Effects of geometrical nonlinearities on the acoustic black hole effect

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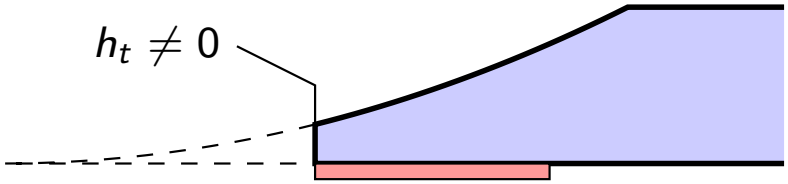
Introduction : Acoustic Black Hole (ABH)

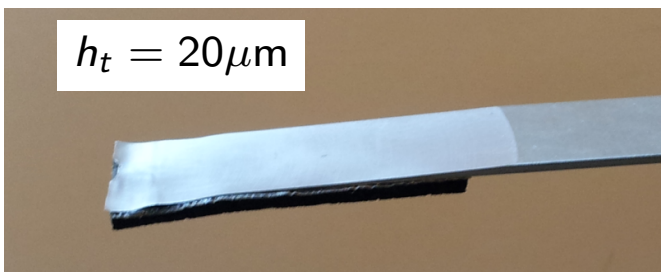
Acoustic Black Hole effet

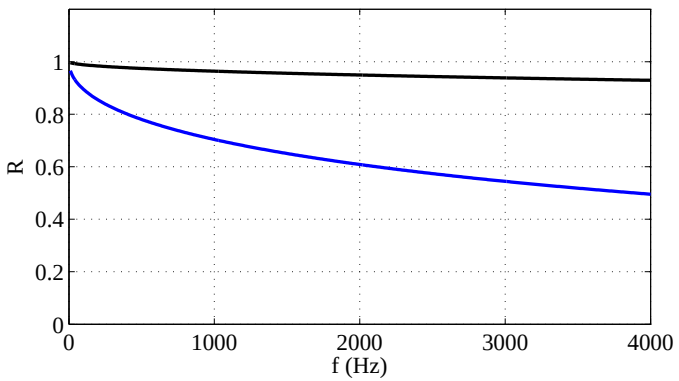
- Thin structure, flexural waves
- Viscoelastic layer
- $\mathrm{Im}(k_f) \nearrow \Rightarrow R \searrow$
- Damping without adding mass

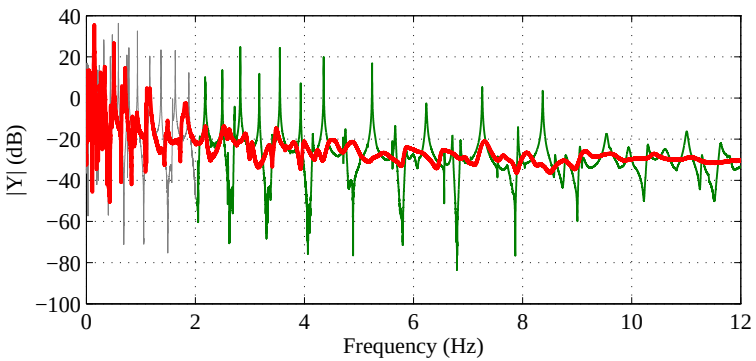
[Mironov, 1988], [Krylov et al., 2004]



$$h(x) = \epsilon x^m, m \geq 2 \quad \Rightarrow \quad \lambda, c \xrightarrow{x \to 0} 0$$







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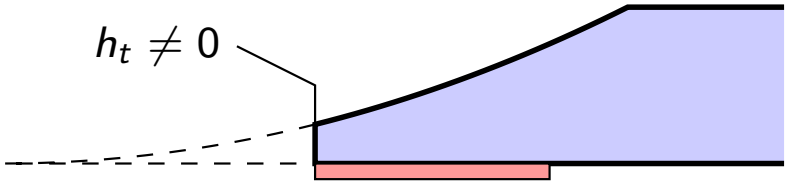

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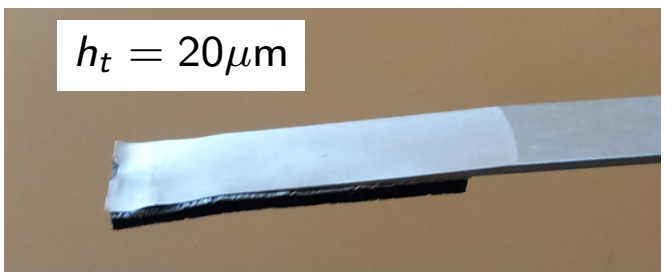
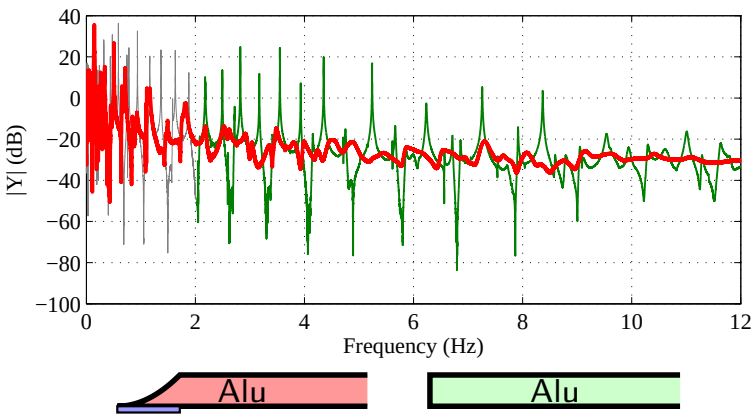
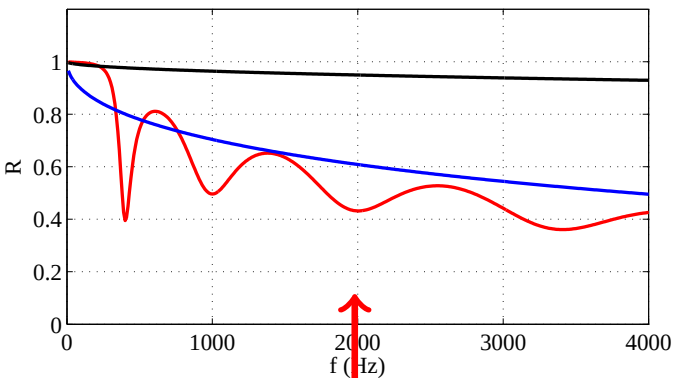


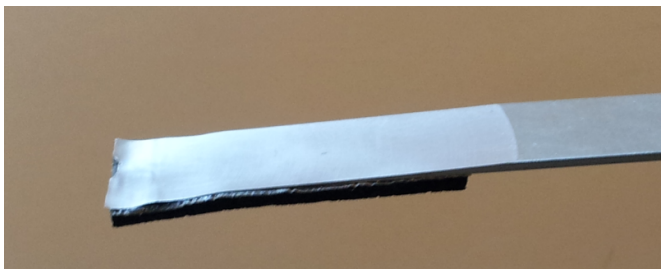


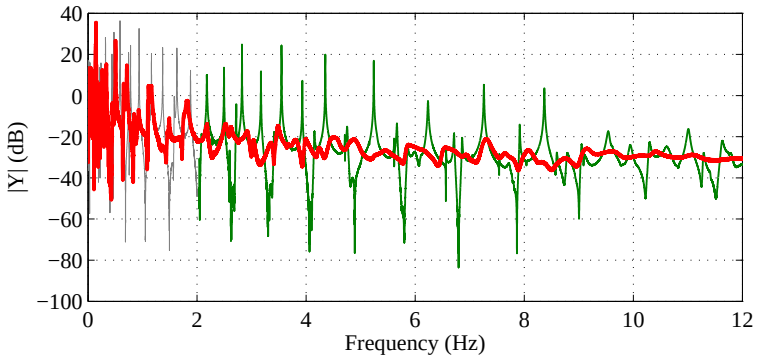


Features

- Complex variations of R [Georgiev et al., 2011]
- Smoothing of FRF $\Rightarrow$ MOF $\nearrow$ [Denis et al., 2014]

Introduction : Acoustic Black Hole (ABH)



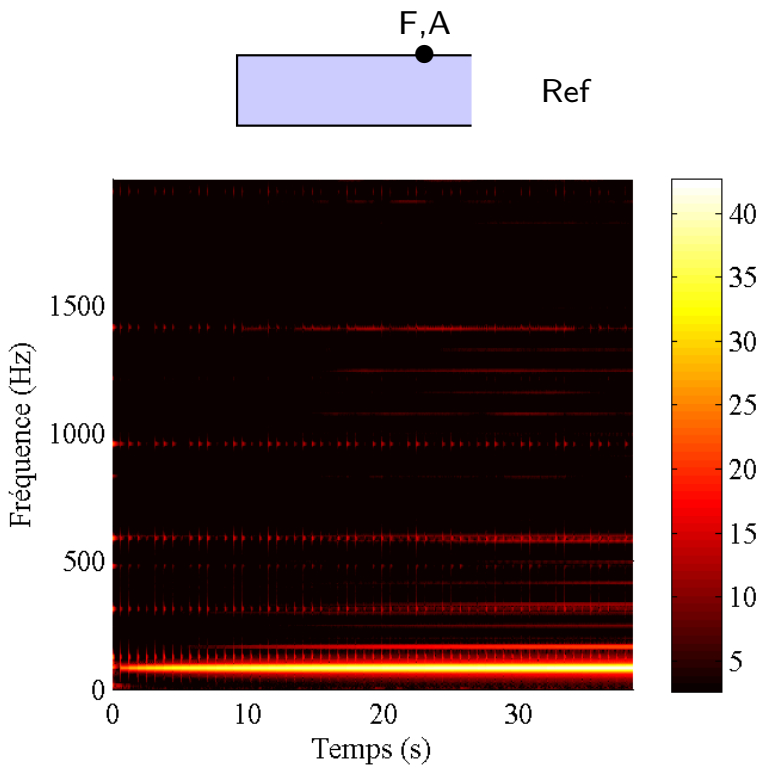


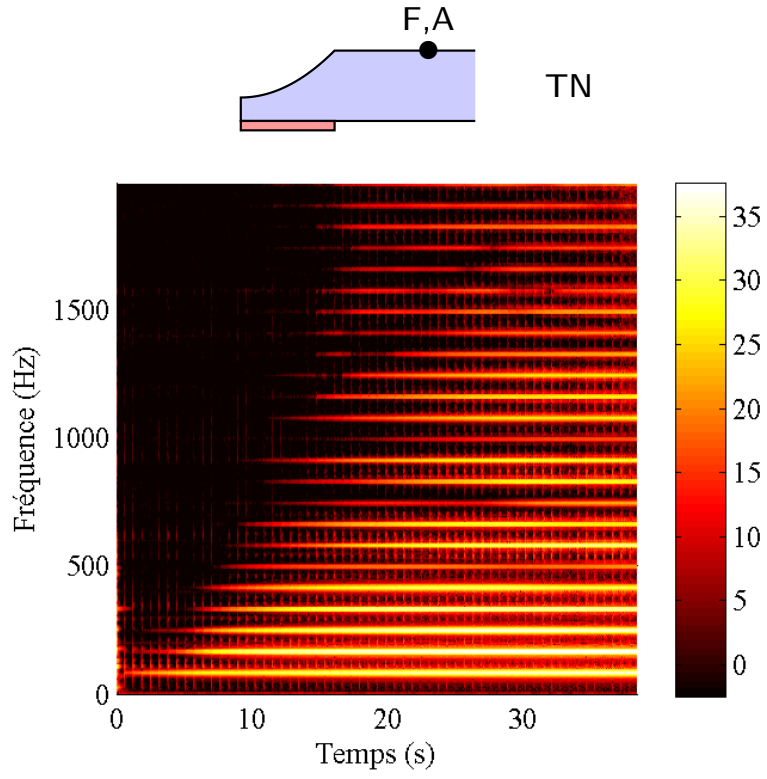
Context

- ABH inefficient at Low Frequency
- How to increase efficiency in LF ?
- Large amplitude at the extremity
- Use of nonlinearities for energy transfer ?

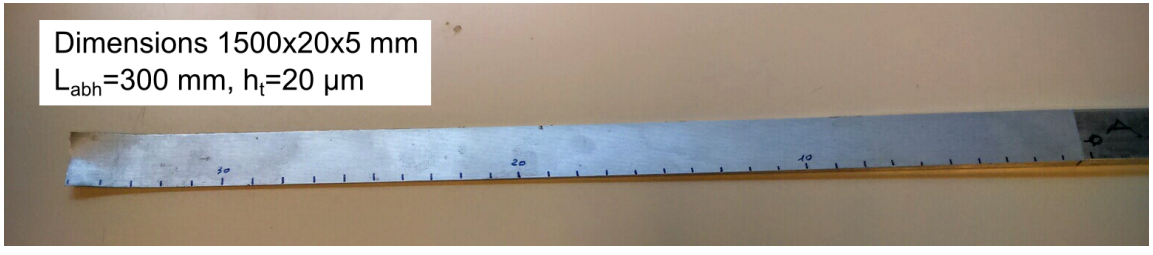
Experiment

Sine excitation f_0=83 Hz, increasing amplitude 0-15 N



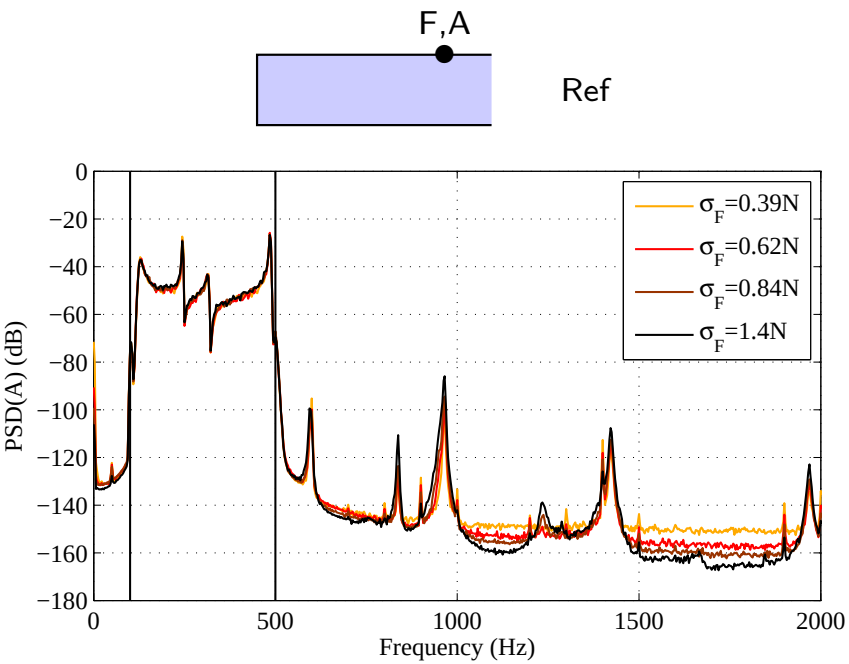


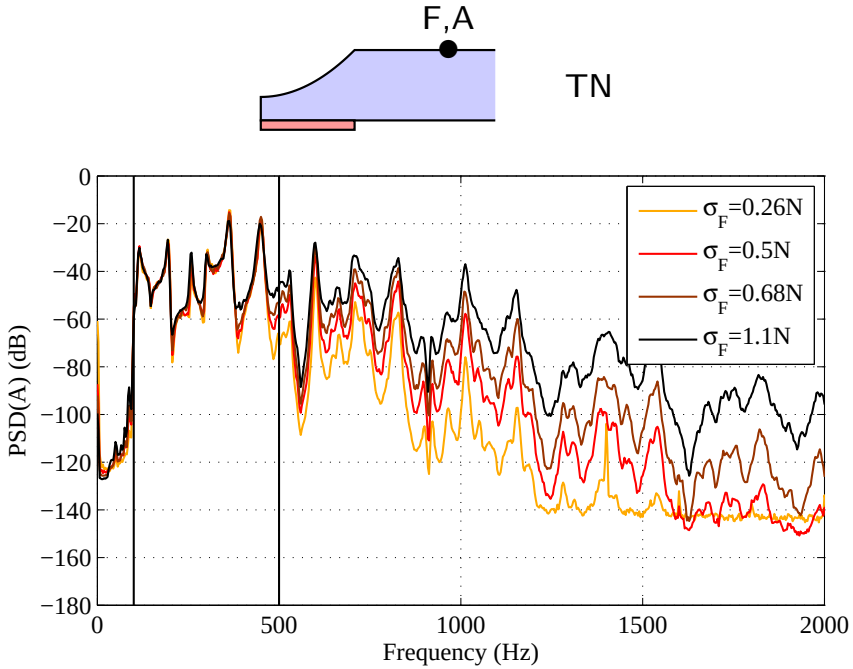
- Uniform beam is linear
- ABH beam is nonlinear
- Spectrum enrichment with amplitude (odd and even harmonics)



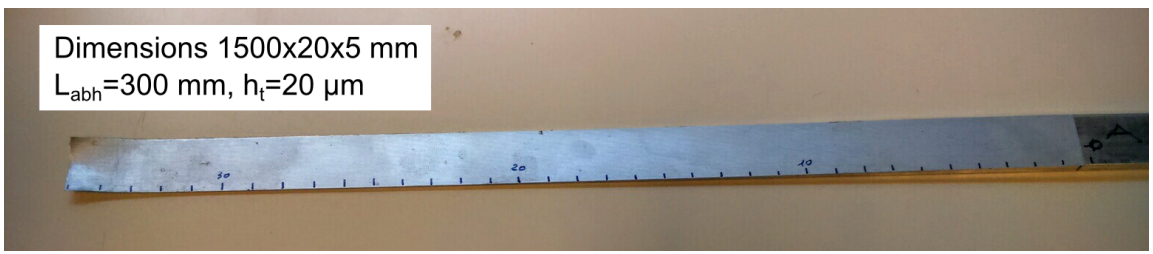
Experiment

Filtered white noise excitation 100-500 Hz.





- Uniform beam : no energy leaks
- ABH beam : energy leaks ⇒ HF
- Transfer not sufficient for resonance peaks reduction



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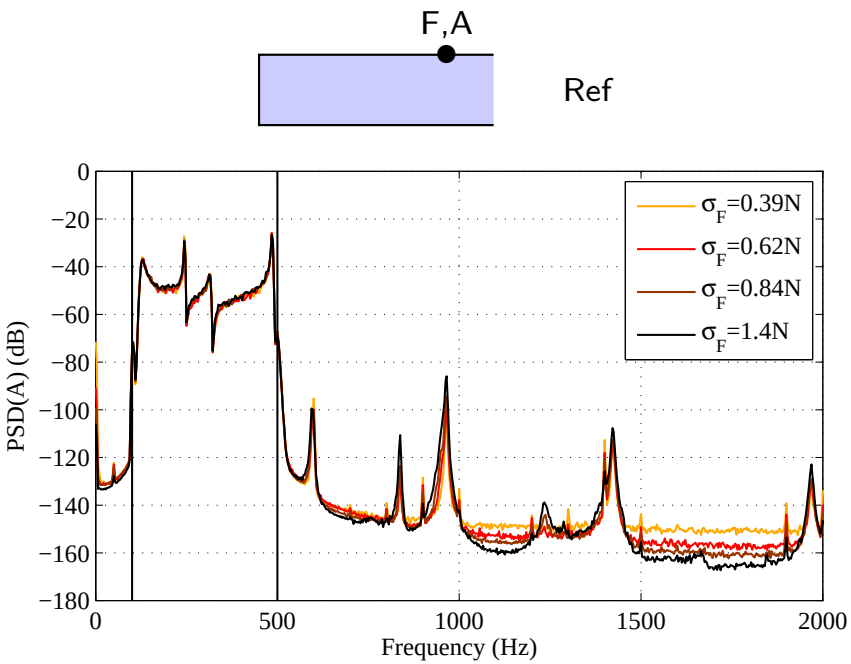
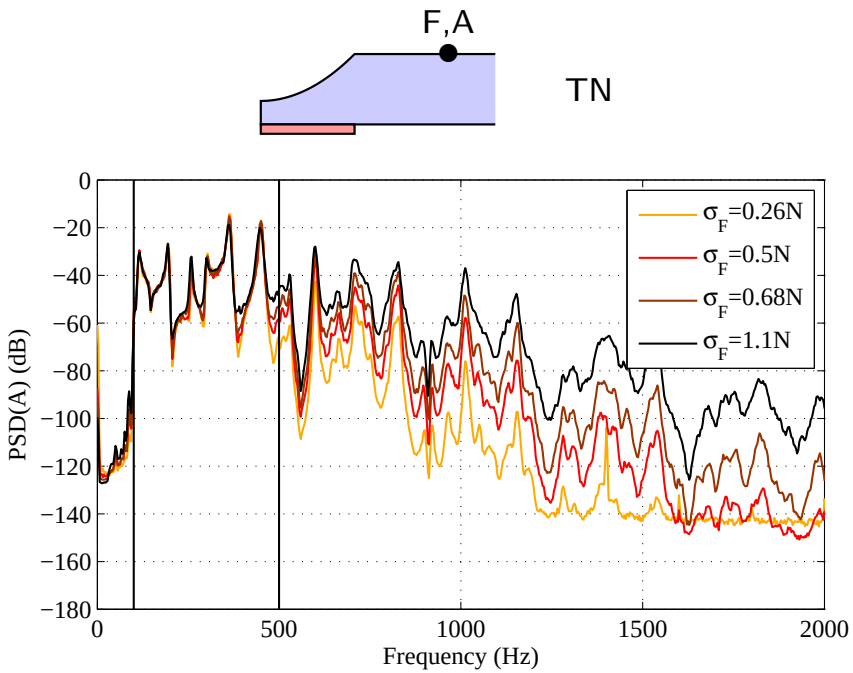


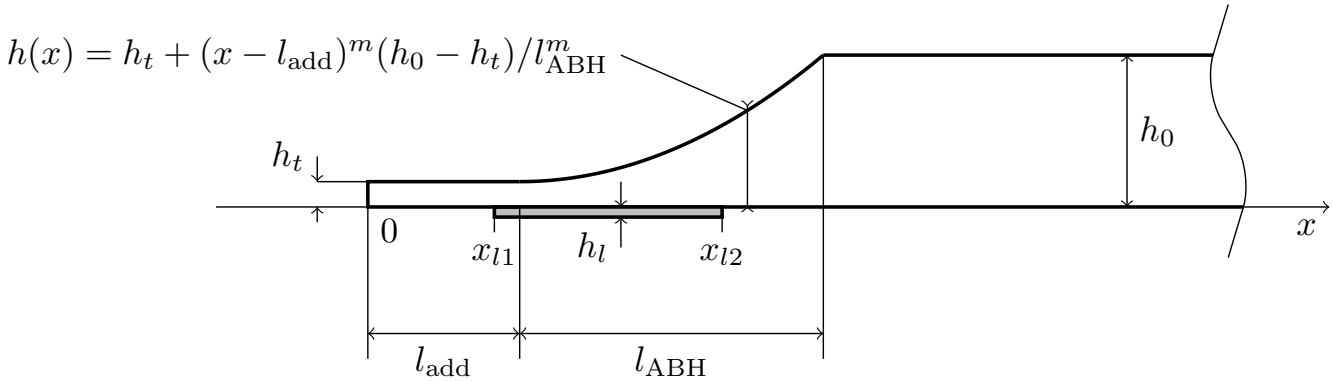


- Uniform beam : no energy leaks
- ABH beam : energy leaks $\Rightarrow$ HF
- Transfer not sufficient for resonance peaks reduction

Conclusions

- ABH : potential for energy transfer LF $\Rightarrow$ HF thanks to nonlinear effects
- Necessity of a model for understanding and use this effect

Model of nonlinear ABH : Development (1)



Thin Von Kármán plate, variable thickness

$$\rho h(\mathbf{x})\ddot{w} + \Delta(D(\mathbf{x})\Delta w) - (1-\nu)L(D(\mathbf{x}), w) = p + L(w, F)$$

$$\Delta(A(\mathbf{x})\Delta F) - (1+\nu)L(A(\mathbf{x}), F) = -\frac{1}{2}L(w, w)$$

+ free boundary conditions.

$w(\mathbf{x}, t)$ transverse displacement, $F(\mathbf{x}, t)$ Airy stress function, $L(f, g) = f_{xx}g_{yy} + f_{yy}g_{xx} - 2f_{xy}g_{xy}$

[Efstathiades, 1971], [Thomas et al., 2008]

$$D(\mathbf{x}) = \frac{Eh(\mathbf{x})^3}{12(1-\nu^2)}$$

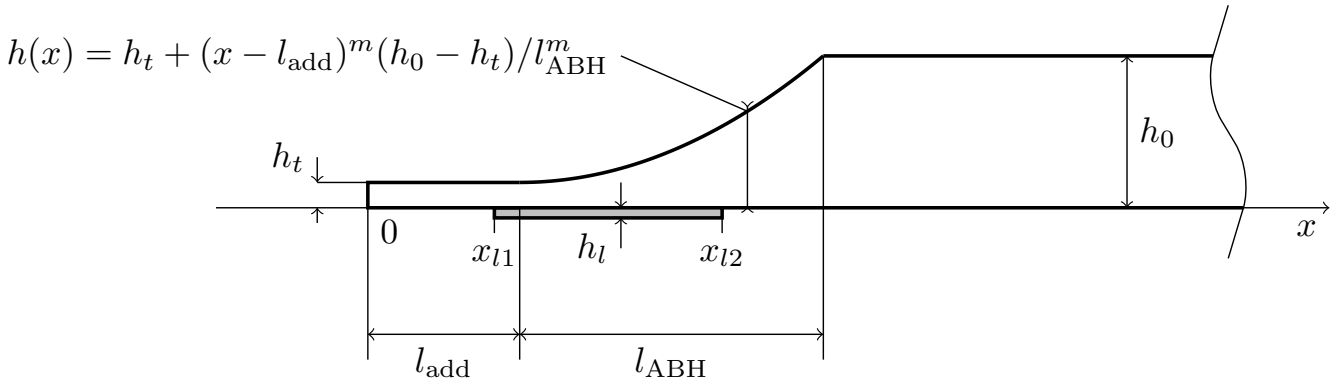
$$A(\mathbf{x}) = 1/Eh(\mathbf{x})$$

Damping

Complex bending stiffness, Ross-Ungar-Kerwin model

$$D^*(x) = D(x)(1 + \mathrm{j}\eta(x))$$

Model of nonlinear ABH : Development (1)



Thin Von Kármán plate, variable thickness

$$\rho h \ddot{w} + \square(D, w) = p + L(w, F)$$
$$\diamond(A, F) = -L(w, w)/2$$

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$$\square(D, w) = \Delta(D(\mathbf{x})\Delta w) - (1 - \nu) L(D(\mathbf{x}), w),$$
$$\diamond(A, F) = \Delta(A(\mathbf{x})\Delta F) - (1 + \nu) L(A(\mathbf{x}), F).$$

Damping

Complex bending stiffness, Ross-Ungar-Kerwin model

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Model of nonlinear ABH : Development (2)

Linear modes

$$-\rho h\omega^2 w + \Box(D, w) = 0 \quad \Rightarrow \quad (\Phi_k, \omega_k)$$

$$\diamond(A, F) = \zeta^4 F \quad \Rightarrow \quad (\Psi_n, \zeta_n)$$

$$w = S_w \sum_{k=1}^{N_\Phi} q_k(t)\Phi_k(\mathbf{x})$$

$$F = S_F \sum_{n=1}^{N_\Psi} \beta_n(t)\Psi_n(\mathbf{x})$$

Projection on the linear modes

Quadratic formulation

$$\beta_p(t) = -\frac{S_w^2}{2\zeta_p^4 S_F} \sum_{i=1}^{N_\Phi}\sum_{j=1}^{N_\Phi} H_{i,j}^p q_i(t)q_j(t),$$

$$\ddot{q}_s(t) + \omega_s^2 q_s(t) = F_s(t) + \frac{S_F}{M_s} \sum_{k=1}^{N_\Phi}\sum_{n=1}^{N_\Psi} E_{k,n}^s q_k(t)\eta_n(t),$$

with

$$H_{i,j}^p = \int_S \Psi_p L(\Phi_i, \Phi_j) dS \quad \text{et} \quad E_{k,l}^s = \int_S L(\Phi_k, \Psi_l)\Phi_s dS.$$

$\Rightarrow$ can be solved with a stable numerical scheme

Model of nonlinear ABH : Development (2)

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Projection on the linear modes

Cubic formulation

$$\ddot{q}_s(t) + \omega_s^2 q_s(t) = F_s(t) - \frac{S_w^2}{M_s} \sum_{k=1}^{N_\Phi}\sum_{m=1}^{N_\Phi}\sum_{n=1}^{N_\Phi} \Gamma^s_{k,m,n} q_k(t) q_m(t) q_n(t)$$

avec

$$\Gamma^s_{k,m,n} = \sum_{l=1}^{N_\Psi} \frac{H^l_{m,n} E^s_{k,l}}{2\zeta_l^4}.$$

$\Rightarrow$ useful for evaluating the convergence

Model of nonlinear ABH : Development (2)

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Dissipative problem

$$-\rho h\omega^2 w + \square(D^*, w) = 0$$
$$\Rightarrow \quad (\Phi_k^*, \omega_k^*) \quad \Rightarrow \quad \xi_k$$

Projection on the linear modes

Quadratic formulation with dissipation

$$\beta_p(t) = -\frac{S_w^2}{2\zeta_p^4 S_F} \sum_{i=1}^{N_\Phi} \sum_{j=1}^{N_\Phi} H_{i,j}^p q_i(t) q_j(t),$$

$$\ddot{q}_s(t) + 2\xi_s \omega_s \dot{q}_s(t) + \omega_s^2 q_s(t) = F_s(t) + \frac{S_F}{M_s} \sum_{k=1}^{N_\Phi} \sum_{n=1}^{N_\Psi} E_{k,n}^s q_k(t) \eta_n(t),$$

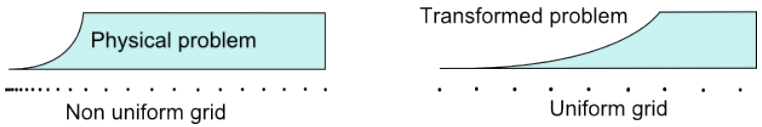
$\Rightarrow$ can be solved with a stable numerical scheme

Model of nonlinear ABH : Numerical resolution, modes

Finite difference method

Grid adapted to wavelength variation

$$d\tilde{x} = \frac{1}{\sqrt{h(x)}} dx$$

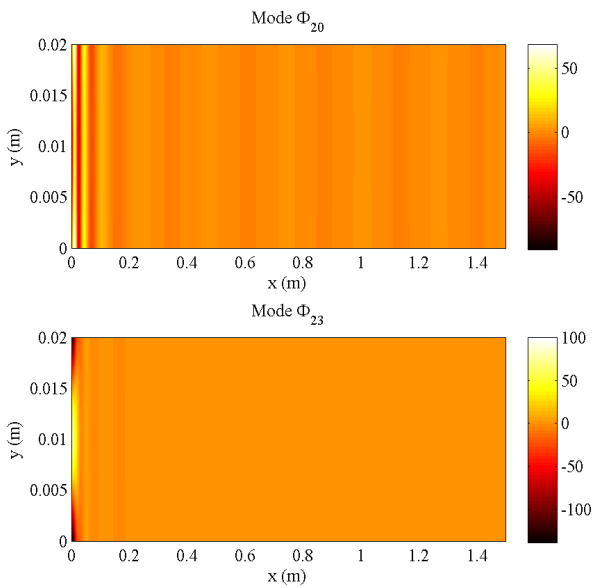


Discrete problem

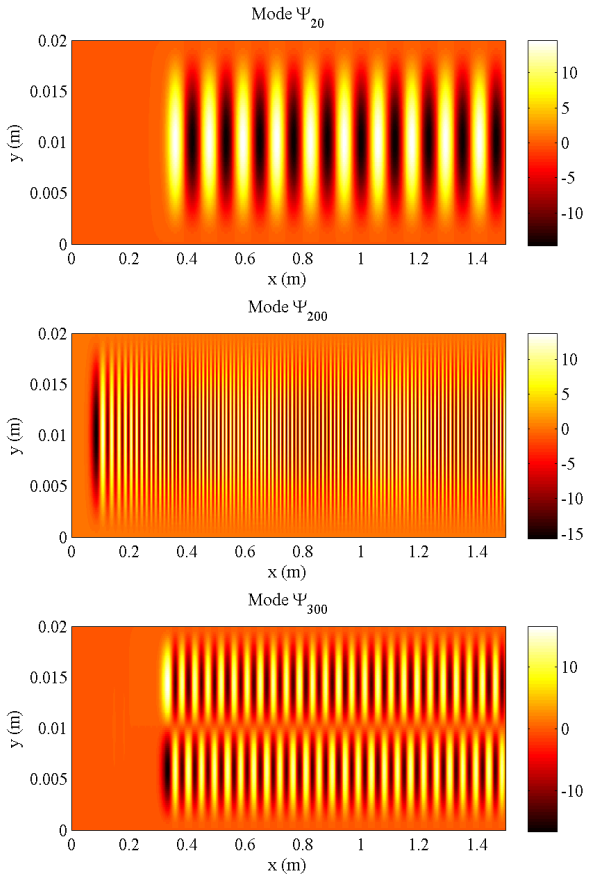
$$(\mathbf{K} - \alpha^2 \mathbf{M}) W = 0$$

[Denis et al., 2014]

Transverse modes Φ



In-plane modes Ψ



$\Rightarrow$ Ψ modes are sorted by families N ($N+1$ nodal lines along y)

Model of nonlinear ABH : Numerical resolution, integration

Conservative scheme

$$\delta_{tt} q_s(n) + 2\xi_s \omega_s \delta_{t.} q_s(n) + \omega_s^2 q_s(n) = \frac{S_F}{M_s} \sum_{k=1}^{N_\Phi} \sum_{l=1}^{N_\Psi} E^s_{k,l} q_k(n) [\mu_{t.} \eta_l(n)] + F_s(n)$$

$$\mu_{t-} \beta_p(n) = \frac{-S_w^2}{2\zeta_p^4 S_F} \sum_{i=1}^{N_\Phi} \sum_{j=1}^{N_\Phi} H^p_{i,j} q_i [e_{t-} q_j(n)]$$

[Ducceschi et al., 2015]

Finite difference operators

$$\delta_{t+} w_n = \frac{w_{n+1} - w_n}{\Delta t},$$

$$\delta_{tt} = \frac{w_{n+1} - 2w_n + w_{n-1}}{\Delta t^2},$$

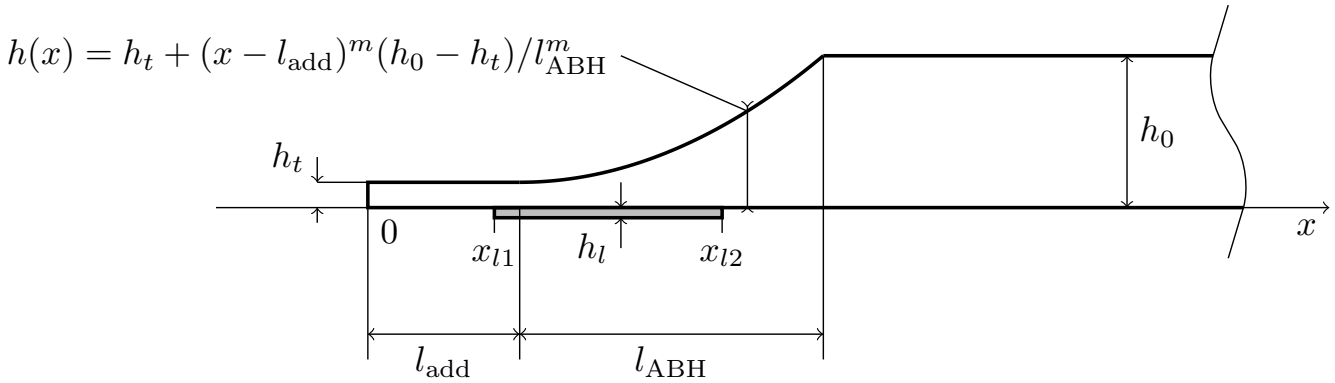
$$\mu_{t+} w_n = \frac{w_{n+1} + w_n}{2}.$$

Stability condition

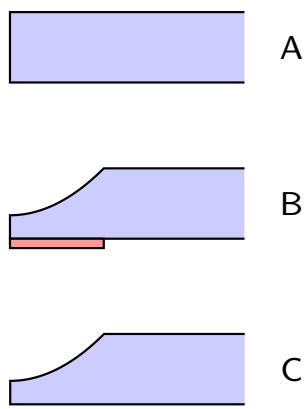
$$f_e > \pi f_{max}$$

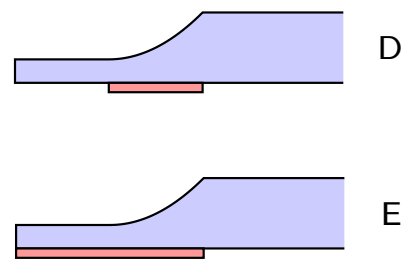
$f_e = 1/\Delta t$ sampling rate
f_{max} frequency of highest mode

Simulated beams



Geometry	
L	1.5 m
b	20 mm
h_0	5 mm
h_t	50 μm
l_{ABH}	30 cm
l_{add}	0 ou 10 cm
h_l	100 μm
Material	
E	70 GPa
ρ	2700 kg.m^{-3}
ν	0.3
E_l	1 GPa
ρ_l	1000 kg.m^{-3}
η_l	0.4



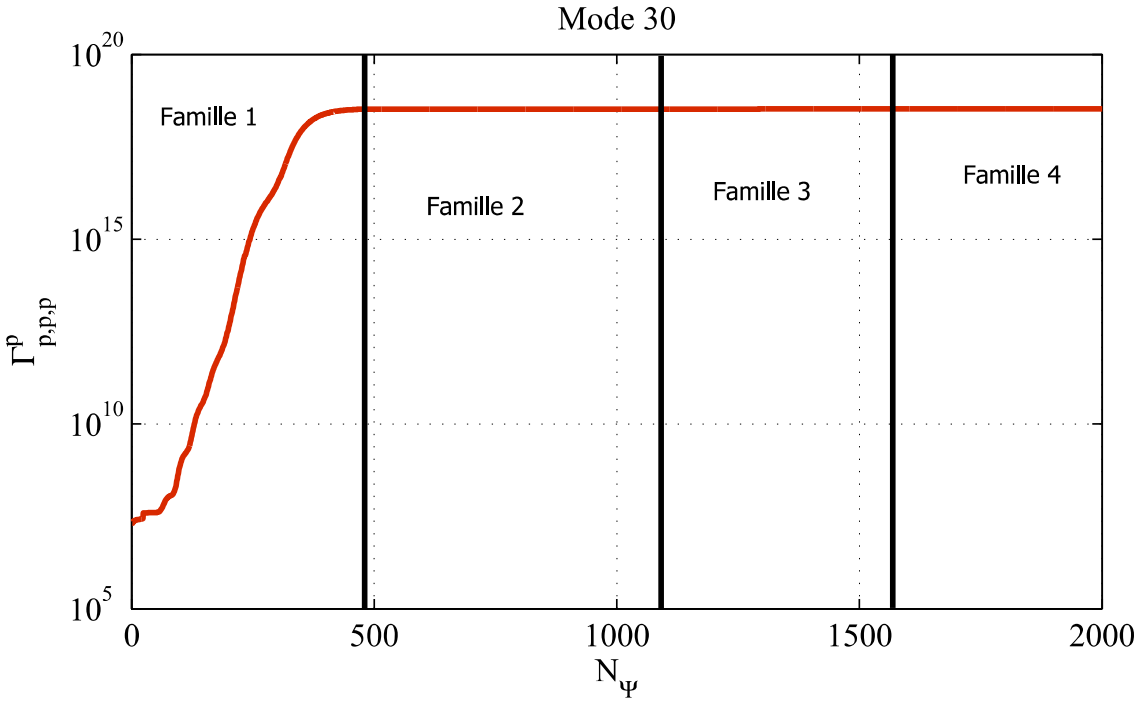


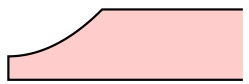
Grid and modes

N_X=2000, N_y=100
N_Φ=100, N_Ψ=2000

Numerical results : Convergence

Convergence of coupling terms $\Gamma^s_{k,m,n}$ with N_Ψ





- Ψ modes sorted by families $\Rightarrow$ Faster convergence

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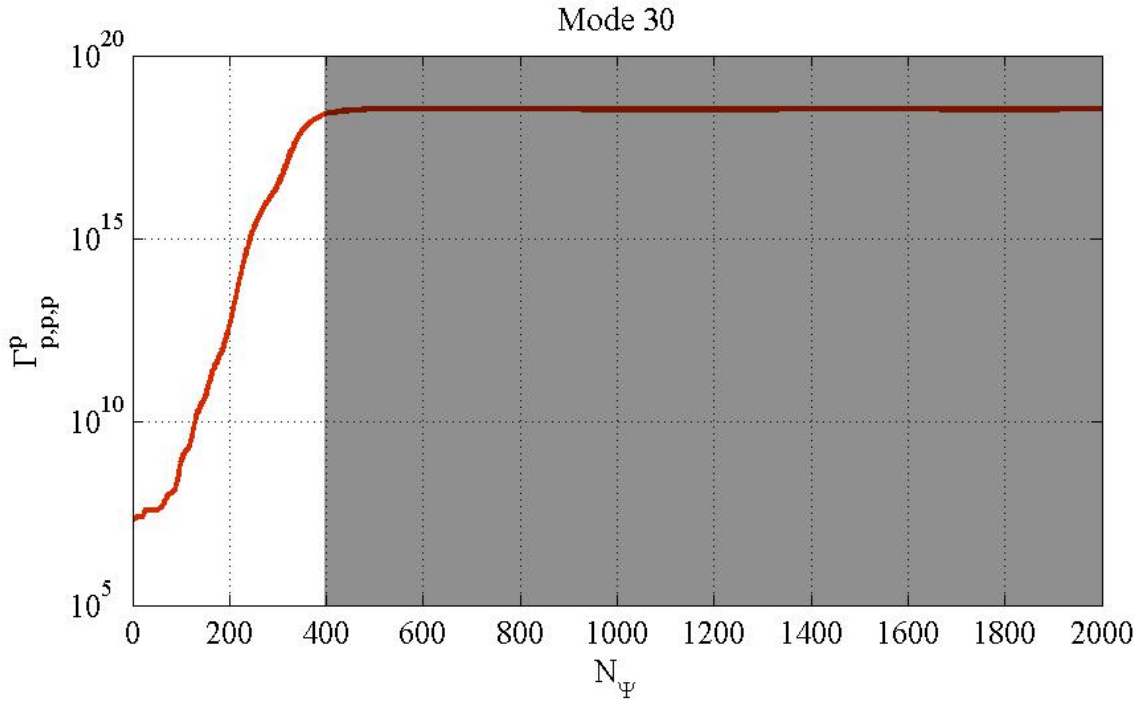

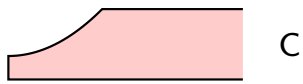

- Ψ modes sorted by families $\Rightarrow$ Faster convergence
- Elimination of the useless modes (only family 1 contributes)

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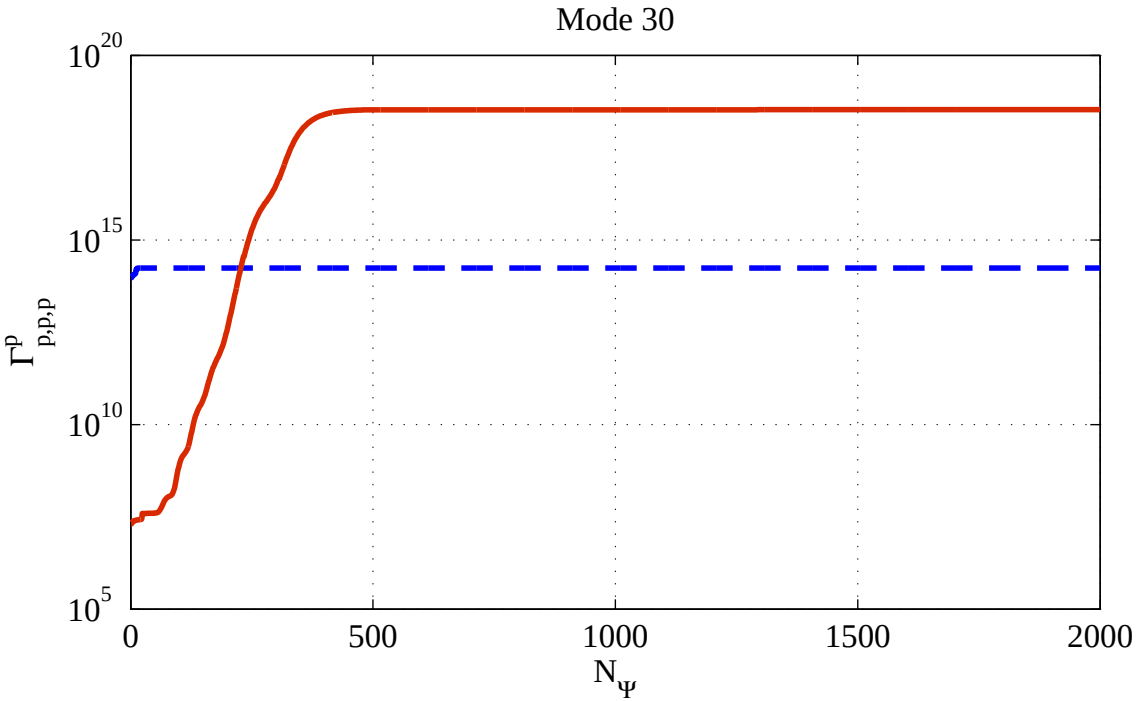

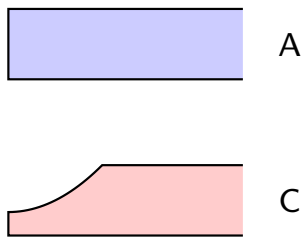

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- Higher coupling coefficient for ABH beam

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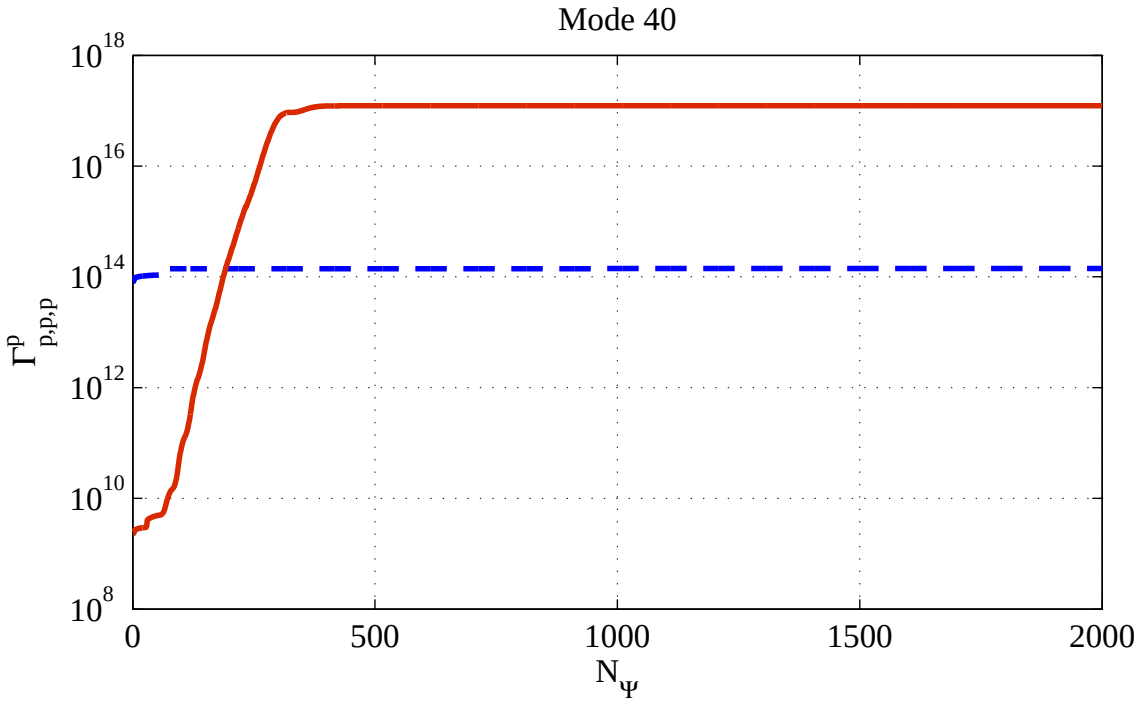

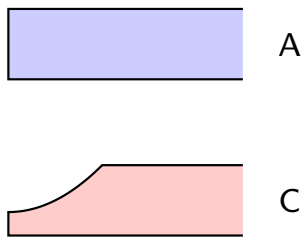

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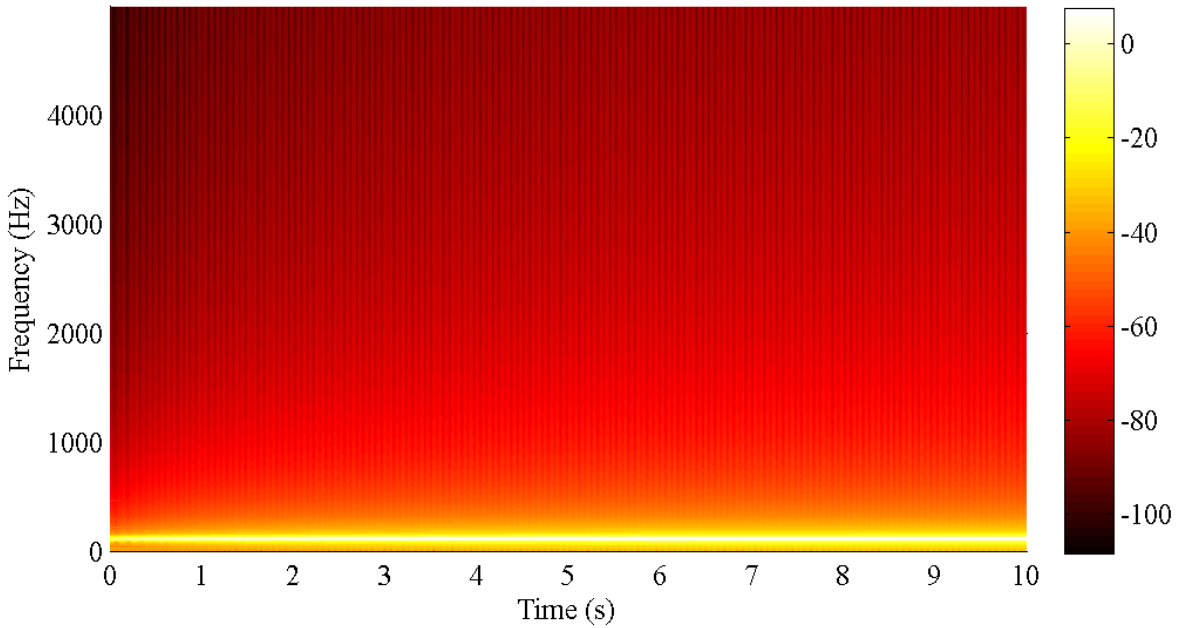
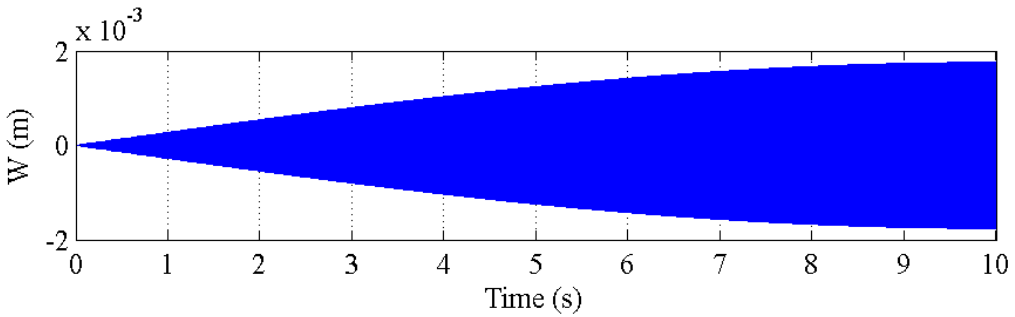
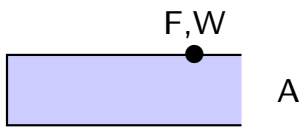
Numerical results : Nonlinear regimes

Sine excitation f_0=111 Hz, increasing amplitude 0-20 N, point (0.6,0.01) m, f_e=100 kHz



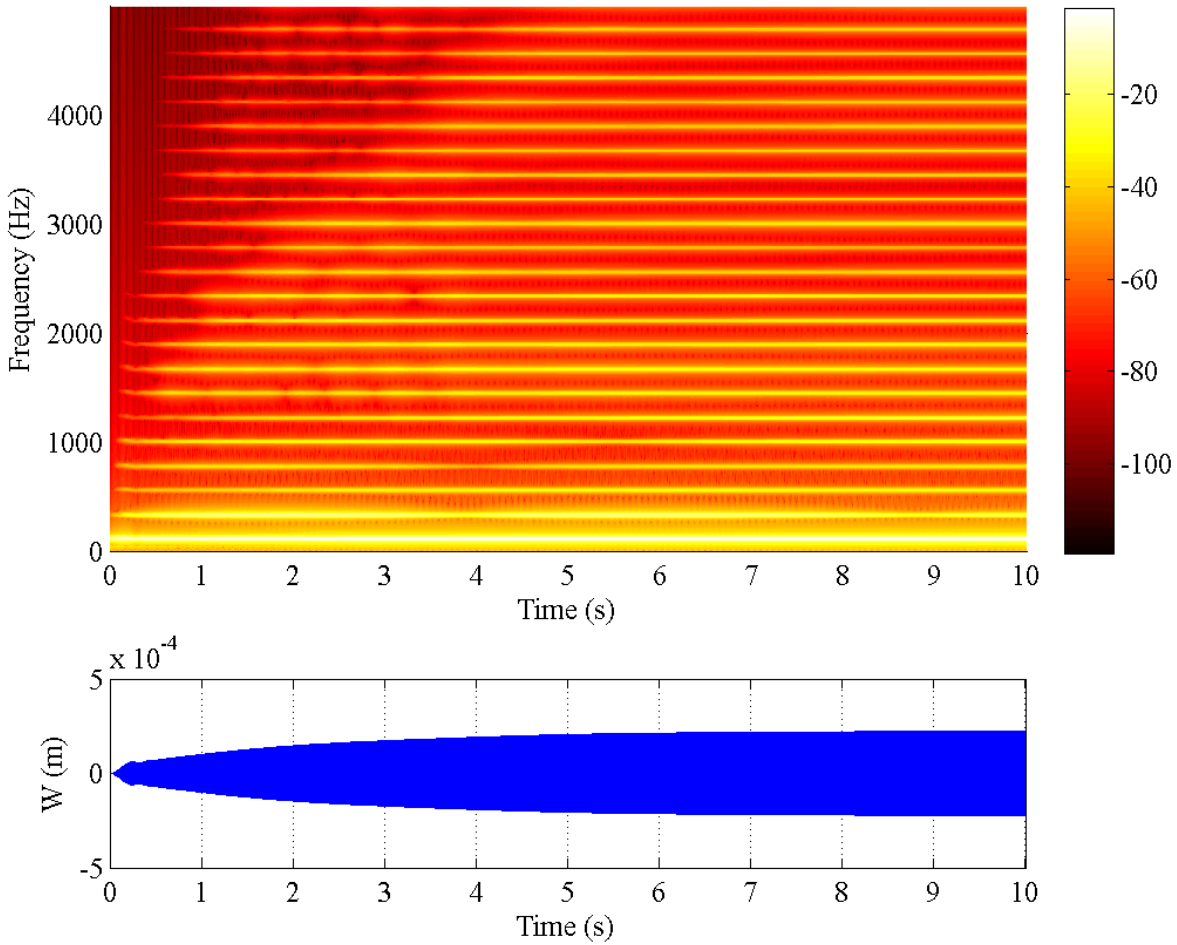


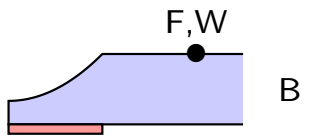


- Linear regime for uniform beam

Numerical results : Nonlinear regimes

Sine excitation f_0=111 Hz, increasing amplitude 0-20 N, point (0.6,0.01) m, f_e=100 kHz

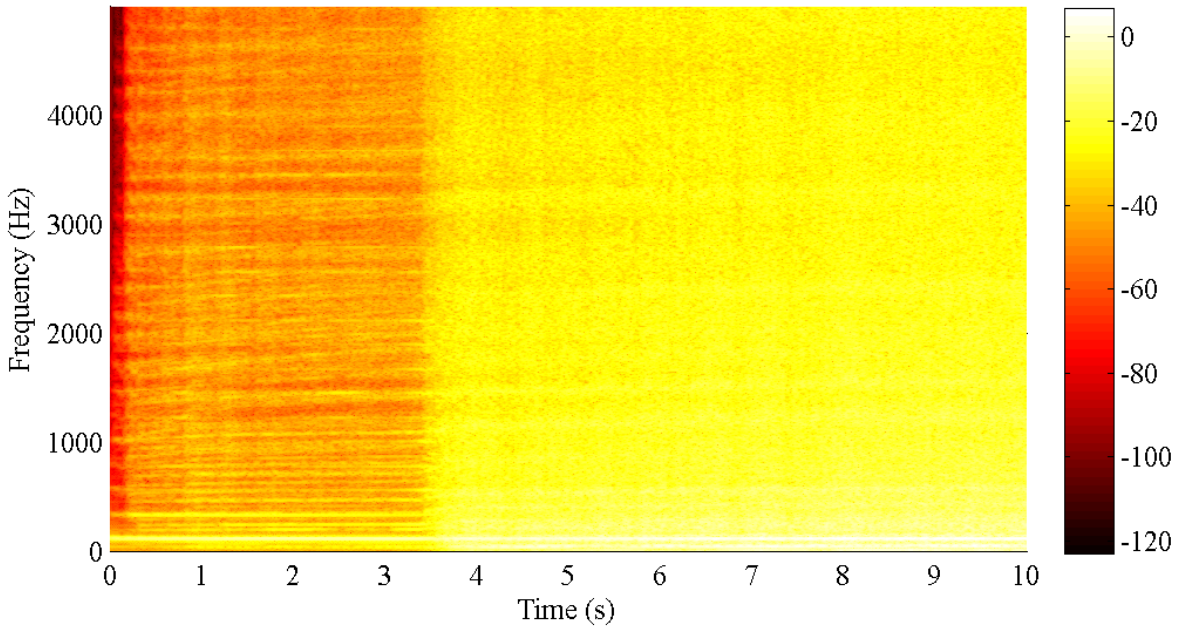


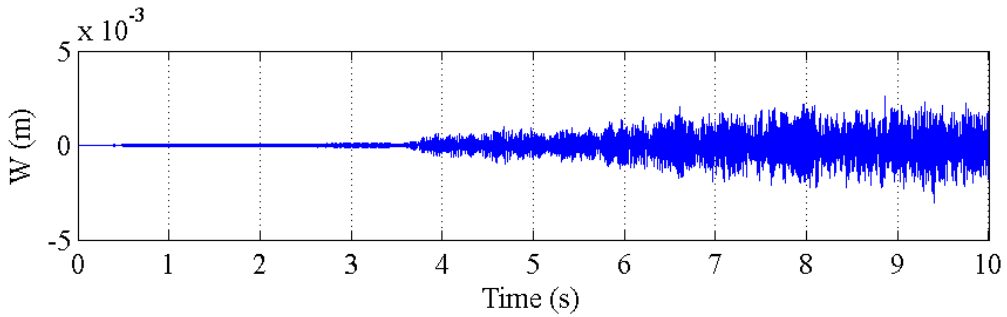


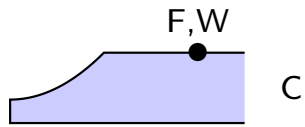
- Linear regime for uniform beam
- Rich spectrum with odd harmonics

Numerical results : Nonlinear regimes

Sine excitation f_0=111 Hz, increasing amplitude 0-20 N, point (0.6,0.01) m, f_e=100 kHz



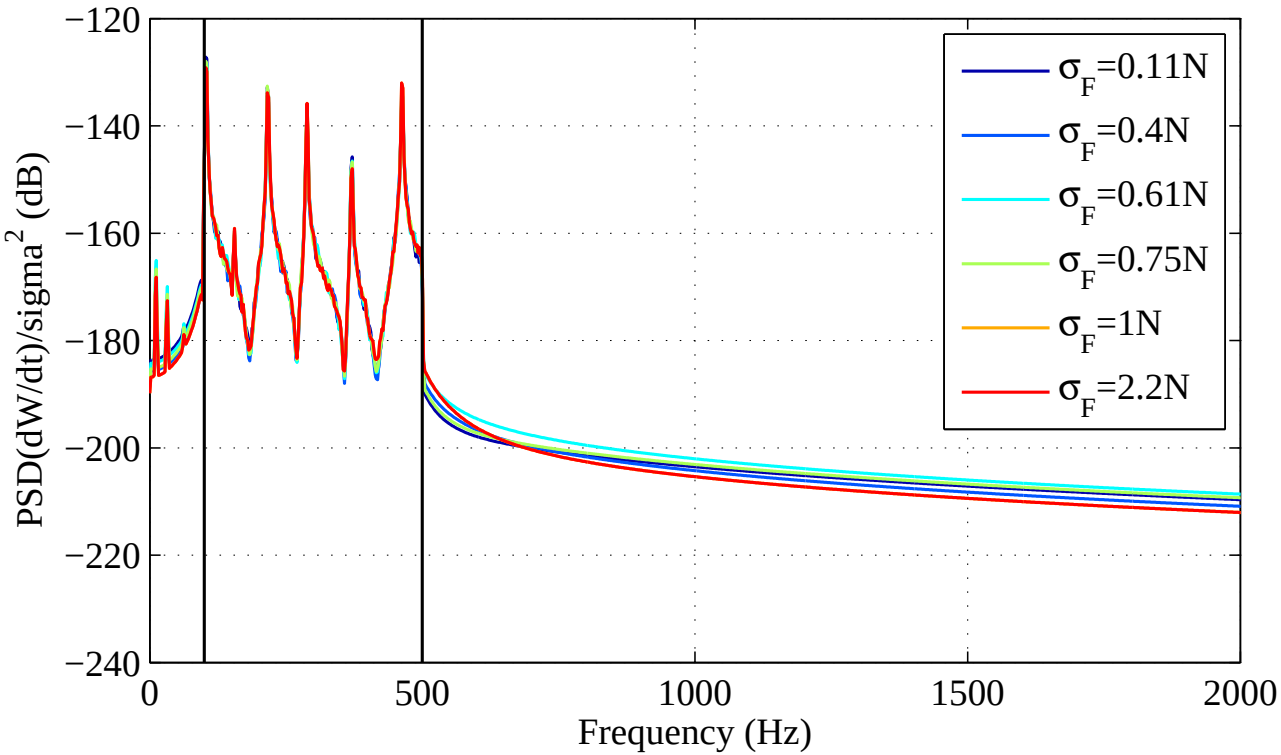


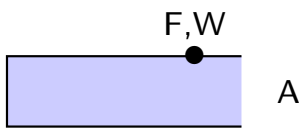


- Linear regime for uniform beam
- Rich spectrum with odd harmonics
- Chaotic regime

Numerical results : Energy transfer

Filtered white noise excitation 100-500 Hz, point (0.6,0.01) m, f_e=100 kHz

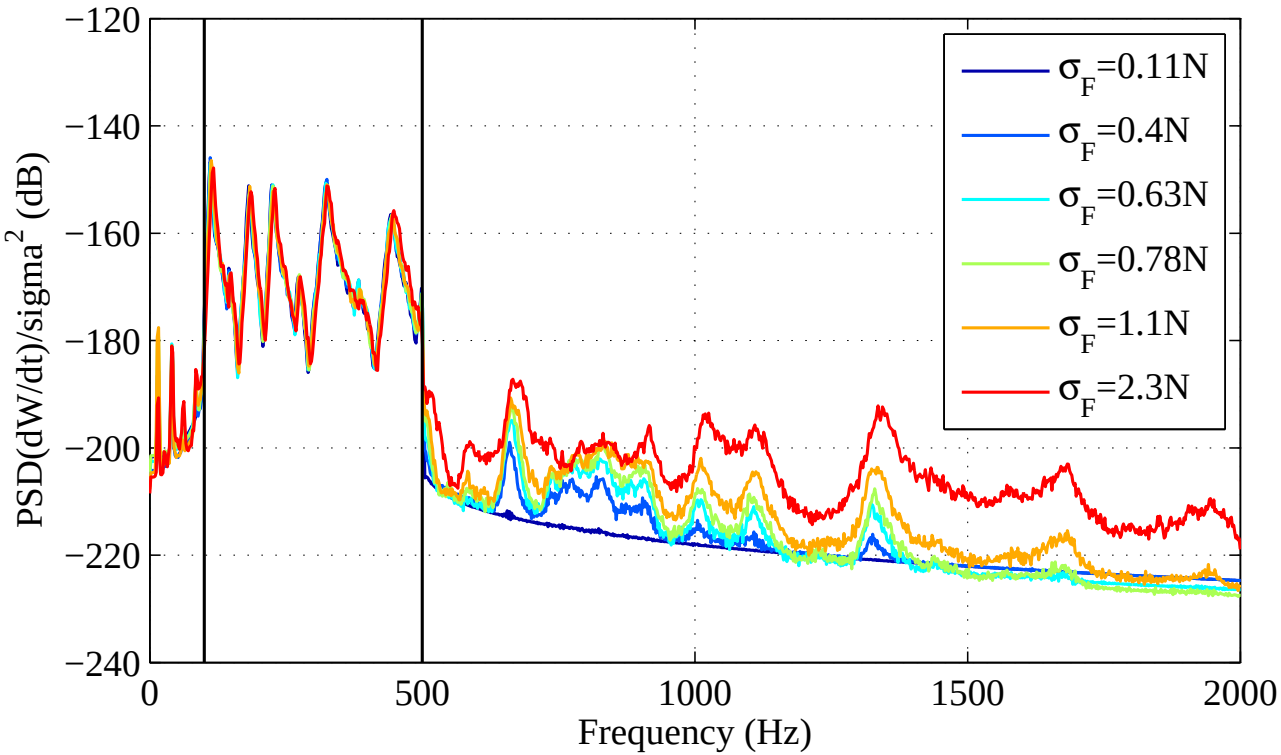


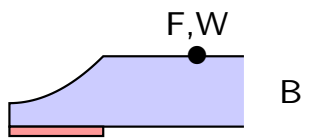


- Linearity of the uniform beam

Numerical results : Energy transfer

Filtered white noise excitation 100-500 Hz, point (0.6,0.01) m, f_e=100 kHz

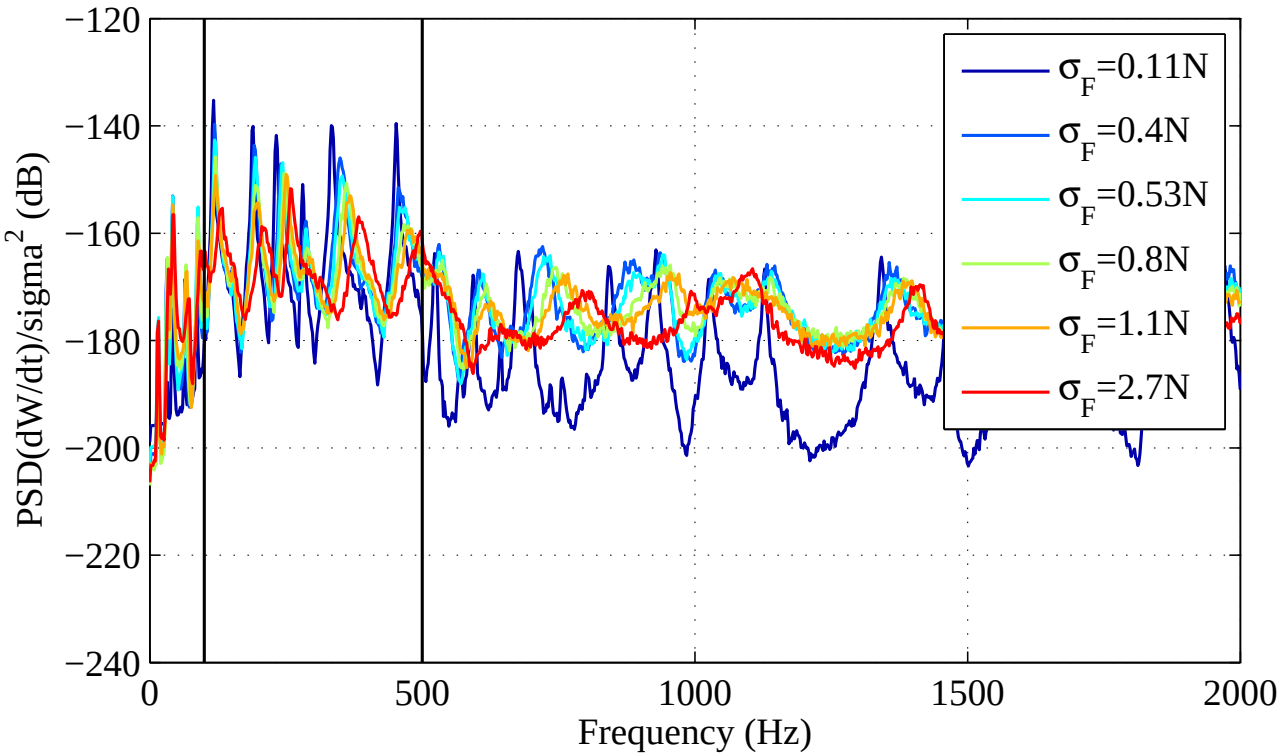


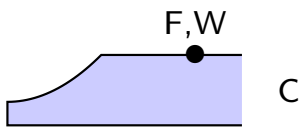


- Energy leaks with amplitude increase
- Slight reduction of resonances peaks at LF with amplitude
- Increase of resonance frequencies (hardening)

Numerical results : Energy transfer

Filtered white noise excitation 100-500 Hz, point (0.6,0.01) m, f_e=100 kHz

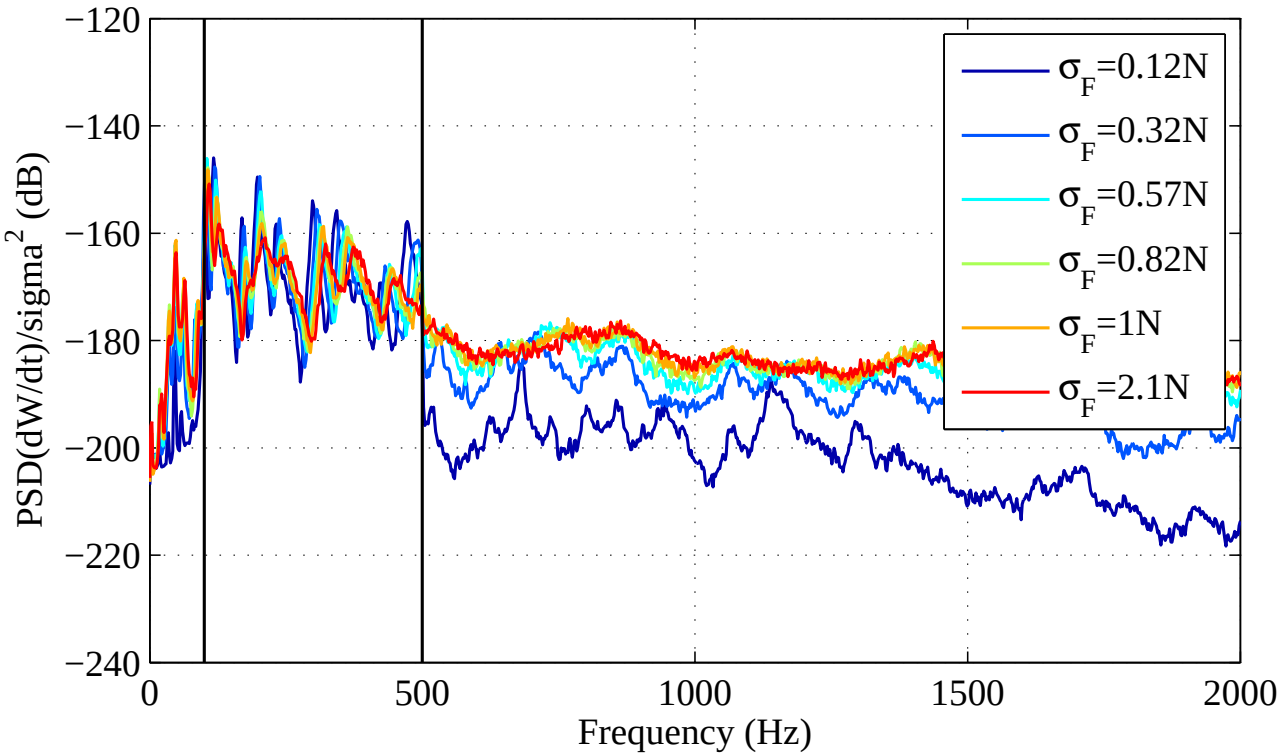


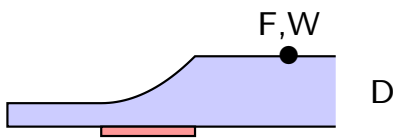


- Important energy transfer ⇒ HF
- LF reduction even without damping

Numerical results : Energy transfer

Filtered white noise excitation 100-500 Hz, point (0.6,0.01) m, f_e=100 kHz

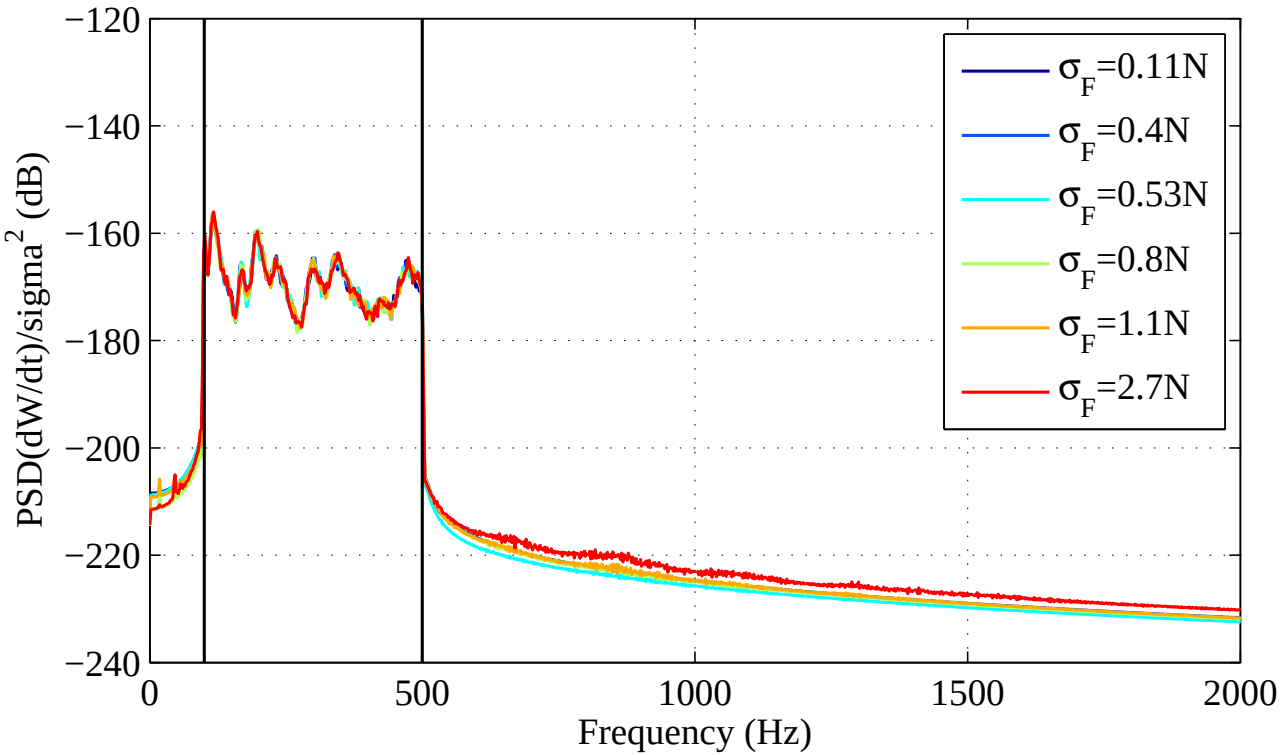


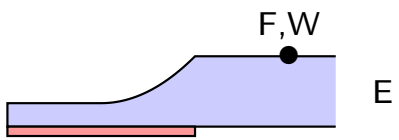


- Increase nonlinear effects with the extension
- Important LF vibration reduction

Numerical results : Energy transfer

Filtered white noise excitation 100-500 Hz, point (0.6,0.01) m, f_e=100 kHz



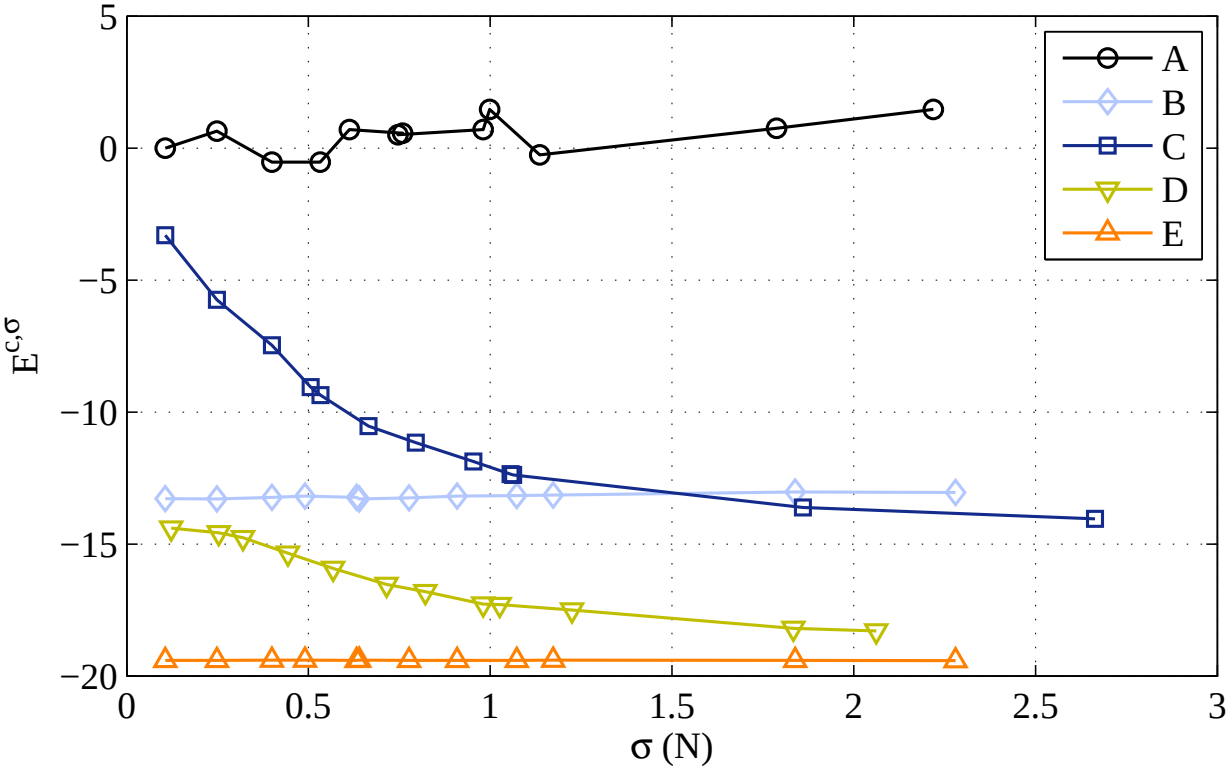


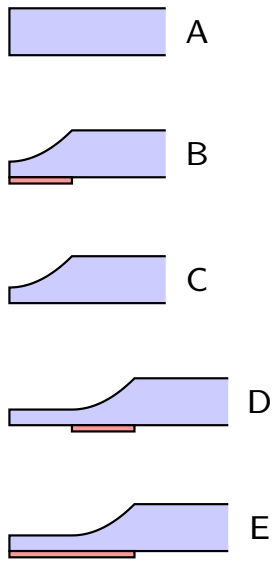
- Too much damping kills nonlinearities

Numerical results : Energy transfer

Indicator of ABH performance in low-frequency

$$I^{c,\sigma} = \int_{100}^{500} \frac{S_{aa}^{c,\sigma}(f)}{S_{pp}^{c,\sigma}(f)} \mathrm{d}f \qquad E^{c,\sigma} = 10 \log_{10}\left(\frac{I^{c,\sigma}}{I^{\mathrm{ref}}}\right)$$





- Reduction of LF energy due to nonlinearity
- Damping kills nonlinear effects
- Right balance between damping and nonlinear extension has to be adjusted

Conclusions

Conclusions

- Numerical model for nonlinear ABH
- ABH profile gives nonlinear behaviour to the beam
- LF to HF energy transfer : better performance of ABH in LF
- Viscoelastic layer calms down NLs
- Extension increases nonlinear effects
- Indicator of ABH performance

Future works

- Right balance between damping and nonlinear extension has to be adjusted
- Obtain nonlinearity for small excitation amplitude
- Observe benefits experimentally

Effects of geometrical nonlinearities on the acoustic black hole effect

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