (d) Shaw

• Kinetic criteria



(a,b) velocity weakening (c) velocity weakening-strengthening

(d) Linear slip weakening

- f_s static and f_k kinetic coefficients of friction.

• History dependent friction

- **Rate and state friction law**
  - Rate $v_t = |\underline{v}_t|$ – relative slip velocity
  - State θ – $\approx$ internal time
  - Dieterich–Ruina–Perrin (1979, 83, 95)

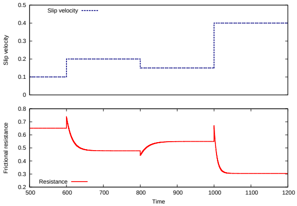
    Frictional resistance

    $$\sigma_t^c = |\sigma_n| \left[f_s + b\theta + a \ln(v_t/v_0) \right]$$

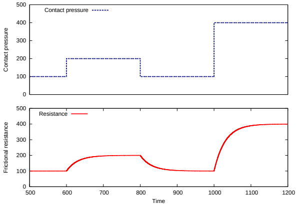
    Evolution of the state variable

    $$\dot{\theta} = -\frac{v_t}{L} \left[\theta + \ln\left(\frac{v_t}{v_0}\right) \right]$$

- **Prakash-Clifton friction law (1992,2000)**
  - Viscous type evolution of frictional resistance σ_t
  - $\dot{\sigma}_t = -\frac{v_t}{L}(\sigma_t + f\sigma_n)$

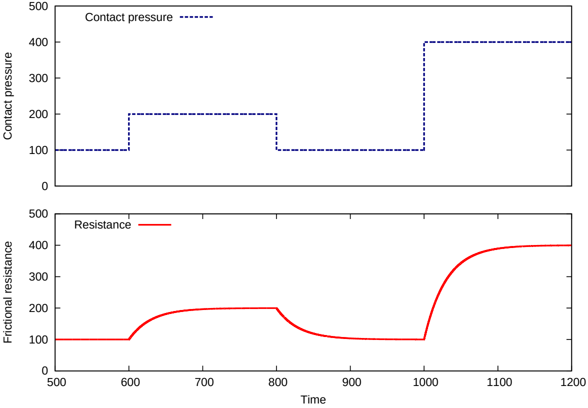


Rate and state friction law



Prakash-Clifton regularization

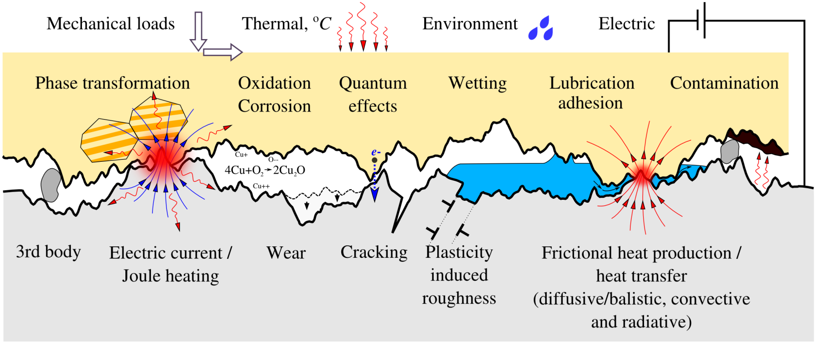
• History dependent friction



• Contact interface physics

Interface constitutive laws are determined by:

- Two distinct materials
- Two distinct roughness
- Thermal/electro-magnetic/mechanical loads
- Fluid/3[rd] body/tribofilm in the interface



V. Yastrebov "Contact Mechanics and Elements of Tribology", DMS master course.

• Example 1: contact between rough surfaces

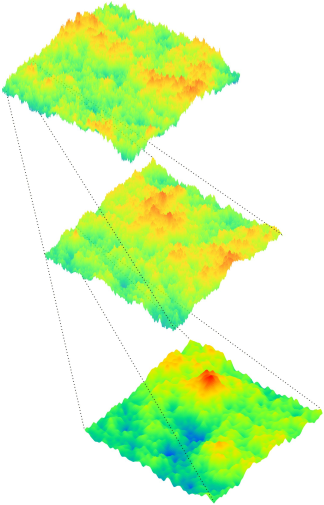


Fig. Example of a rough surface for $H = 0.3$

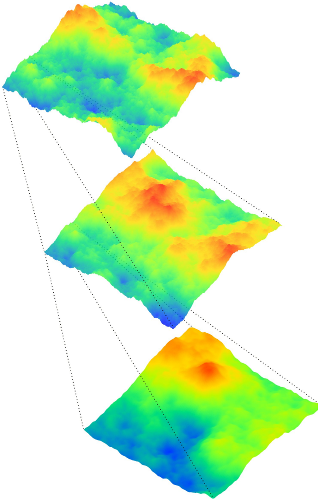


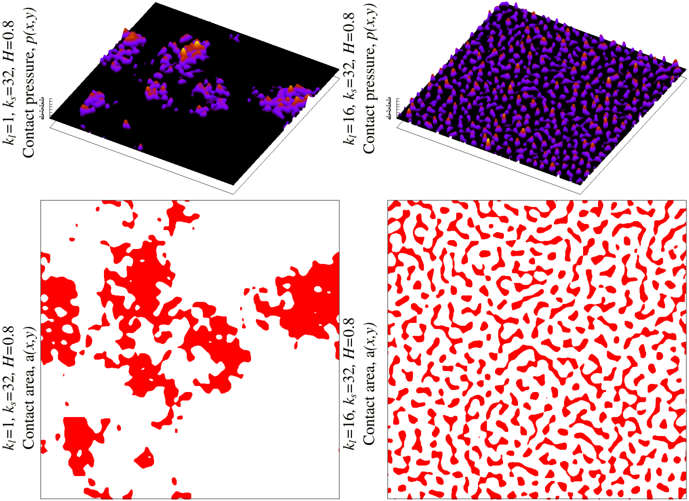
Fig. Example of a rough surface for $H = 0.8$

Recall: the Hurst exponent H and the fractal dimension D for 2-manifolds are interconnected via $D = 3 - H$

[1] Yastrebov, Anciaux, Molinari, Phys Rev E 86 (2012)

• Example 1: contact between rough surfaces

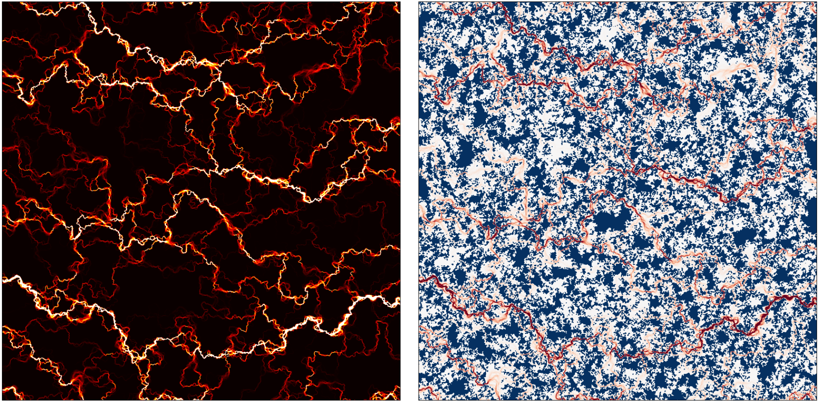
- Normal frictionless and non-adhesive contact between rough surfaces
- Dynamics of contact clusters



[1] Yastrebov, Anciaux, Molinari, Int. J. Solids Struct. 52 (2015)

• Example 2: creeping fluid transport

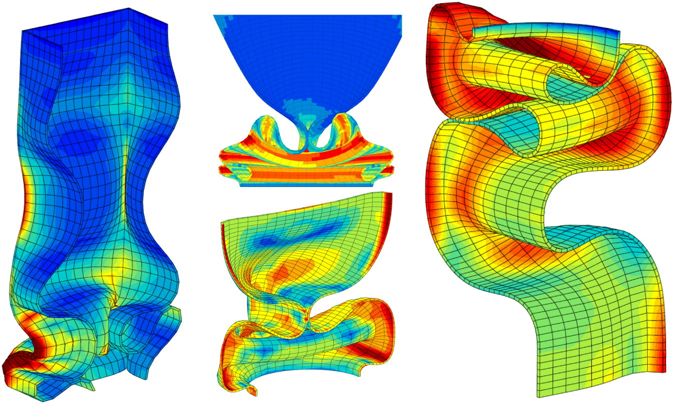
- Normal contact between rough surfaces (2048 $\times$ 2048 elements in BEM)
- Pressure driven creeping fluid flow simulation (FEM)
- Dynamics of flux chanels



Yastrebov, Anciaux, Molinari, Cailletaud, ICTAM 2016

• Example 3: post-buckling behavior

- Finite strain plasticity
- Buckling + self-contact



Finite-element analysis of post-buclking behavior of a thin walled structures
Z-set/ZéBuLoN

[1] Yastrebov, Numerical Methods in Contact Mechanics, Wiley/ISTE, 2013

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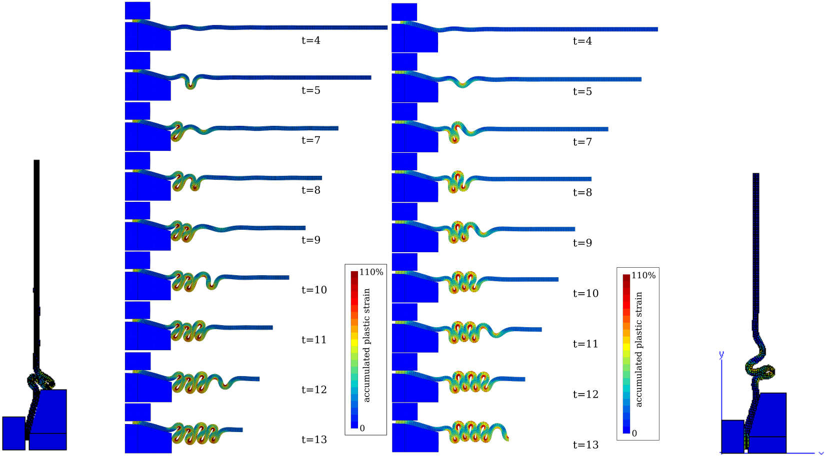
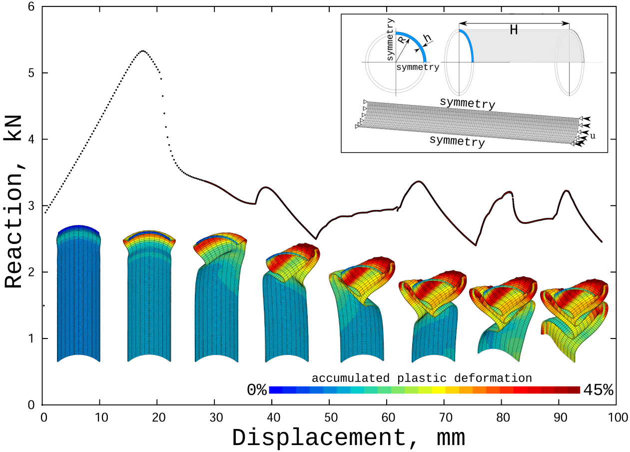


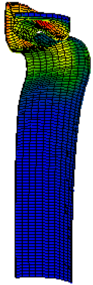
Axisymmetric finite-element analysis of post-buclking behavior of a thin walled tube (self-contact, finite strain plasticity)
Z-set/ZéBuLoN

[1] Yastrebov, Numerical Methods in Contact Mechanics, Wiley/ISTE, 2013

• Example 3: post-buckling behavior

- Finite strain plasticity
- Buckling + self-contact



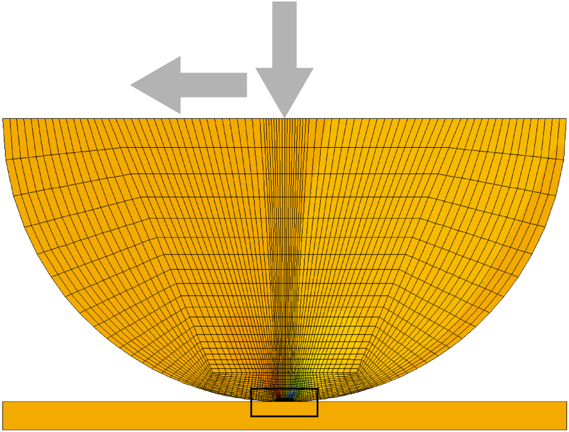


Finite-element analysis of post-buclking behavior of a thin walled tube (self-contact, finite strain plasticity)
Z-set/ZéBuLoN

[1] Yastrebov, Numerical Methods in Contact Mechanics, Wiley/ISTE, 2013

• Example 4: frictional contact

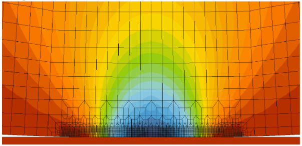
- Non-conservative problem, path dependence
- Coulomb's friction, infinitesimal deformation



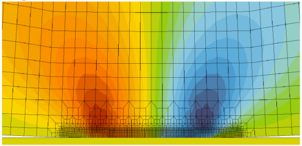
• Example 4: frictional contact

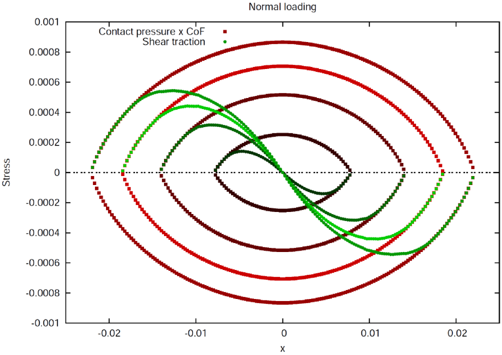
- Non-conservative problem, path dependence
- Coulomb's friction, infinitesimal deformation

sig22 at maximal normal load



sig12 at maximal normal load





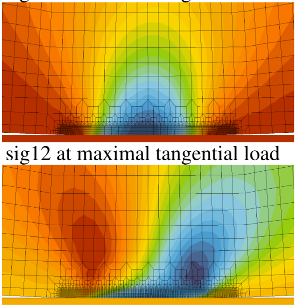
Squeezed in 100 increments, $u_z \sim t^2$

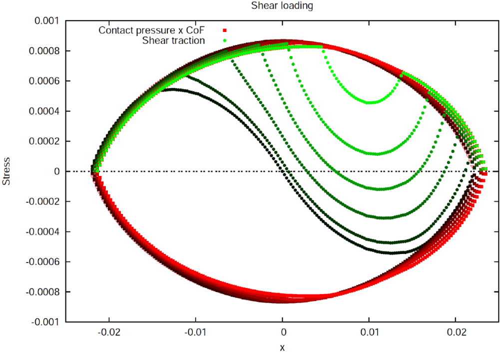
• Example 4: frictional contact

- Non-conservative problem, path dependence
- Coulomb's friction, infinitesimal deformation

sig22 at maximal tangential load

sig12 at maximal tangential load

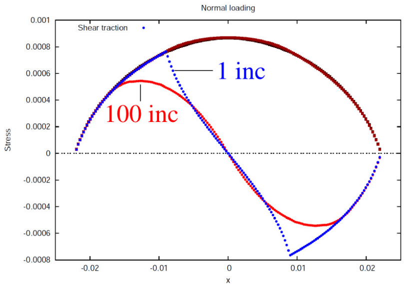


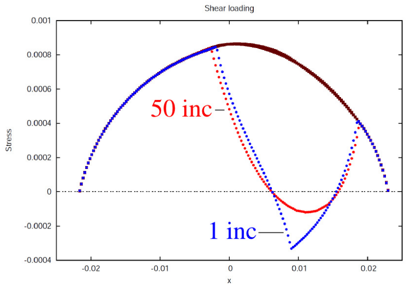


Shifted in 100 increments, $u_x \sim t$

• Example 4: frictional contact

- Non-conservative problem, path dependence
- Coulomb's friction, infinitesimal deformation

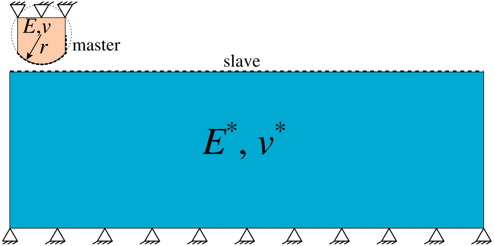




Compare with a fast loading:
squeezed in 1 increment, shifted in 1 increments

• Example 5: shallow ironing test

- Frictional sliding between two deformable solids
- Problem solved by different groups[1,2,3,4,5,6]
- Considerable dispersion of results



[1] Fischer K. A., Wriggers P., "Mortar based frictional contact formulation for higher order interpolations using the moving friction cone", Computer Methods in Applied Mechanics and Engineering, 195:5020-5036, 2006.

[2] Hartmann S., Oliver J., Cante J. C., Weyler R., Hernández J. A., "A contact domain method for large deformation frictional contact problems. Part 2: Numerical aspects", Computer Methods in Applied Mechanics and Engineering, 198:2607-2631, 2009.

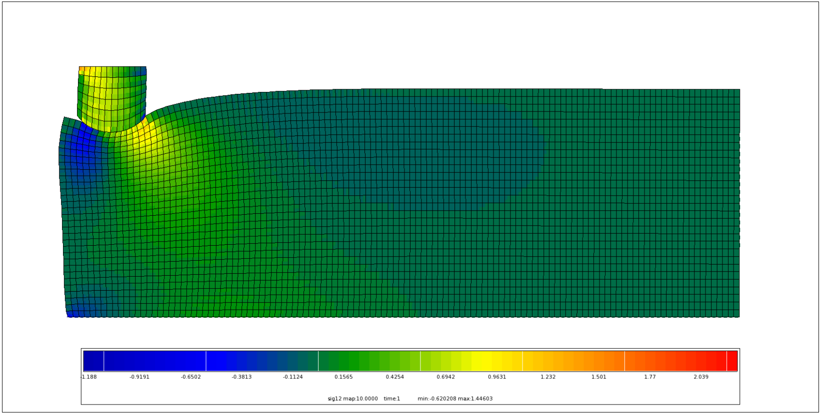
[3] Yastrebov V. A., "Computational contact mechanics: geometry, detection and numerical techniques", Thèse MINES ParisTech, 2011.

[4] Kudawoo A. D., "Problèmes industriels de grande dimension en mćanique numérique du contact : performance, fiabilité et robustesse", Thèse @ LMA & LAMSID, 2012.

[5] Poulios K., Renard Y., "A non-symmetric integral approximation of large sliding frictional contact problems of deformable bodies based on ray-tracing", Computers & Structures 153:75-90, 2015

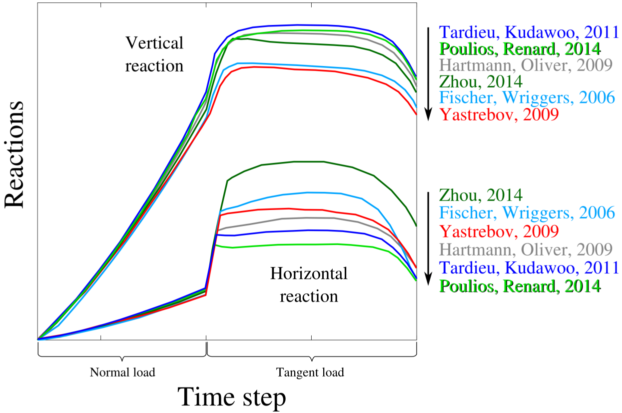
[6] Zhou Lei's blog, http://kt2008plus.blogspot.de

• Example 6: shallow ironing test



• Example 6: shallow ironing test

- No good agreement between results of different groups
- Different meshes/interpolations/large deformation formalisms



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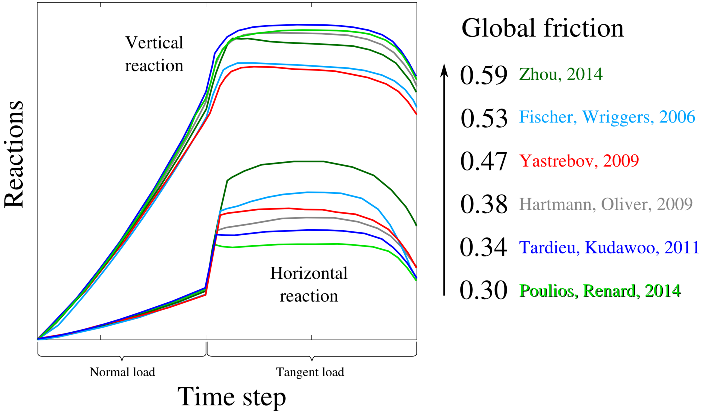


Local coefficient of friction $f_l = 0.3$

Elastodynamic frictional sliding on bi-material interface

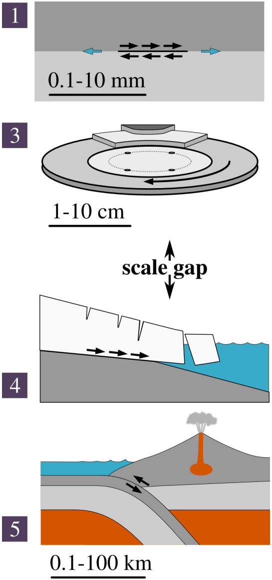
Applications

Relevant applications & phenomena

1. Interface cracks in mixed mode
2. Touch interfaces
3. Brake systems

scale gap

4. Glacier basal slip
5. Slip in faults



Applications

Relevant applications & phenomena

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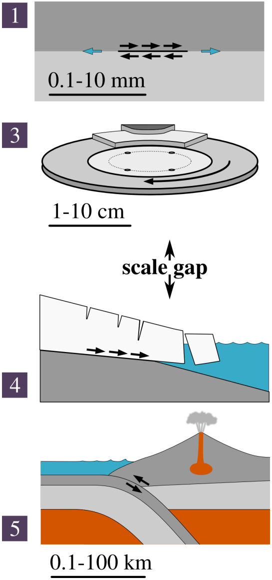
Scalable phenomena

1. Earthquakes can be reproduced in laboratory



Physics of frictional sliding

- **Local** friction depends on
  - materials
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Coulomb-Amontons law

$$\boxed{\begin{cases} |\tau| < f|\sigma|, & |\dot u| = 0 \\ |\tau| = f|\sigma|, & |\dot u| > 0 \end{cases}}$$

τ - tangential traction,
σ - contact pressure,
$\dot u$ - slip velocity,
f - coefficient of friction.

Physics of frictional sliding

- **Local** friction depends on
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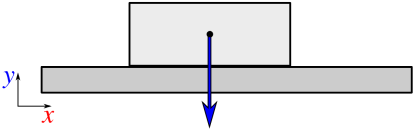
- **Global** friction depends on
  - scale
  - loads
  - geometry
- Local vs global friction



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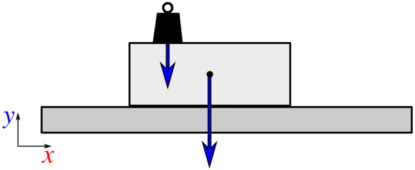

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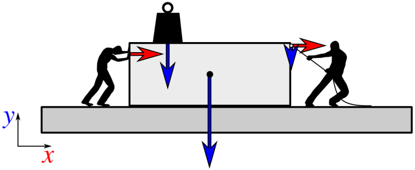

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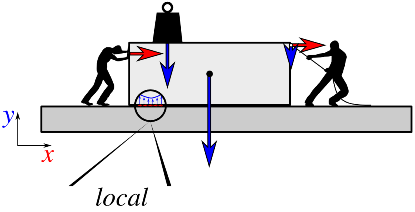

$$f_{loc} = \max(|\tau/\sigma|)$$

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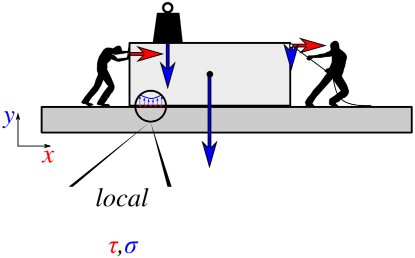

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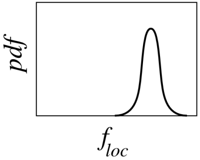

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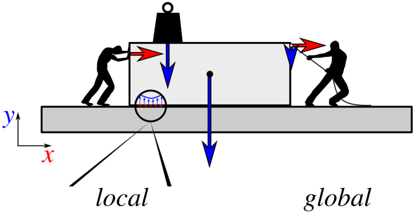
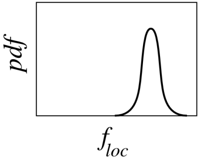


τ, σ $\quad$ $T = \Sigma F_x,\ N = \Sigma F_y$

$f_{loc} = \max(|\tau / \sigma|)$ $\quad$ $f_{gl} = \max(|T / N|)$



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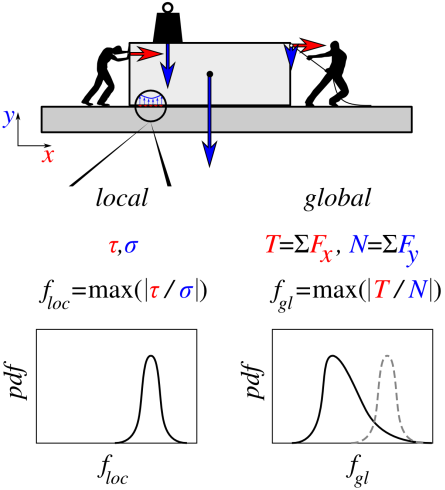

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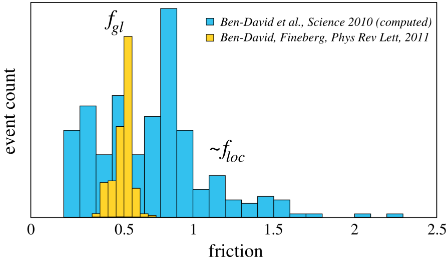

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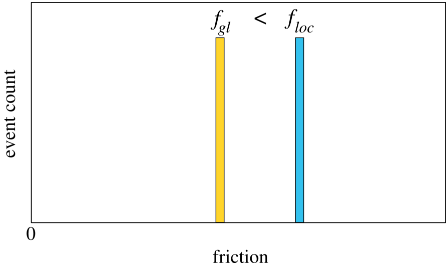

(Weertman, 1980) Unstable slippage across a fault that separates elastic media of different elastic constants, *J Geo Research*

(Adams, 1995) Self-excited oscillations of two elastic half-spaces sliding with a constant coefficient of friction, *J Appl Mech*

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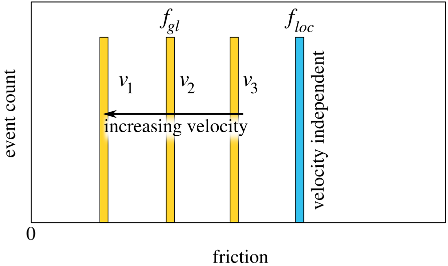

(Adams, 2000) Radiation of body waves induced by the sliding of an elastic half-space against a rigid surface, *J Appl Mech*

(Moirot, Nguyen, Oueslati, 2002) An example of stick-slip and stick-slip-separation waves, *Europ J Mech A Solids*

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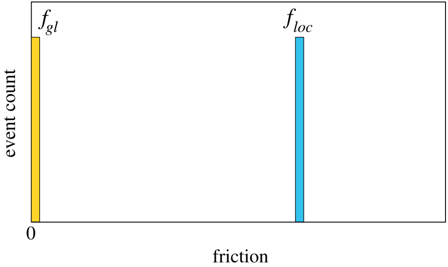

(Cochard & Rice, 2001) Fault rupture between dissimilar materials: Ill-posedness, regularization, and slip-pulse response, J Geophys Res

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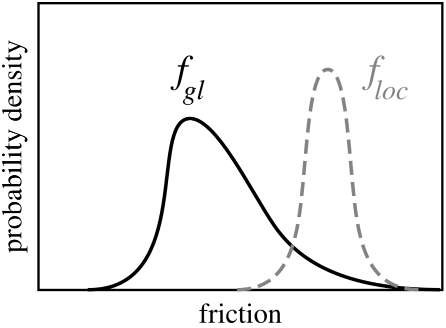


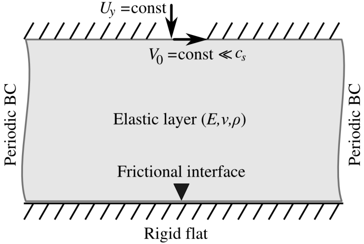
- What are the relevant local friction laws?

- How does the dynamics affect the frictional slip?

- How can it be used for practice?

Problem set-up

- **Problem:** Elastic layer sliding on a rigid flat under Coulomb friction.



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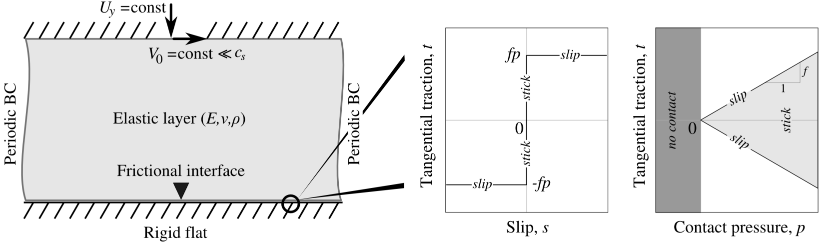

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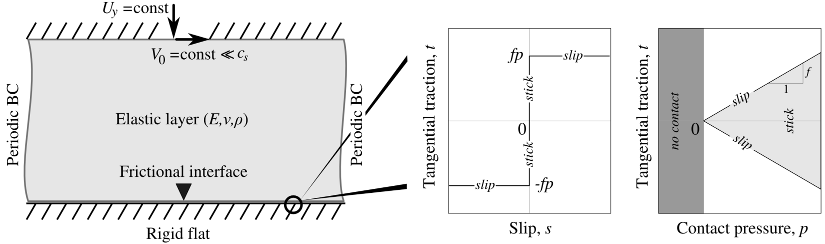

- **Parameters:** Poisson's ratio ν, friction f and sliding velocity V_0.

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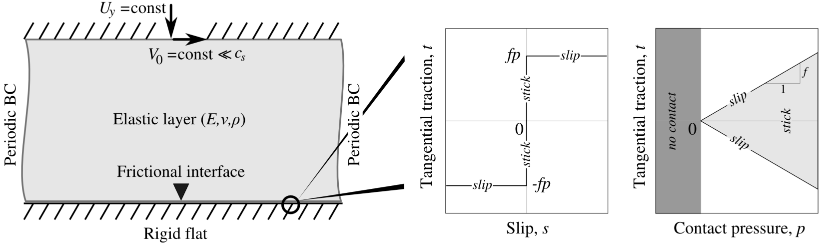


- **Parameters:** Poisson's ratio ν, friction f and sliding velocity V_0.
- **Methods:** Implicit dynamic finite element simulation, α-method HHT[1], direct method to solve frictional contact[2,3].

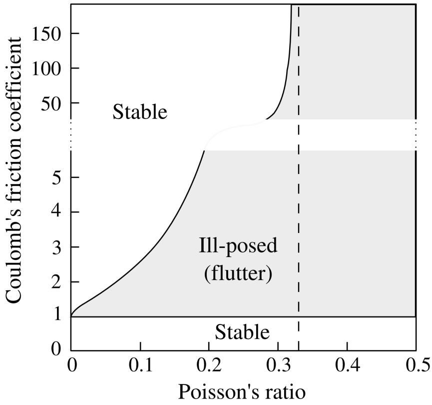
  FE mesh: 33 000 quadrilateral elements with reduced integration.

[1] **(Hilber, Hughes and Taylor, 1977)** Improved numerical dissipation for time integration algorithms in structural dynamics, *Earthq Eng Struct D*

[2] **(Francavilla & Zienkiewicz, 1975)** A note on numerical computation of elastic contact problems, Int J Num Meth Eng.

[3] **(Jean, 1995)** Frictional contact in collections of rigid or deformable bodies: numerical simulation of geomaterial motions, *Stud Appl Mech*

Limitations and expectations



(Renardy, 1992 [1989]) Ill-posedness at the boundary for elastic solids sliding under Coulomb friction, *J Elast*

(Martins, Guimarães & Faria, 1995 [1993]) Dynamic Surface Solutions in Linear Elasticity and Viscoelasticity With Frictional Boundary Conditions, *J Vibr Acoust*

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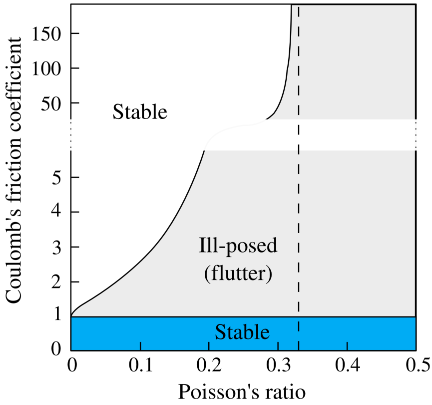

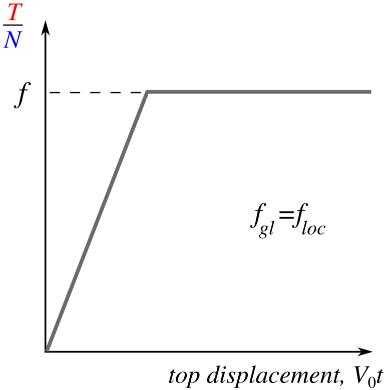

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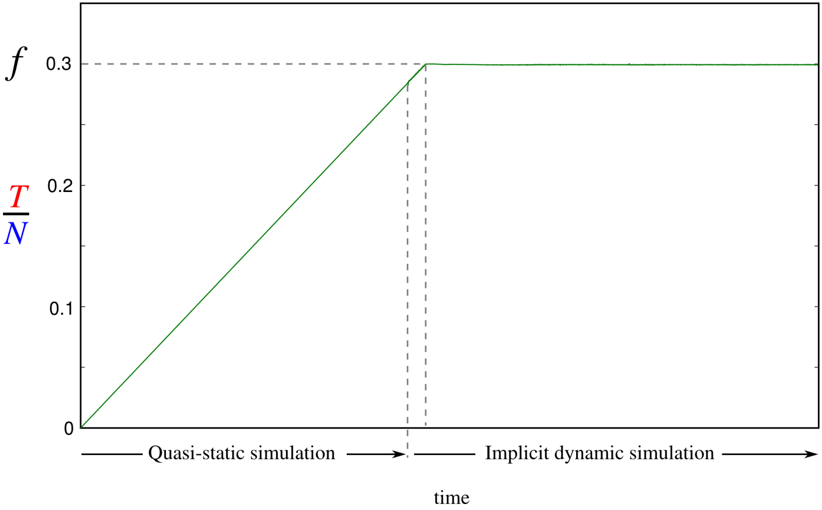
Results I: $f = 0.3$

Parameters: $\nu = 0.2, f = 0.3, V_0 = 10^{-6} c_s$.



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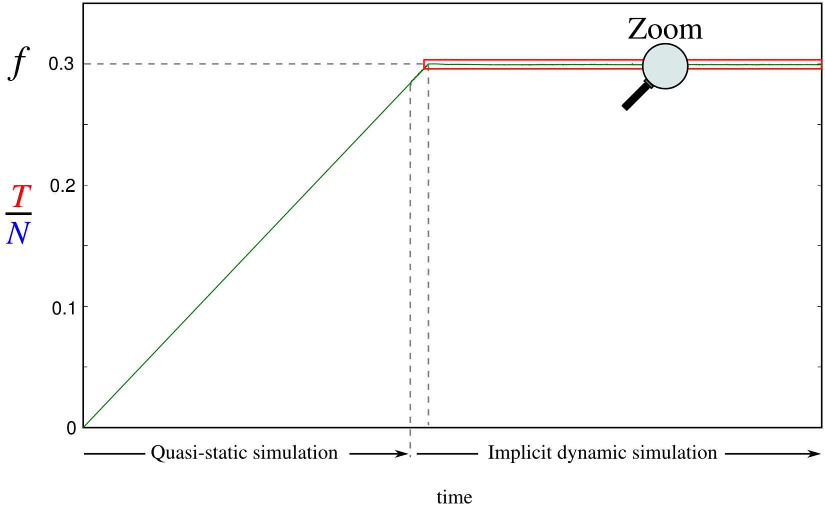

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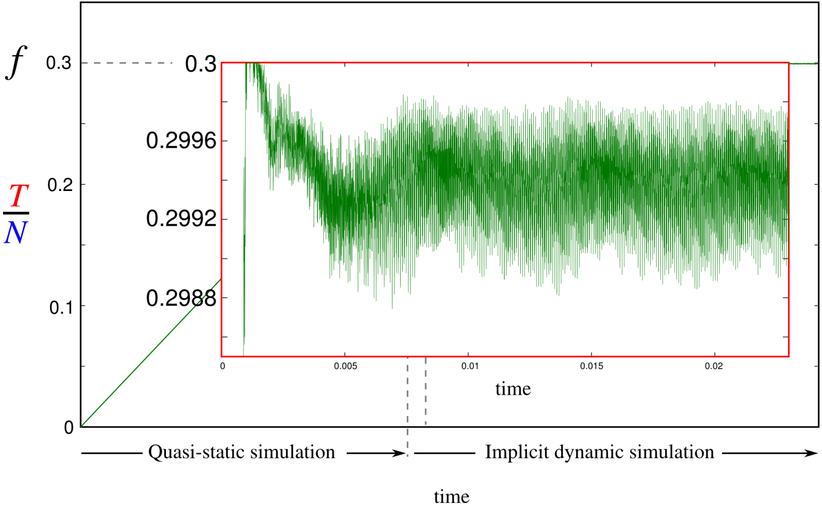

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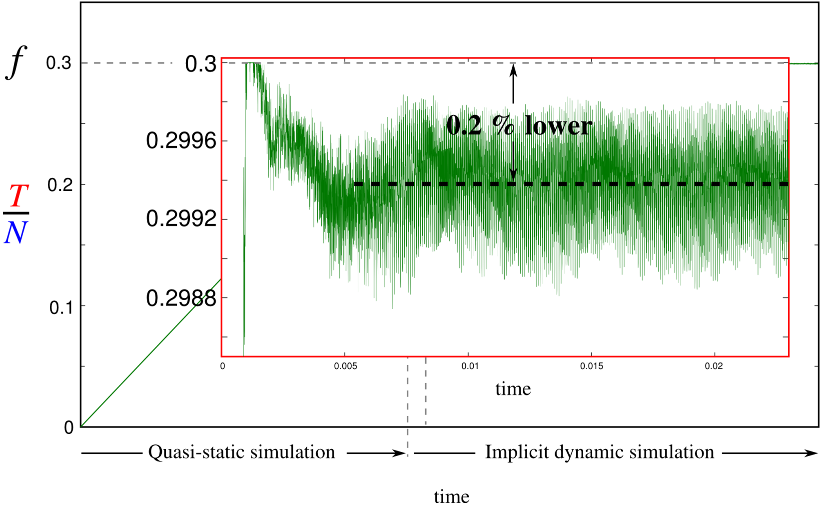

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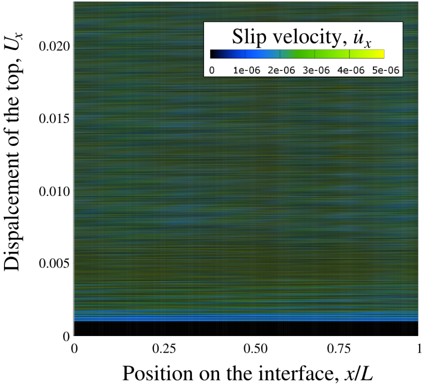

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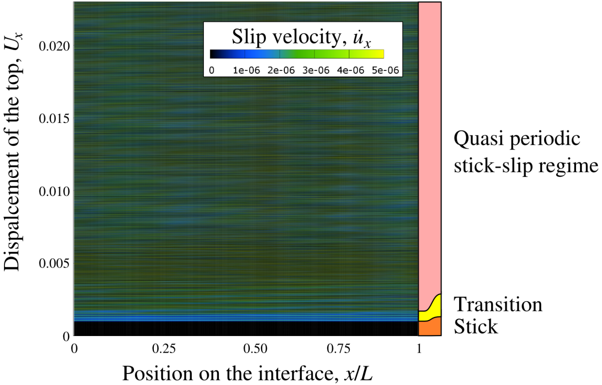

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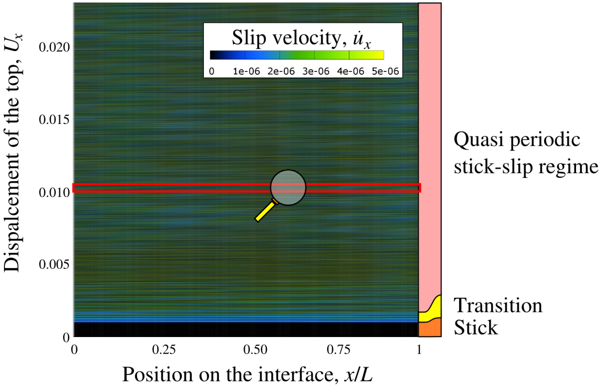

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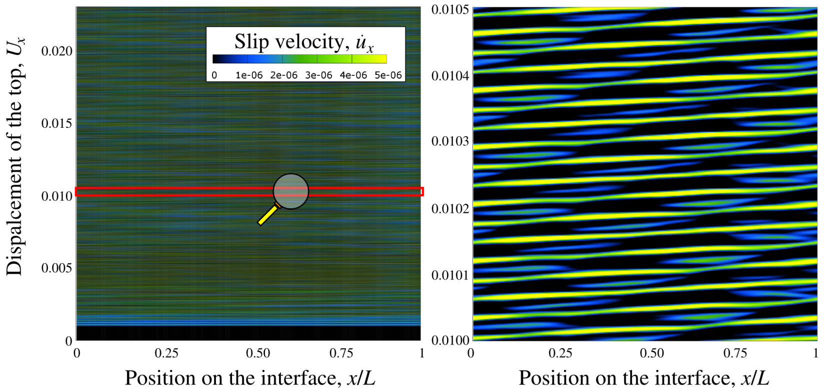

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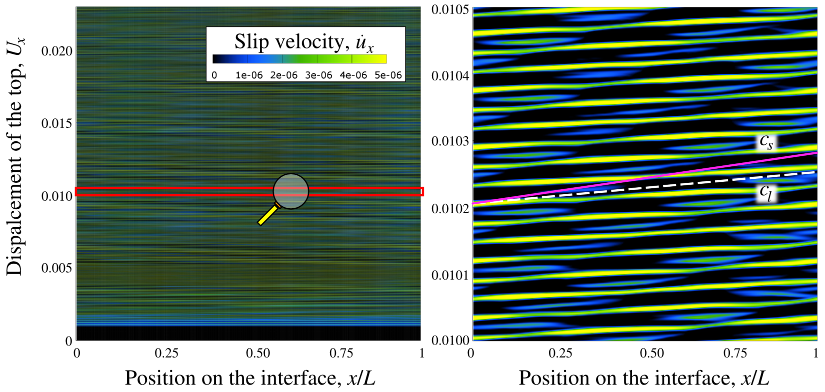

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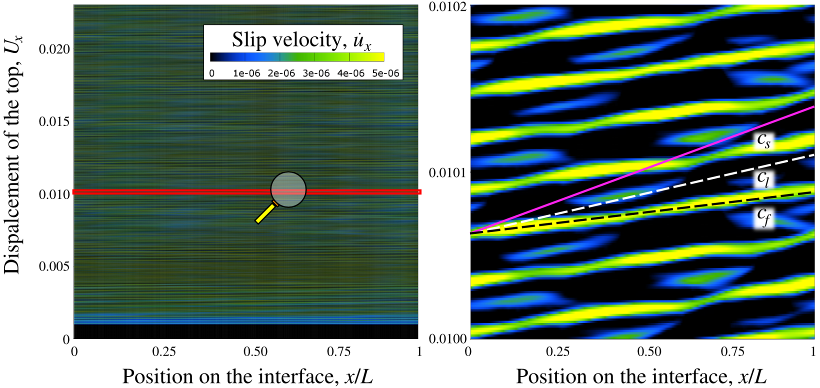
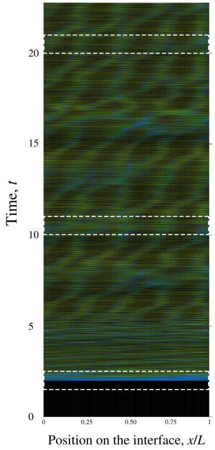


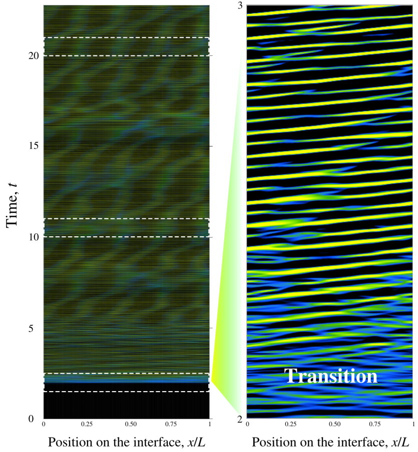
Results II: $f = 0.6$

Parameters: $\nu = 0.2, f = 0.6, V_0 = 10^{-6} c_s$.



Results II: $f = 0.6$

Parameters: $\nu = 0.2, f = 0.6, V_0 = 10^{-6} c_s$.



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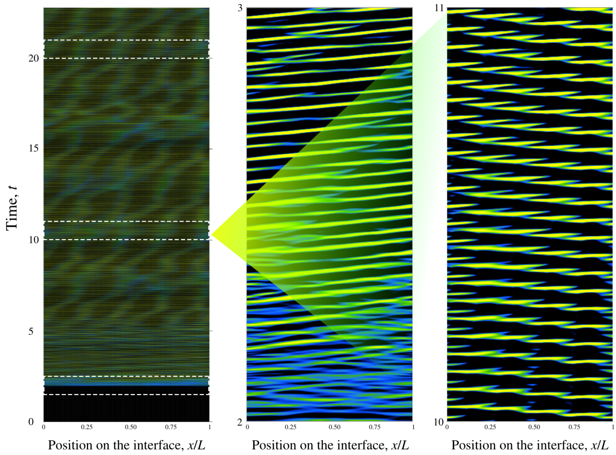

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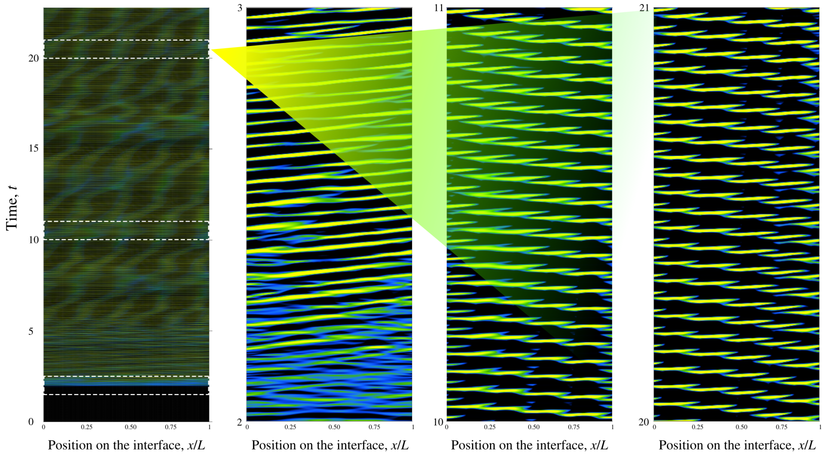
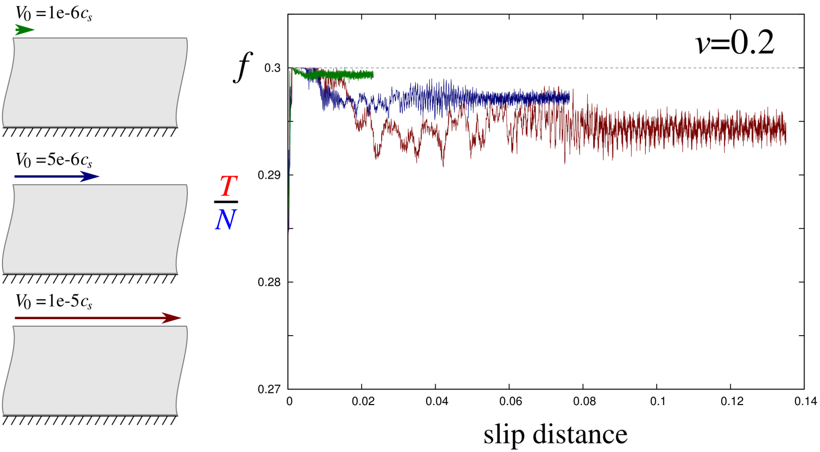
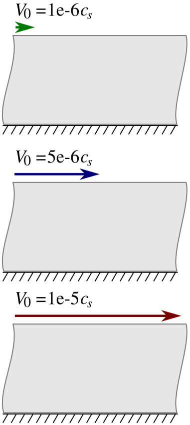
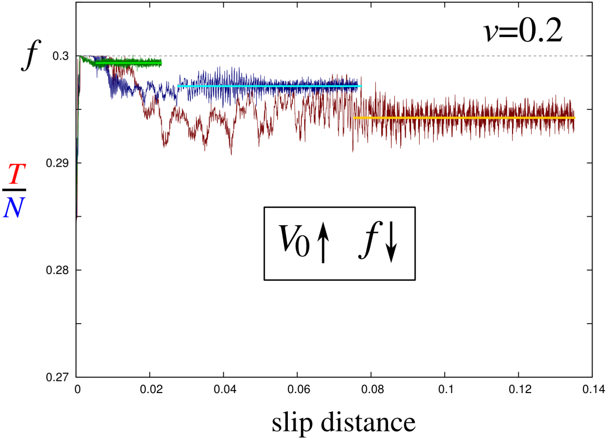


Results III: velocity dependence



Results III: velocity dependence





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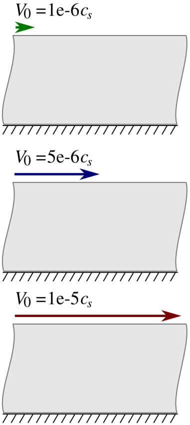
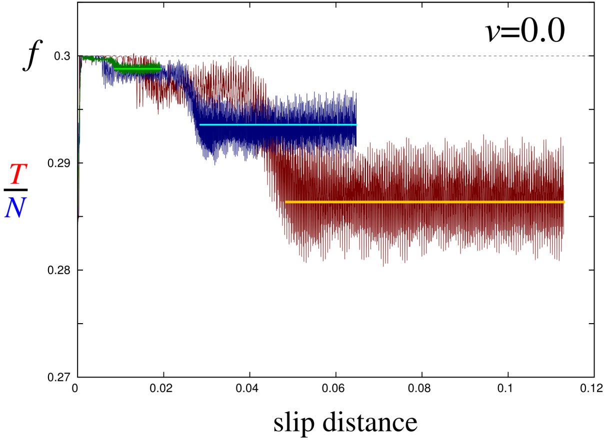
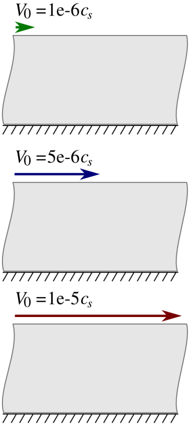
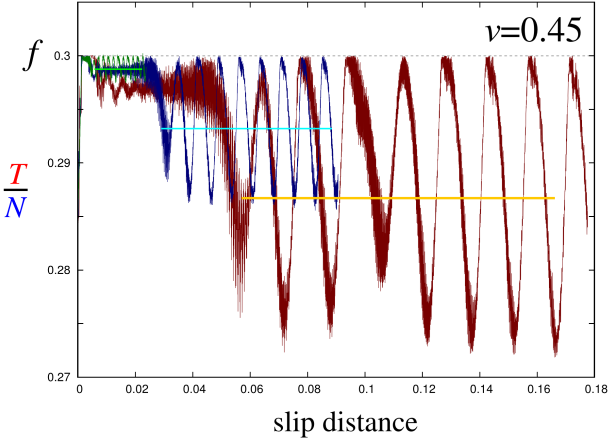
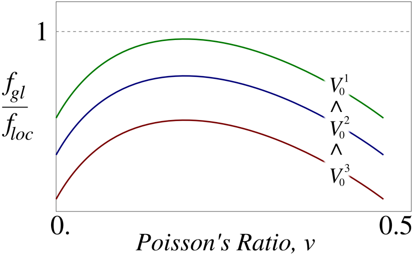




Results III: velocity dependence







- For a train of rectangular stick-slip waves **(Adams, 2000)** predicted velocity dependence:

$$f_{gl} = f_{loc} - \left(\frac{f}{f_{loc}} - 1\right) \frac{S\alpha \dot{u}_x G}{(L-S)\sigma_{yy} c_s},$$

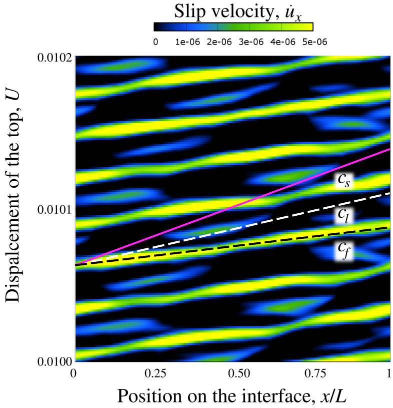
where S is the slip length, L period, G shear modulus, $f = \sigma_{xy}/\sigma_{yy}$ in body waves, $\alpha = \alpha(c_f, c_l, f)$ a coefficient.

(Adams, 2000) Radiation of body waves induced by the sliding of an elastic half-space against a rigid surface, *J Appl Mech*

Intermediate discussion:

- uniform slip in *finite size systems* is unstable
- stick-slip waves
- intersonic $c_s < c_p \le c_l$ and supersonic pulses $c_p > c_l$
- global friction is reduced

$$\boxed{f_{gl} < f_{loc}}$$



Results I: discussion

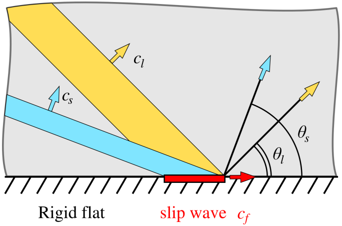
Intermediate discussion:

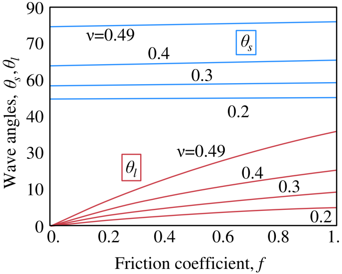
- uniform slip in *finite size systems* is unstable
- stick-slip waves
- intersonic $c_s < c_p \le c_l$ and supersonic pulses $c_p > c_l$
- global friction is reduced

$$\boxed{f_{gl} < f_{loc}}$$

Explanation:

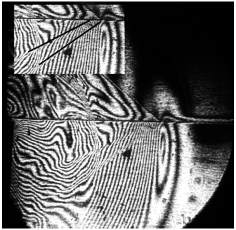
- Waveguide modes[1,2]
- Stick-slip waves[3]
- “Radiation of body waves induced by the sliding”[4]

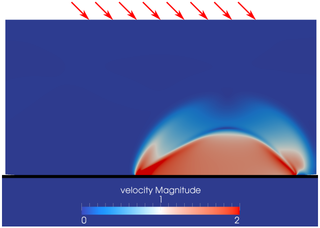




[1] **(Mindlin, 1955)** An introduction to the mathematical theory of vibrations of elastic plates.
[2] **(Brener et al., 2016)** Dynamic instabilities of frictional sliding at a bimaterial interface, J Mech Phys Solids
[3] **(Bui & Oueslati, 2010)** On the stick-slip waves under unilateral contact . . . , *Ann Solid Struct Mech*
[4] **(Adams, 2000)** Radiation of body waves induced by the sliding of an elastic half-space . . . , *J Appl Mech*

Supersonic slip propagation does not violate causality
even though c_l is the maximal signal speed





(Coker, Rosakis, Needleman, 2003) Dynamic crack growth along a polymer composite-Homalite interface, *J Mech Phys Solids*

(Coker, Lykotrafitis, Needleman, Rosakis, 2005) Frictional sliding modes along an interface between identical elastic plates subject to shear impact loading, *J Mech Phys Solids*

(Kammer & Yastrebov, 2012) On the Propagation of Slip Fronts at Frictional Interfaces, *Tribol Lett*

Results I: discussion II

Supersonic slip propagation does not violate causality
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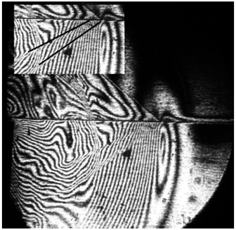

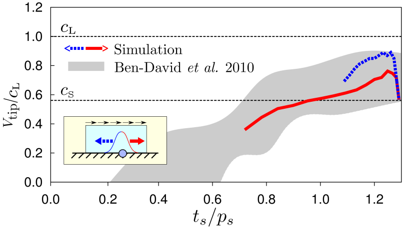

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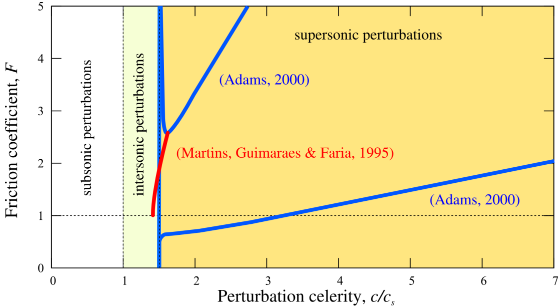
(Kammer & Yastrebov, 2012) On the Propagation of Slip Fronts at Frictional Interfaces, *Tribol Lett*

(Ben-David, Cohen & Fineberg 2010) The dynamics of the onset of frictional slip, *Science*

Results I: discussion III

Inter- and supersonic stable and unstable roots for $\nu = 0.1$



(Martins, Guimarães & Faria, 1995) Dynamic Surface Solutions in Linear Elasticity . . . , *J Vibr Acoust*
(Adams, 2000) Radiation of body waves induced by the sliding of an elastic half-space . . . , *J Appl Mech*

- Neglect supersonic perturbations in half-space

  (Ranjith & Rice, 2001) Slip dynamics at an interface between dissimilar materials, *J Mech Phys Solids*

  (Bui & Oueslati, 2010) On the stick-slip waves under unilateral contact and Coulomb friction, *Ann Solid Struct Mech*

- Neglect supersonic perturbations in an elastic-layer
  (Brener et al., 2016) Dynamic instabilities of frictional sliding at a bimaterial interface, *J Mech Phys Solids*

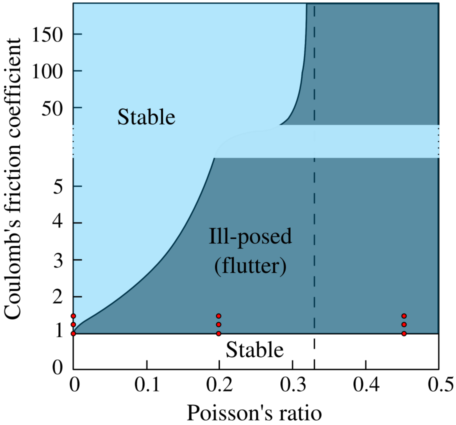
Within the ill-posed region

Preliminary comments:

- Ill-posed[1] if

  $$\sigma_{xy} = f\sigma_{yy}$$

  holds all along the interface
- Exponential growth of amplitude results in a stick-slip or separation
- Critical region $f \geq 1$ for high frequencies $kH \gg 1$[2-4]



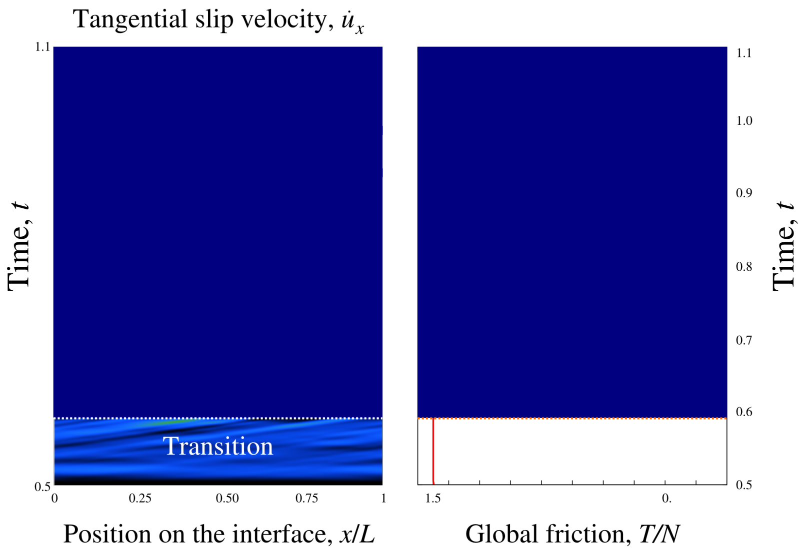
[1] **(Martins, Guimarães & Faria, 1995)** Dynamic Surface Solutions in Linear Elasticity . . . , *J Vibr Acoust*

[2] **(Ranjith & Rice, 2001)** Slip dynamics at an interface between dissimilar materials, *J Mech Phys Solids* 49

[3] **(Cochard & Rice, 2000)** Fault rupture between dissimilar materials: Ill-posedness . . . , *J Geophys Res* 105

[4] **(Kammer, Yastrebov, Anciaux, and Molinari, 2014)** The existence of a critical length scale in regularised friction, *J Mech Phys Solids* 63

Results IV: within the ill-posed region $f = 1.5$



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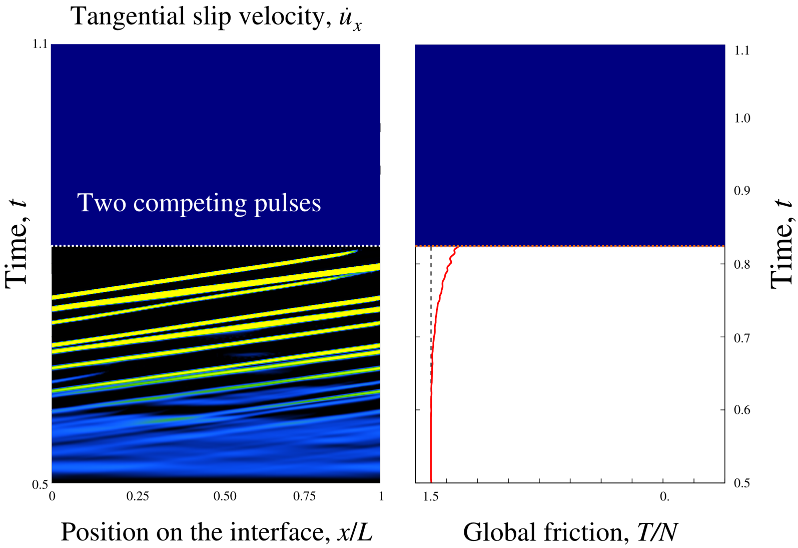

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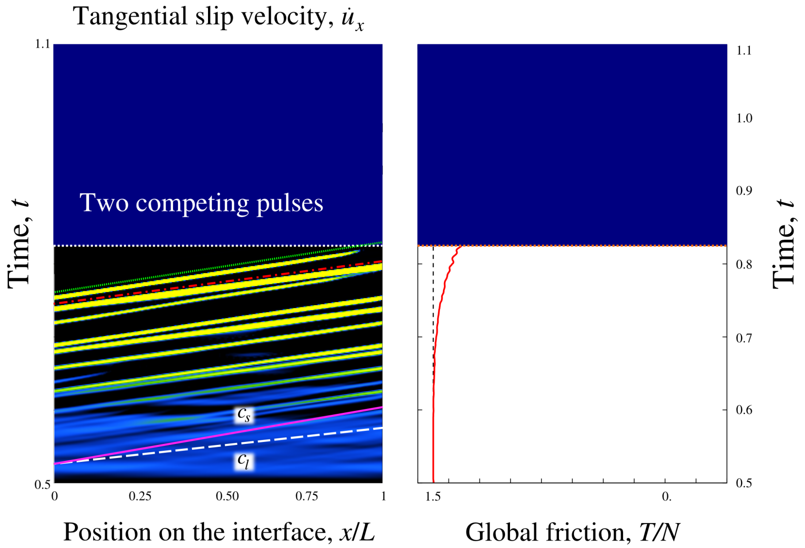

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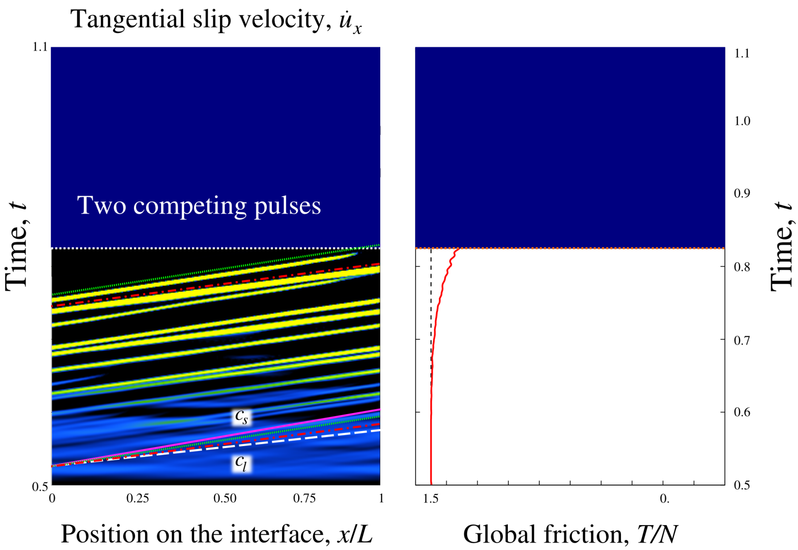

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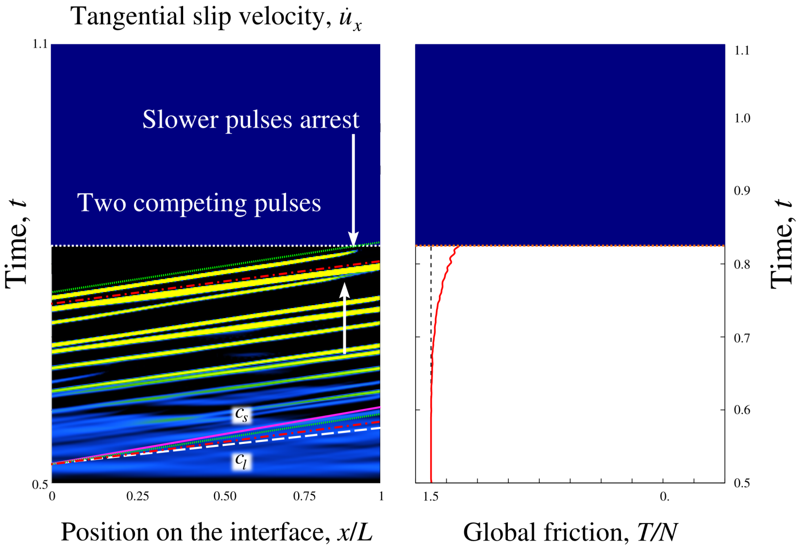

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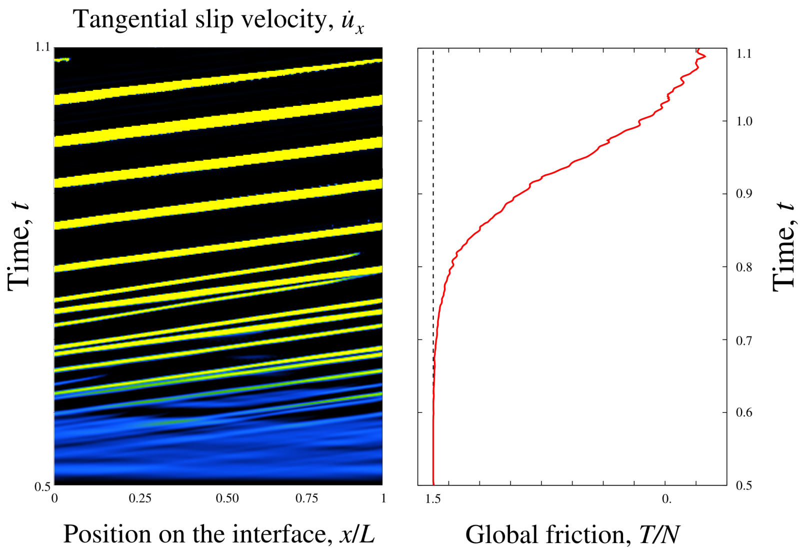

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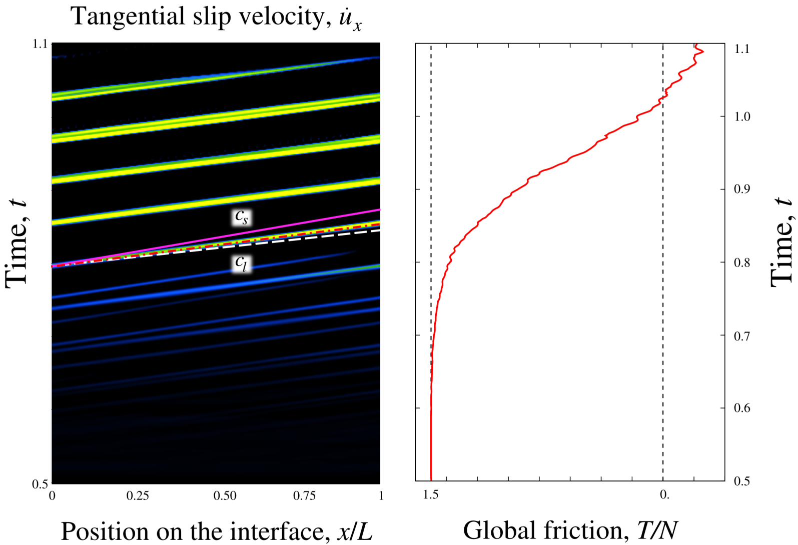

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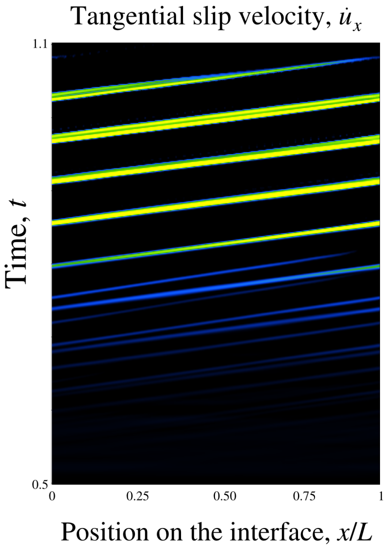

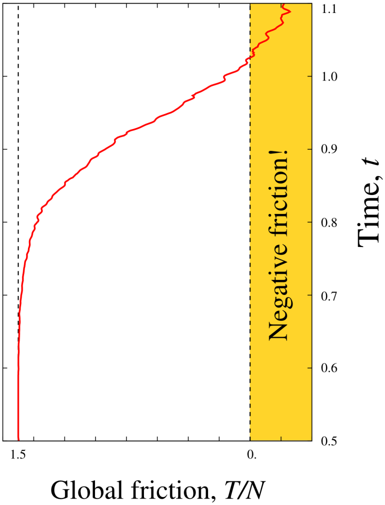

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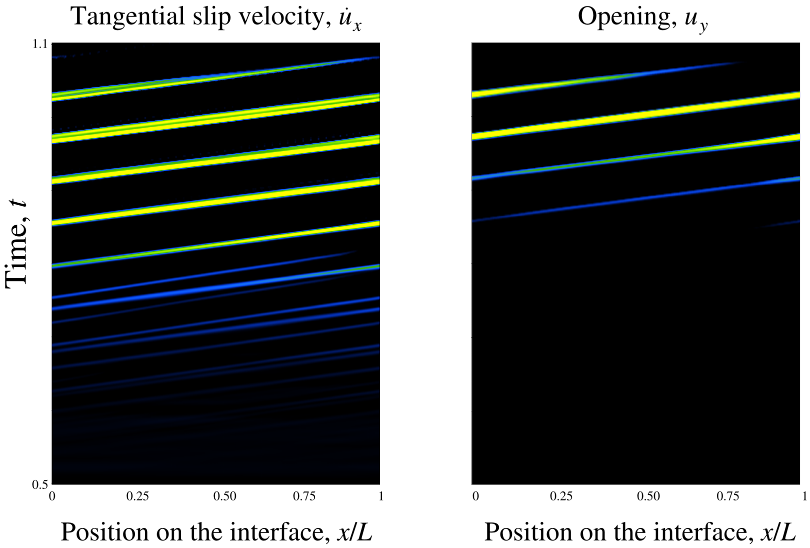

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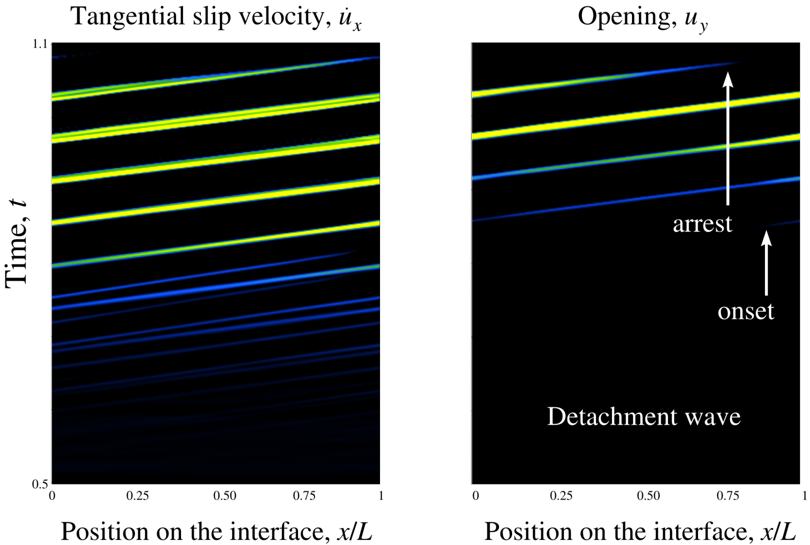

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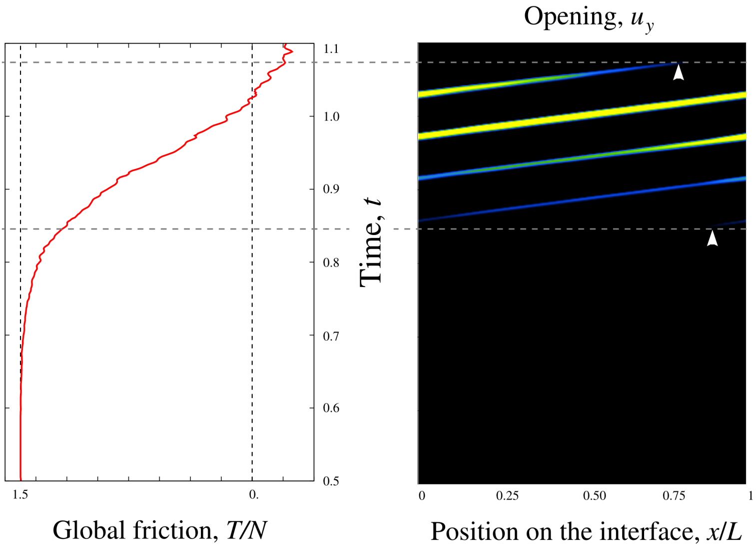
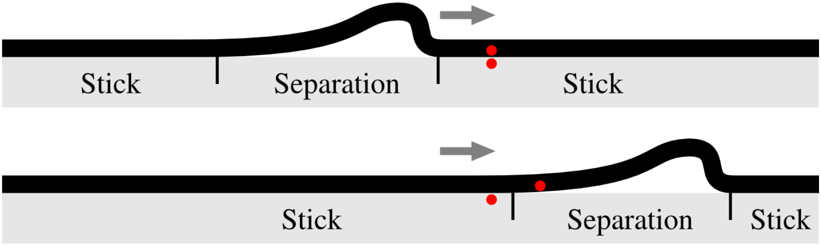


Results IV: animation



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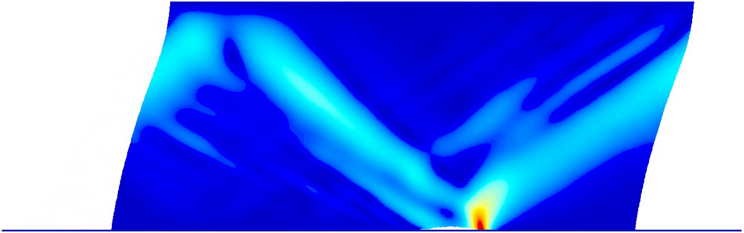

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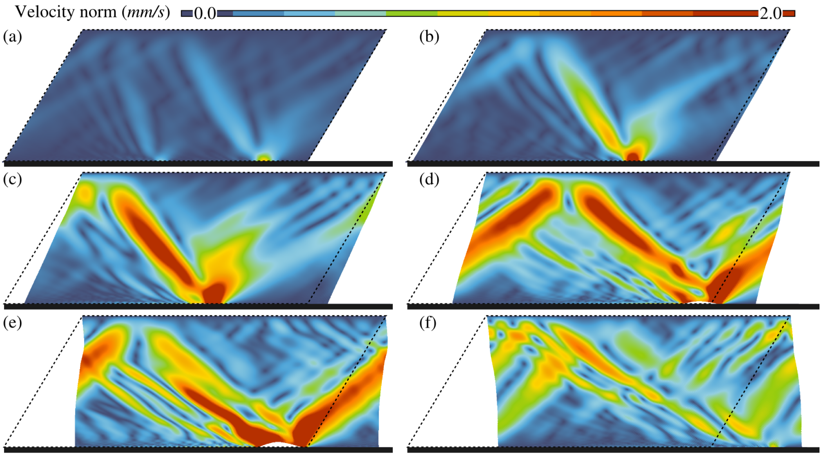
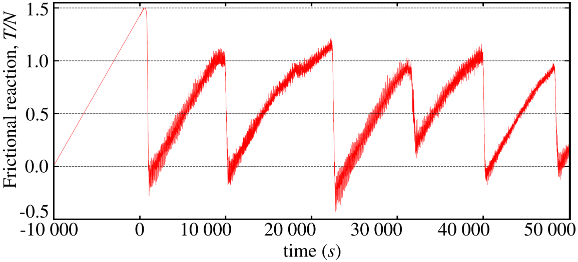


Results IV: behavior at longer periods



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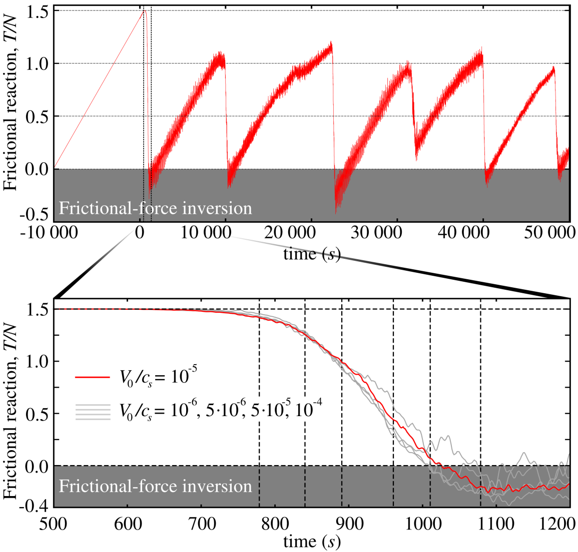
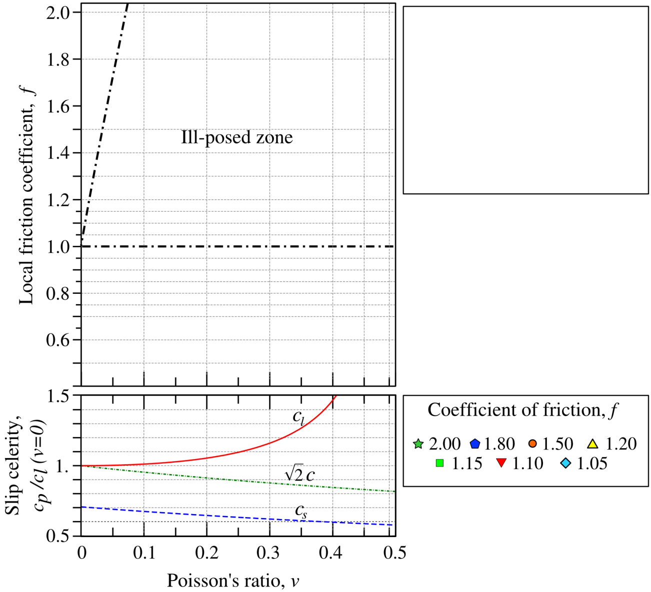


Results IV: parametric study



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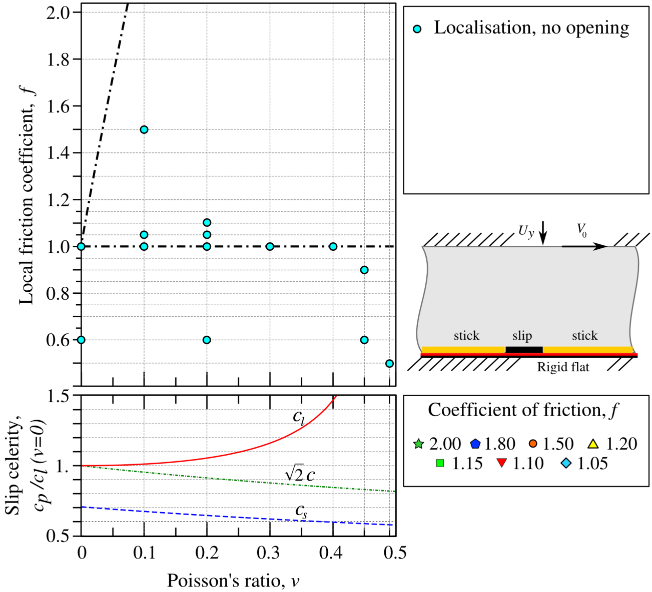

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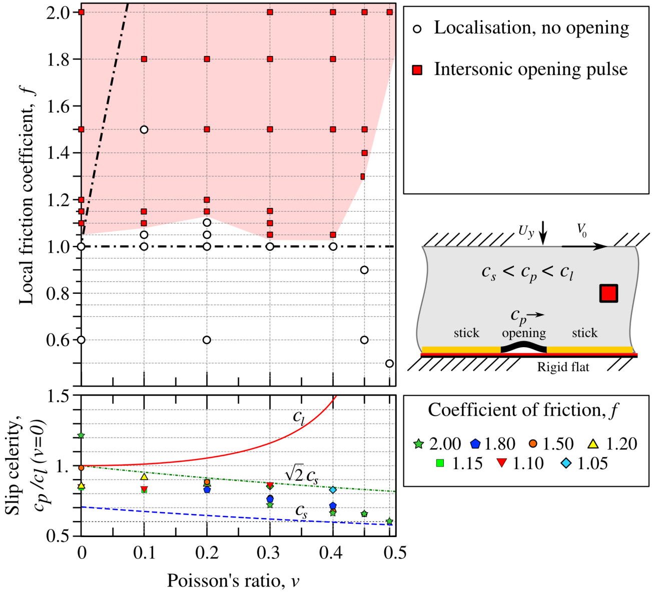

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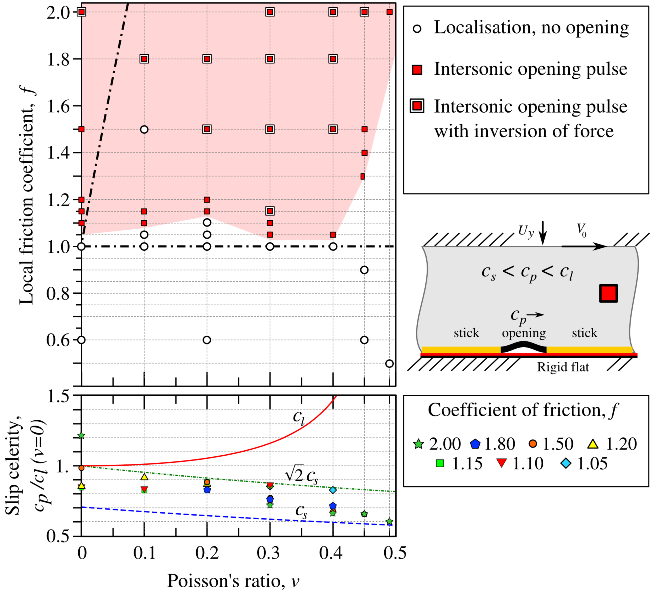

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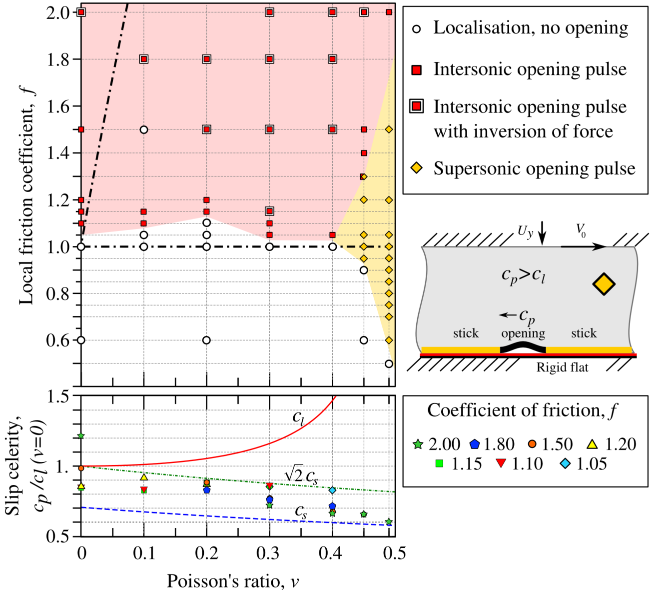


Formal analysis I

- Scalar elastodynamic potentials for dilatational and shear waves[1]:

$$\varphi = f(y)\exp[ik(x-ct)], \quad \psi = g(y)\exp[ik(x-ct)]$$

$$f(y) = A\sin(\eta_d y) + B\cos(\eta_d y), \quad g(y) = C\sin(\eta_s y) + D\cos(\eta_s y)$$

- Horizontal and vertical displacement:

$$u = [ikf(y) - g'(y)]\exp[ik(x-cy)], \quad v = [f'(y) + ikg(y)]\exp[ik(x-cy)]$$

- Stress components:

$$\sigma_{xx} = -\mu\left[(k^2\gamma^2 + \xi^2\eta_d^2)f + 2ikg'\right]\exp[ik(x-cy)]$$

$$\sigma_{yy} = -\mu\left[(\xi^2 k^2 + \gamma^2\eta_d^2)f - 2ikg'\right]\exp[ik(x-cy)]$$

$$\sigma_{xy} = \mu\left[\xi^2(\eta_s^2 - k^2)g + 2ikf'\right]\exp[ik(x-cy)]$$

$$\sigma_{zz} = \nu(\sigma_{xx} + \sigma_{yy}) = -\mu\xi^2 f(k^2 + \eta_d^2)\exp[ik(x-cy)]$$

- Constants:

$$\gamma^2 = c_d^2/c_s^2, \quad \xi^2 = \lambda/\mu$$

λ, μ are Lamé parameters, c_s, c_l are transverse and longitudinal wave celerities.

[1] **(Miklowitz, 1980)** The Theory of elastic waves and waveguides. North-Holland

Formal analysis II

- Prescribed displacements on boundary $y = H$:

$$\begin{cases} u(H) = ikf(H) - g'(H) = 0, \\ v(H) = f'(H) + ikg(H) = 0, \end{cases} \tag{1}$$

- Zero vertical displacement on boundary $y = 0$ (no opening):

$$v(H) = f'(0) + ikg(0) = 0$$

- Frictional relation between normal and shear tractions:

$$\sigma_{xy}(0)+F\sigma_{yy}(0) = \xi^2(\eta_s^2-k^2)g(0)+2ikf'(0)-F\left[(\xi^2k^2 + \gamma^2\eta_d^2)f(0) - 2ikg'(0)\right] = 0$$

- Obtain a linear system of equations for $X = \{A, B, C, D\}$

$$KX = 0$$

- For nontrivial solutions, we require that

$$\det(K) = 0$$

Formal analysis III

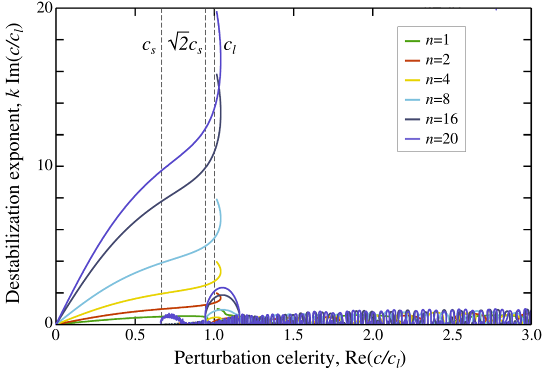
- $\det(K) = 0$ corresponds to this transcendental equation:

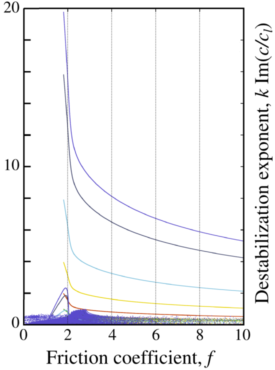
$$\boxed{\begin{aligned} &F\left[\sin(\eta_d H) + \frac{\eta_s\eta_d}{k^2}\sin(\eta_s H)\right]\left(-2\frac{\eta_d\eta_s}{k^2}\sin(\eta_d H) + (\xi^2+\gamma^2\frac{\eta_d{}^2}{k^2})\sin(\eta_s H)\right) - \\ &-F\frac{\eta_s\eta_d}{k^2}\left[\cos(\eta_d H) - \cos(\eta_s H)\right]\left(2\cos(\eta_d H) + (\xi^2+\gamma^2\frac{\eta_d{}^2}{k^2})\cos(\eta_s H)\right) + \\ &+ i\frac{\eta_d}{k}\left[2+\xi^2\left(\frac{\eta_s{}^2}{k^2}-1\right)\right]\left(\cos(\eta_d H)\sin(\eta_s H) + \frac{\eta_s\eta_d}{k^2}\cos(\eta_s H)\sin(\eta_d H)\right) = 0 \end{aligned}}$$

- Fix ν and $k = 2\pi n/L$ with $n \in \mathbb{Z}$
- Take $\eta_d = |k|\sqrt{(c/c_d)^2 - 1}$, $\eta_s = |k|\sqrt{(c/c_s)^2-1}$
- Express $F = F(\mathrm{Re}(c), \mathrm{Im}(c))$
- Search for $\mathrm{Re}(F)$ at lines with $\mathrm{Im}(F) = 0$
- Solution $u \sim \exp[ik(x - \mathrm{Re}(c)t] \cdot \exp[k\mathrm{Im}(c)t]$

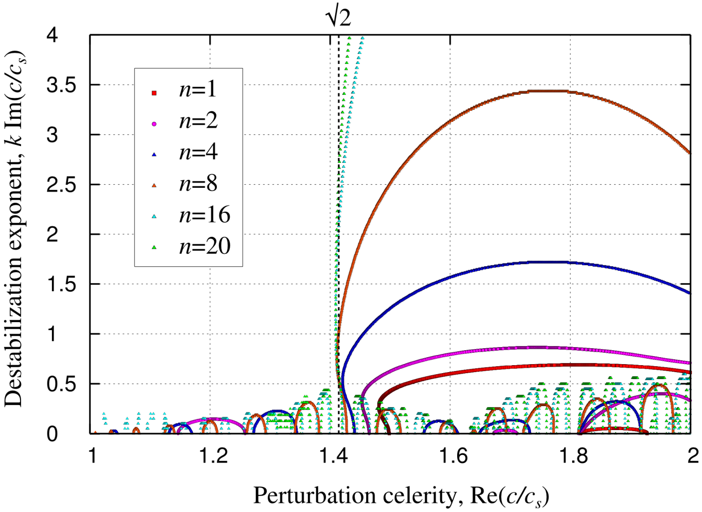
Formal analysis III

- Example: $\nu = 0.1$, $L = 2H$, $k = 2\pi n/L$



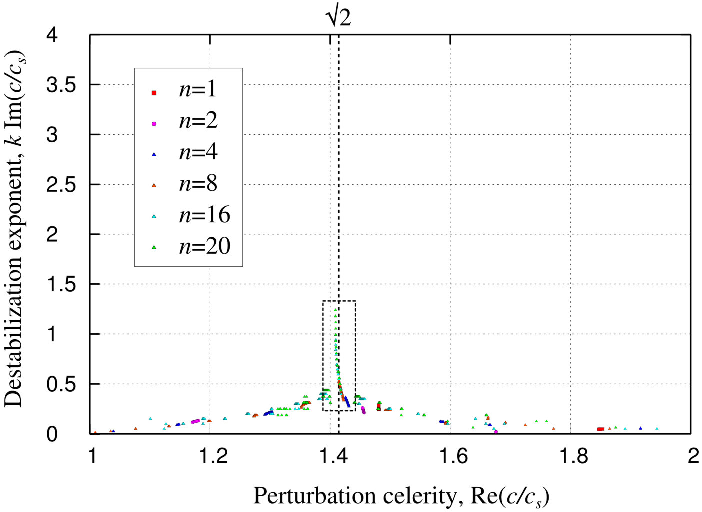


- Example: $\nu = 0.2$, $L = 2H$, $k = 2\pi n/L$

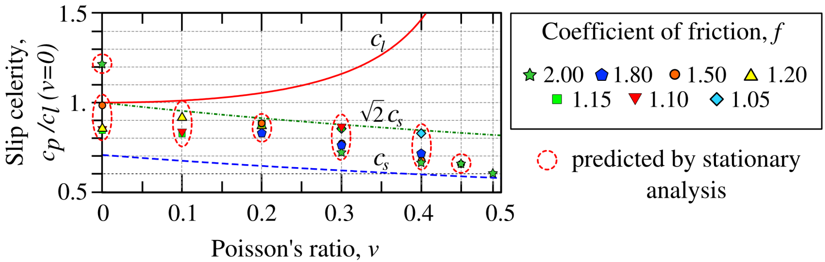


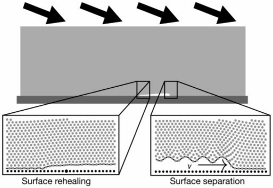
Formal analysis III

- Example: $\nu = 0.2$, $L = 2H$, $k = 2\pi n/L$

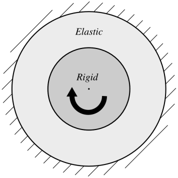


- Example: $\nu = 0.2$, $L = 2H$, $k = 2\pi n/L$

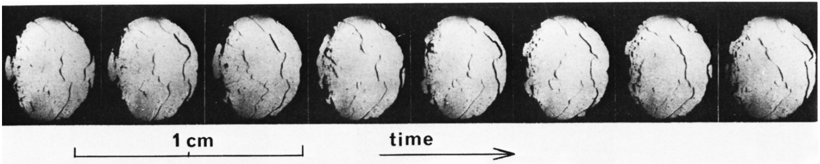




(Gerde & Marder, 2001) Friction and Fracture, *Nature*



(Moirot, Nguyen, Oueslati, 2002) An example of stick-slip and stick-slip-separation waves, *Eur J Mech A-Solid*



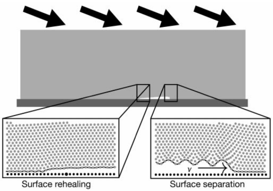
(Schallamach, 1971) How does rubber slide?, *Wear*

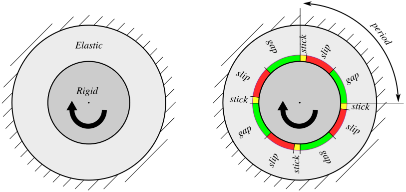
Remark: Schallamach waves are much slower.

(Comninou & Dundurs, 1977, 1978) Elastic interface waves involving separation

(Freund, 1978) Discussion: elastic interface waves involving separation

Results IV: references





(Gerde & Marder, 2001) Friction and Fracture, *Nature*

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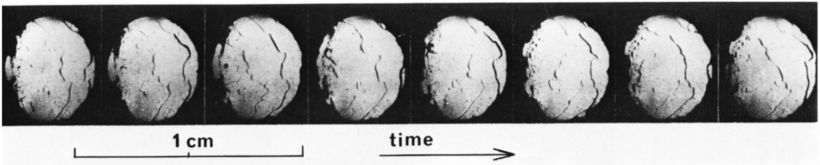

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(Freund, 1978) Discussion: elastic interface waves involving separation

Conclusion

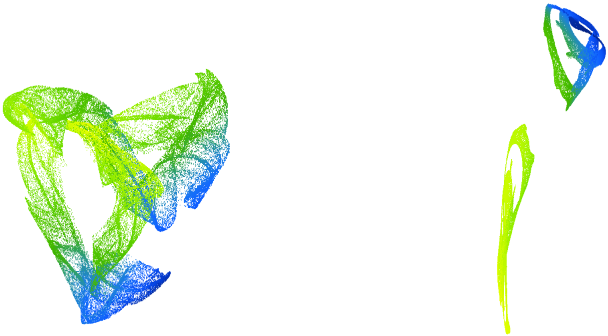
- **Sub-critical regime**
  - stationary solution
  - supersonic stick-slip
  - or standing waves stick-slip
  - velocity dependent global friction
- **Critical regime**
  - intersonic transient slip pulse
  - transforms into opening pulse
  - ★ sliding without slipping(!)
  - ★ inversion of frictional force(!)
  - stationary waveguide analysis predicts instability and slip velocity

[1] V.A. Yastrebov, Sliding Without Slipping Under Coulomb Friction: Opening Waves and Inversion of Frictional Force, *Tribology Letters* 62:1-8 (2016)

[2] V.A. Yastrebov, Elastodynamic frictional sliding of an elastic layer on a rigid flat, *in preparation*

Vibration of asymmetric materials with internal contacts

Examples of 4D strange attractor projected on 3D ($x_1, \dot{x}_1, x_2, \dot{x}_2$)



Merci de votre attention!

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P.S. Looking for a motivated postdoc ☺