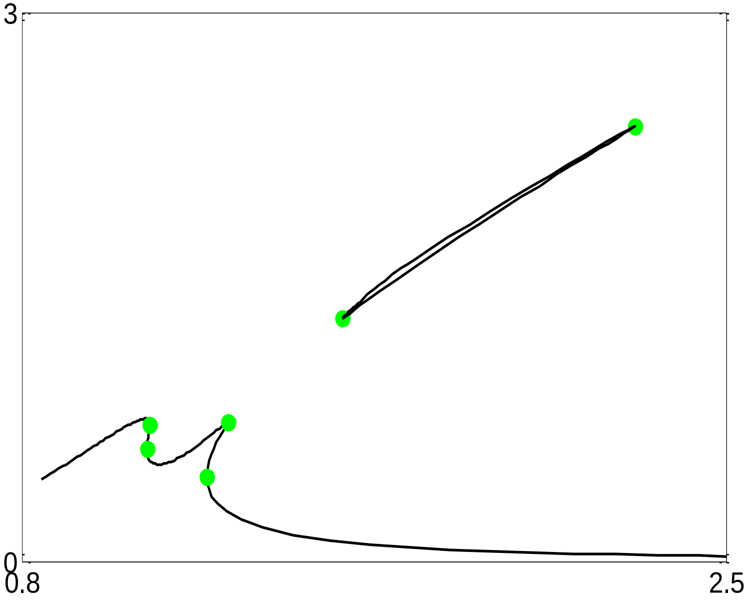
Nonlinear Vibration Absorbers: Pros and Cons



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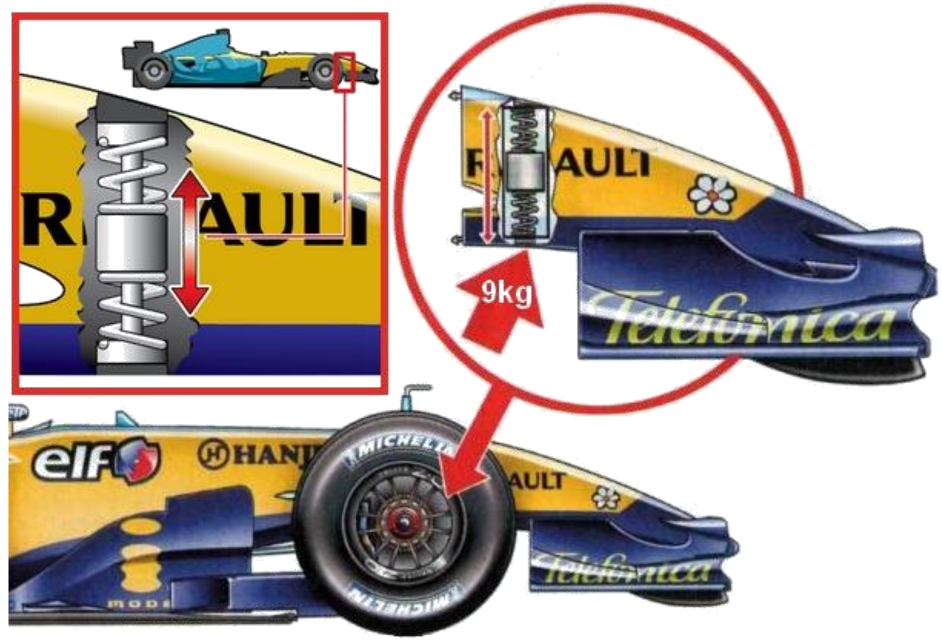
NES: L.A. Bergman, A.F. Vakakis
NLTVA: T. Detroux, C. Grappasonni, G. Habib
Robust LTVA: L. Dell'Elce, E. Gourc, G. Aridon, G. Michon, A. Hot

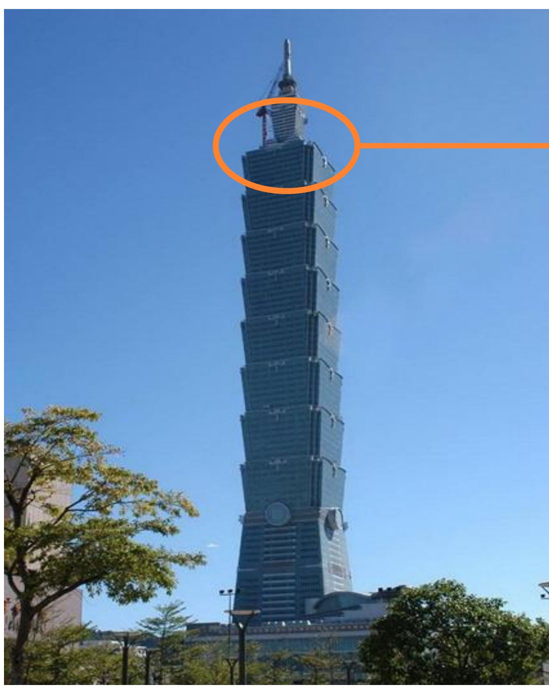
Outline of the First Part

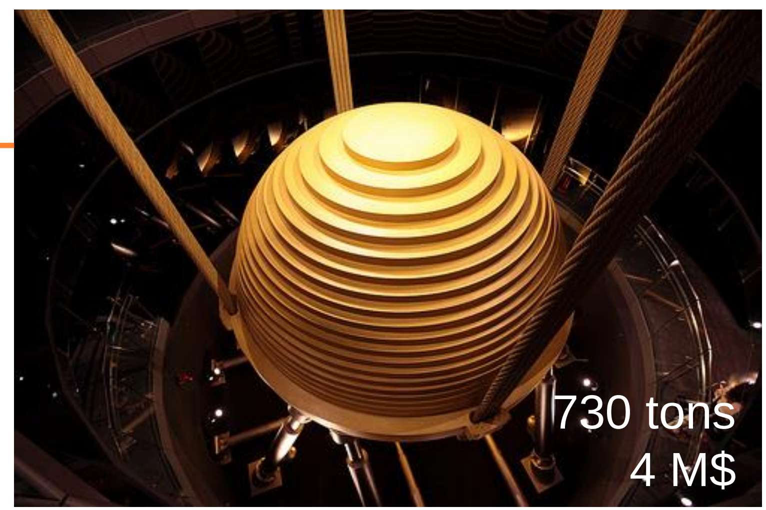
1. The linear tuned vibration absorber (LTVA)

2. The nonlinear energy sink (NES)

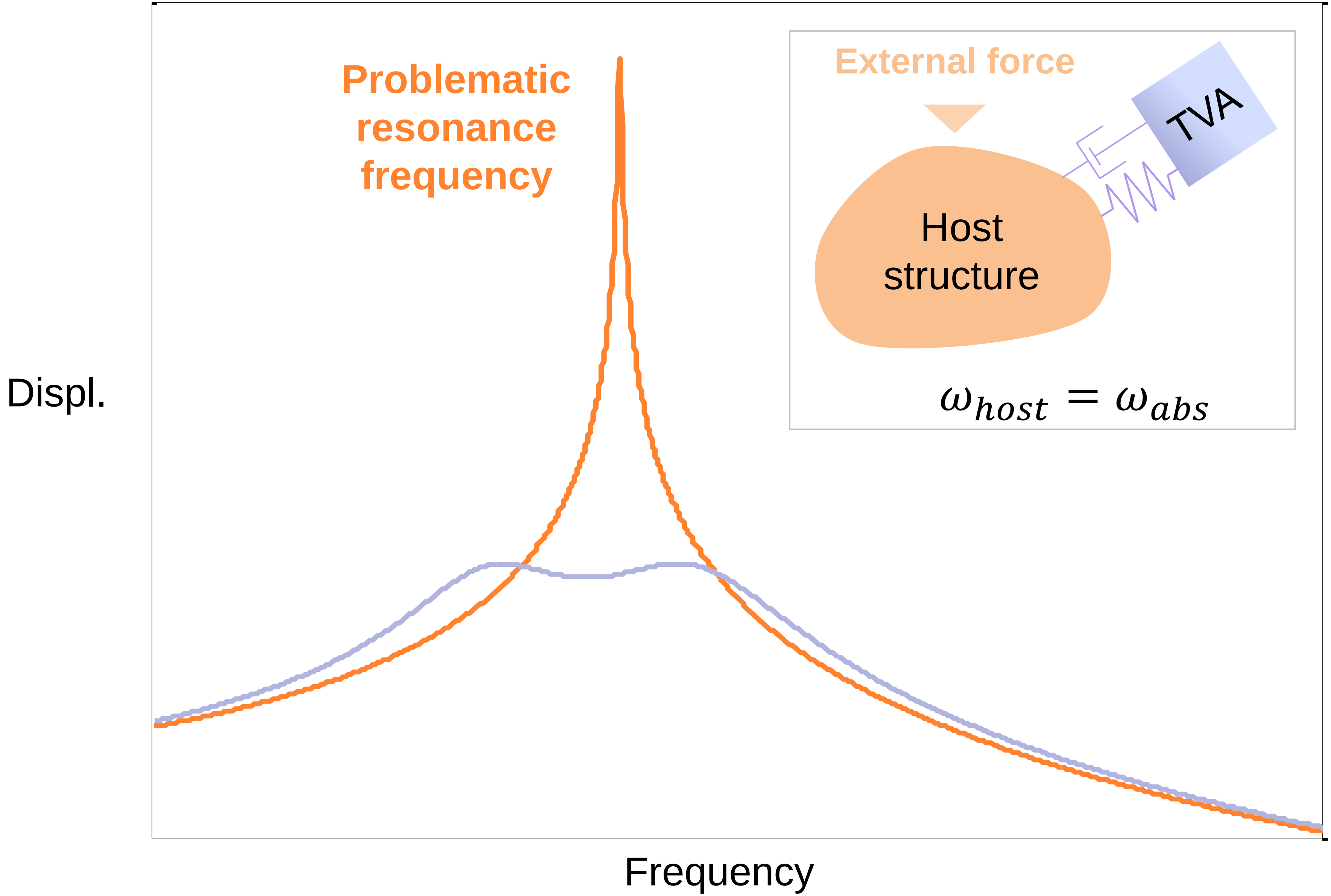
3. The nonlinear tuned vibration absorber (NLTVA)


9kg

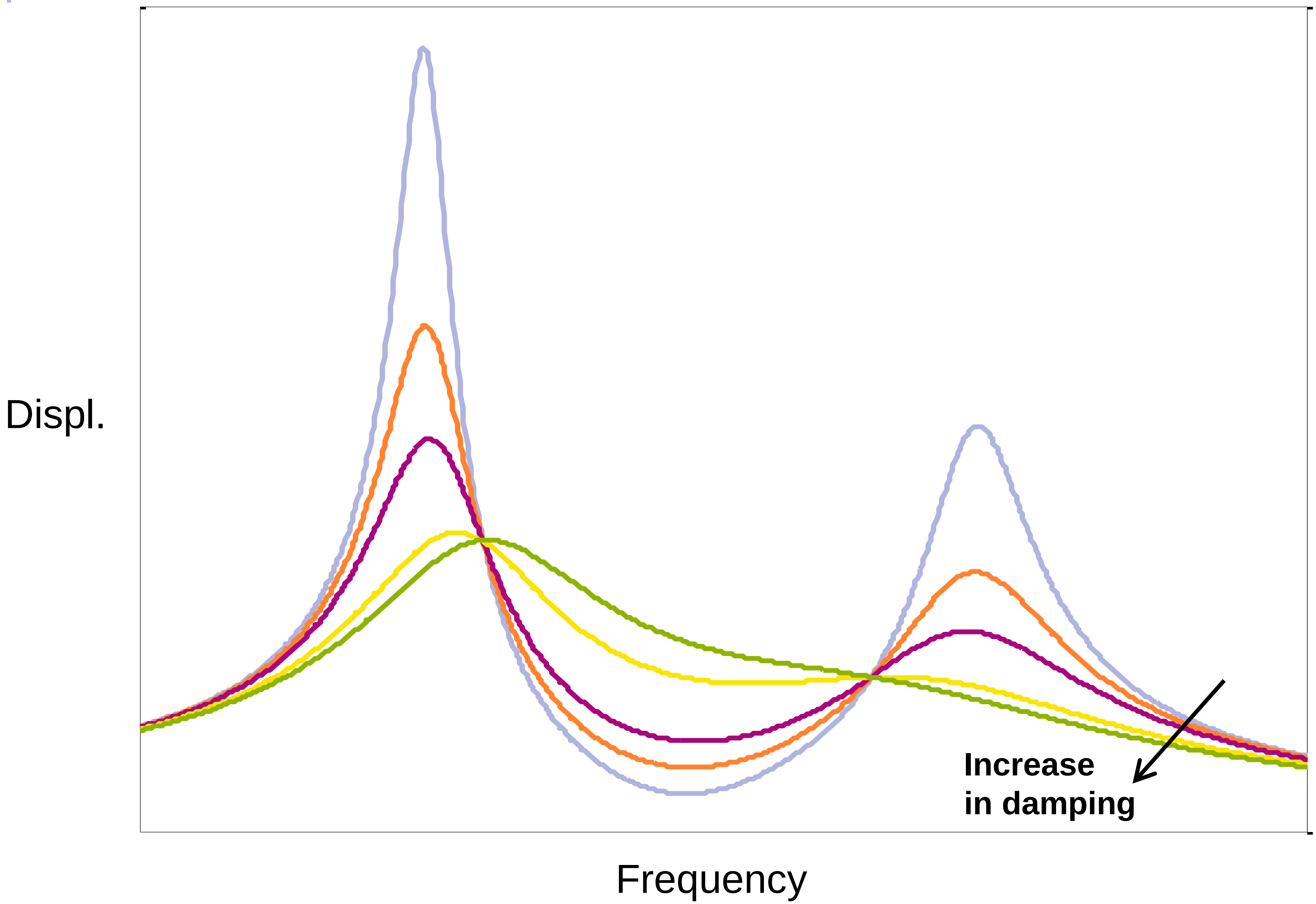



730 tons
4 M$

The *"Classical"* Linear Tuned Vibration Absorber



Existence of Invariance Points for the LTVA



(Approximate) Tuning Rule Proposed by Den Hartog

Optimum frequency ratio:

- The invariant points must have the same amplitude.

$$\omega_{abs} = \frac{\omega_{host}}{1+\epsilon}$$

(Den Hartog, 1928)

Optimum damping ratio:

- The receptance maxima must occur at the invariant points.

$$\xi_{abs} = \sqrt{\frac{3}{8}\frac{\epsilon}{1+\epsilon}}$$

(Brock, 1946)

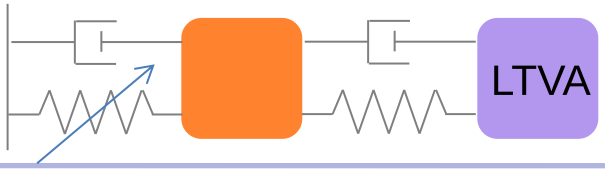
Summary for the LTVA

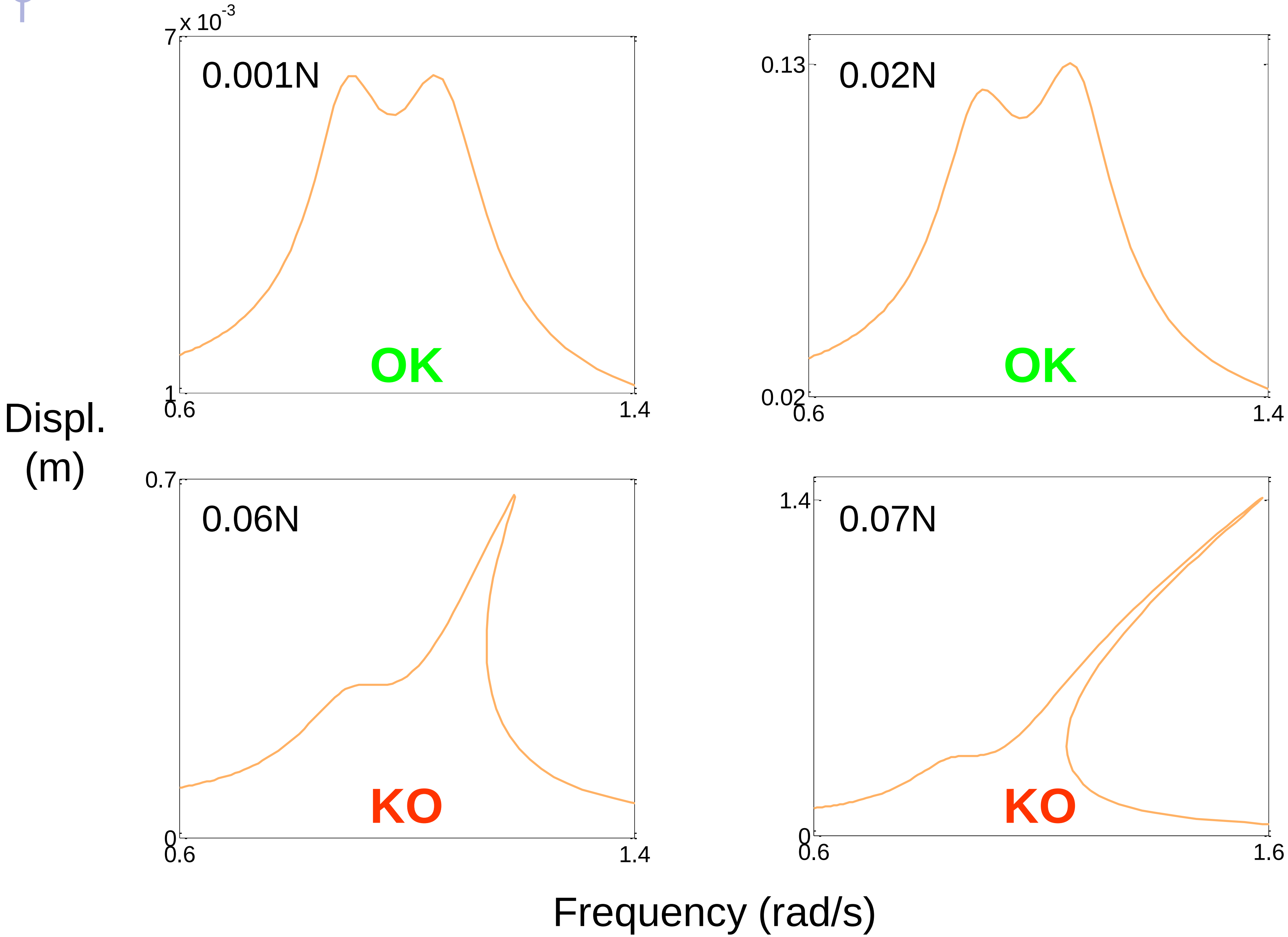
Easy practical realization.

Insensitive to the forcing amplitude.

Narrow-band device:
broadband and nonlinear instabilities ?

Nonlinear Host Structures ?





Outline of the First Part

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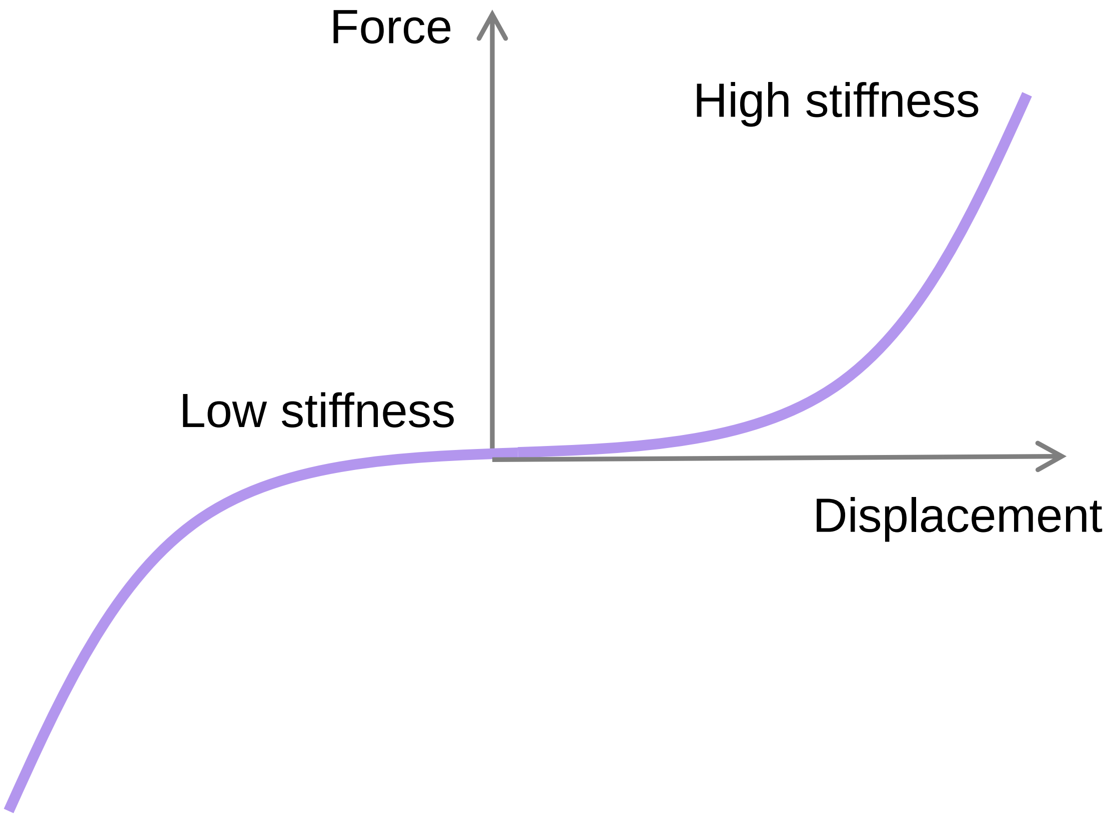
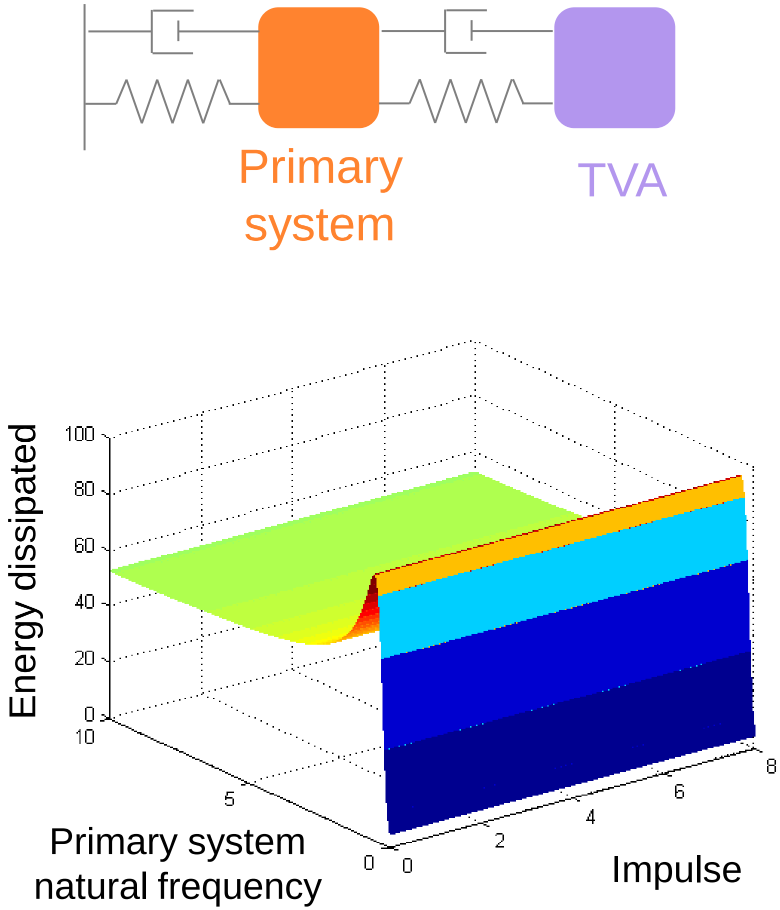
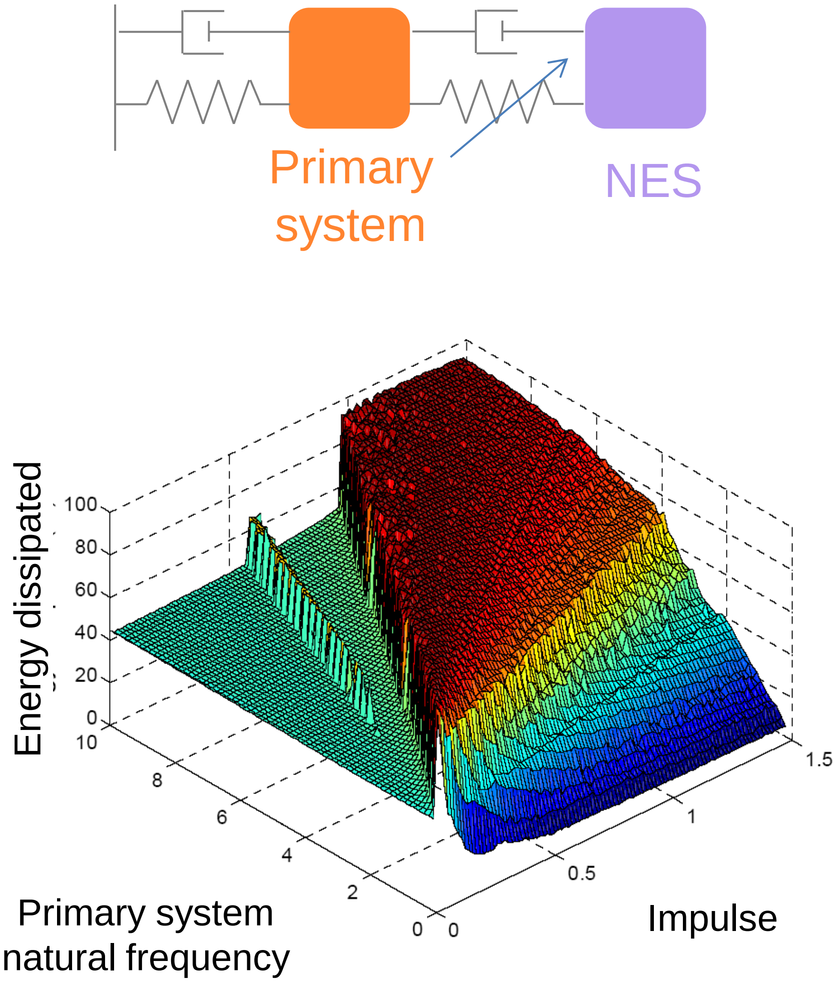
Basic Idea of the Nonlinear Energy Sink



An essential nonlinearity gives rise to an absorber which has no preferential resonance frequency (broadband device).

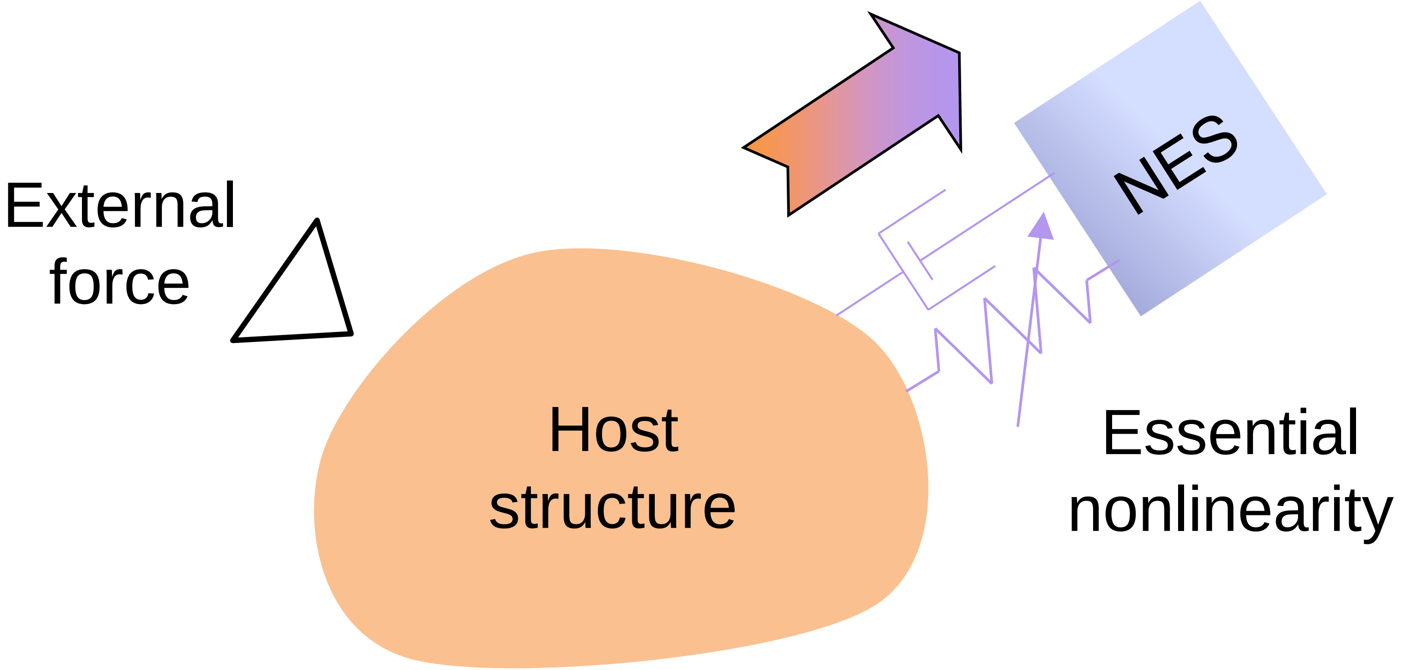
Linear TVA vs. NES: Completely Different Picture



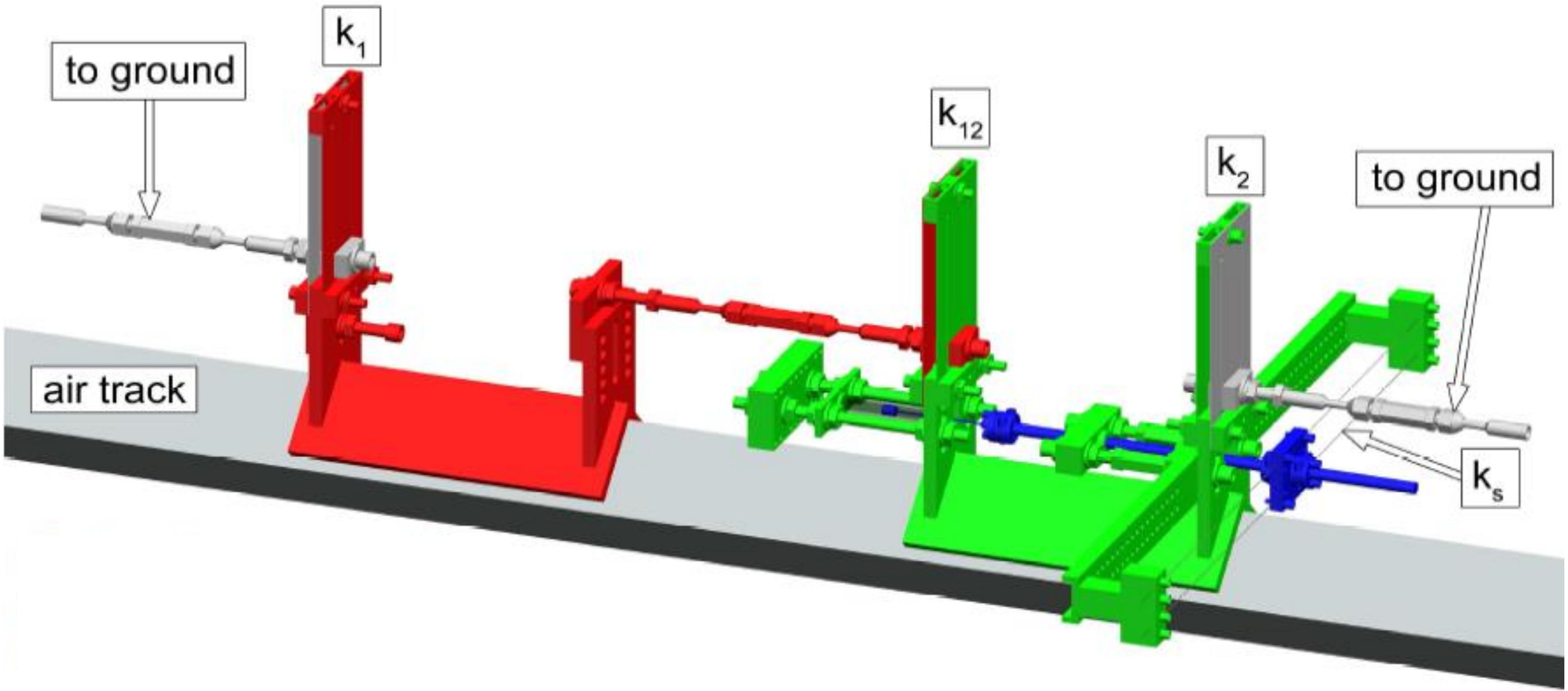


NES Has Two Salient Features

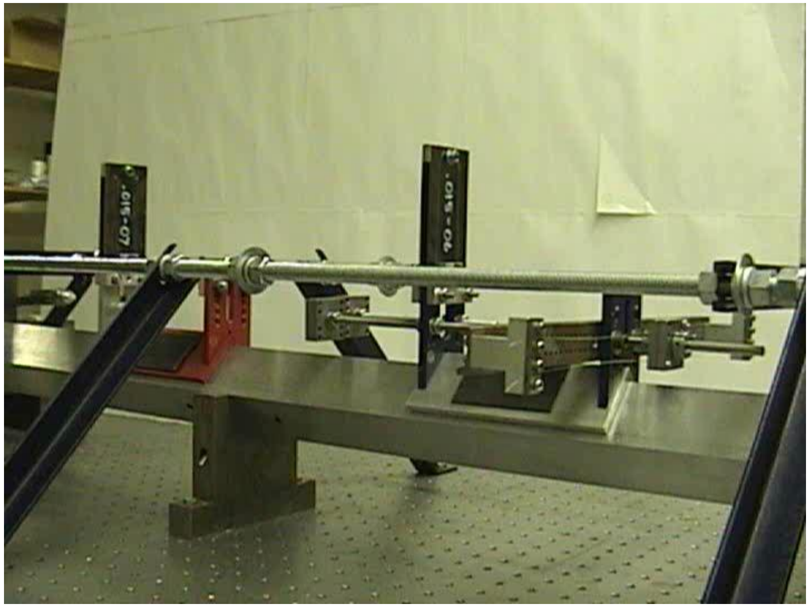
1. IRREVERSIBLE (not discussed)
2. MULTIMODAL

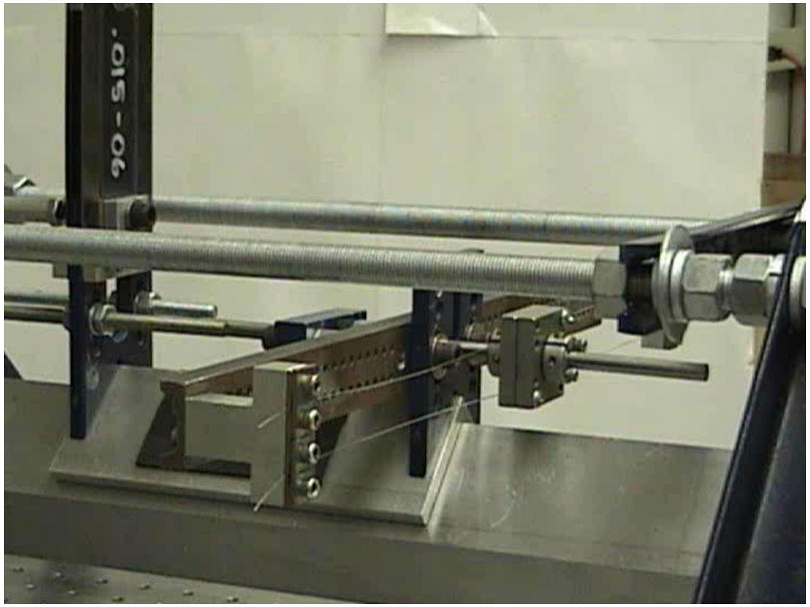


Multimodal Energy Transfer Using an NES

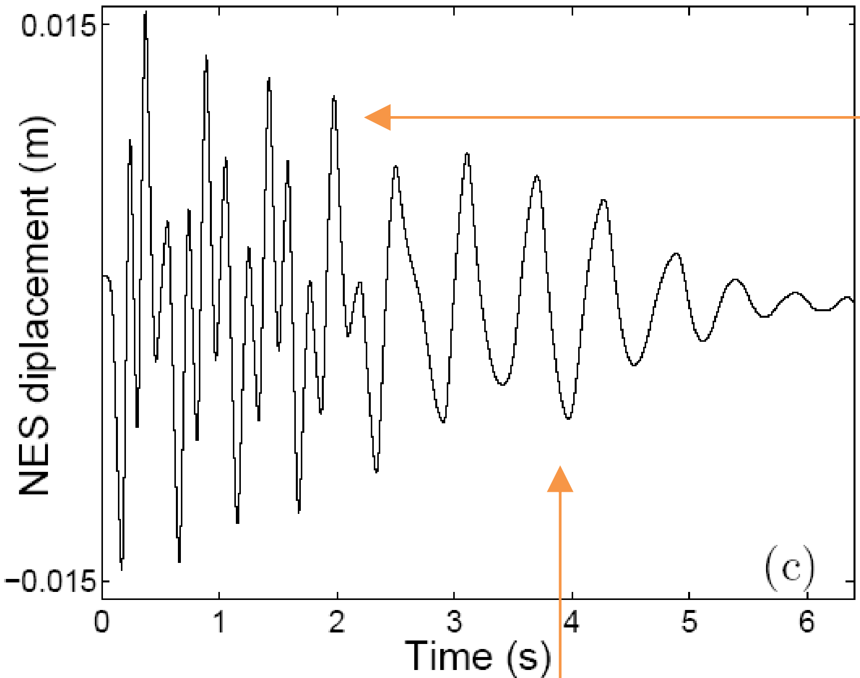


Multimodal Energy Transfer: Experimental Evidence

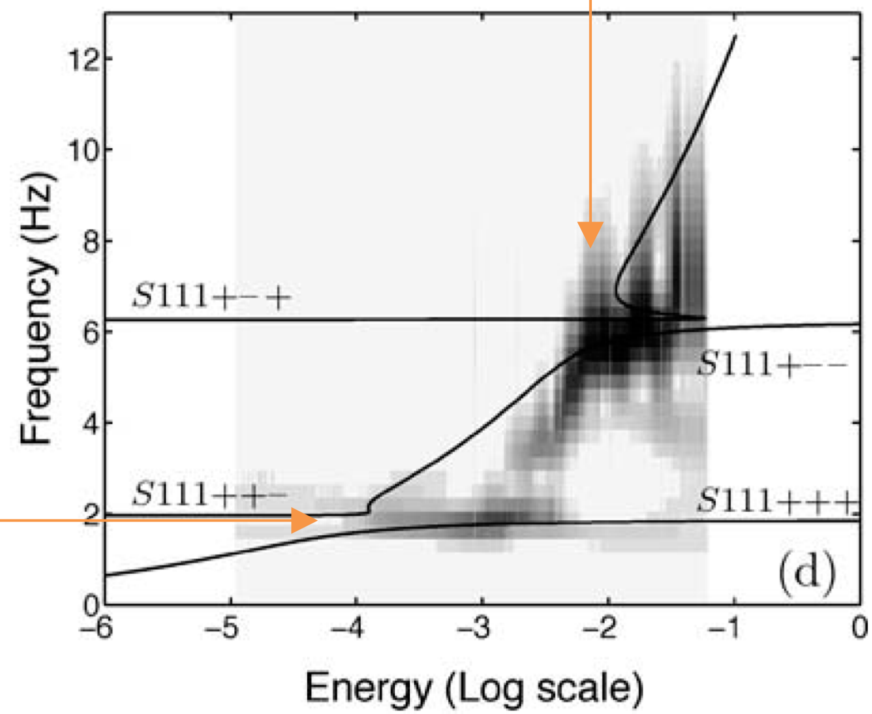




Multimodal Energy Transfer: Experimental Evidence



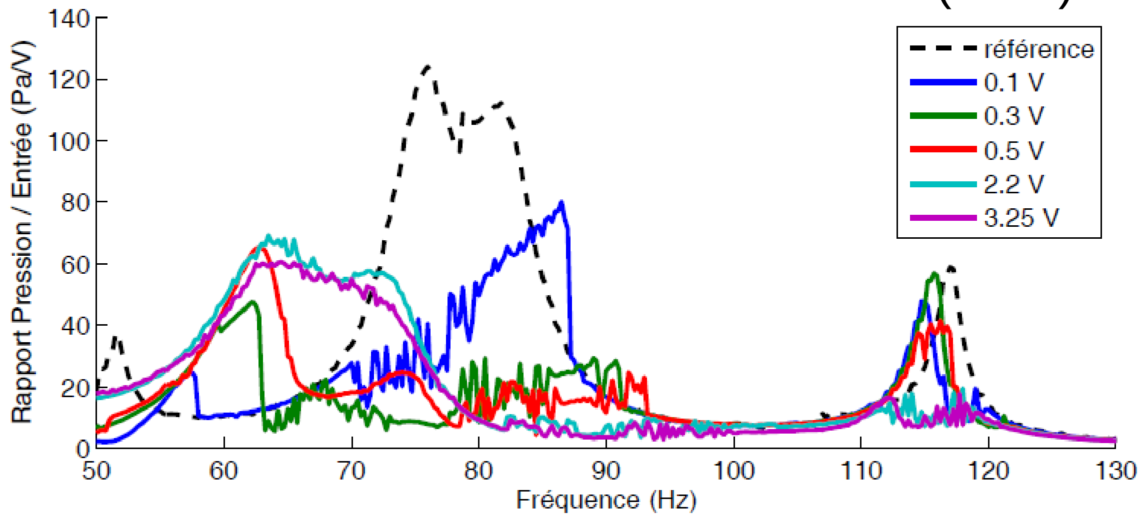
Resonance with the out-of-phase mode



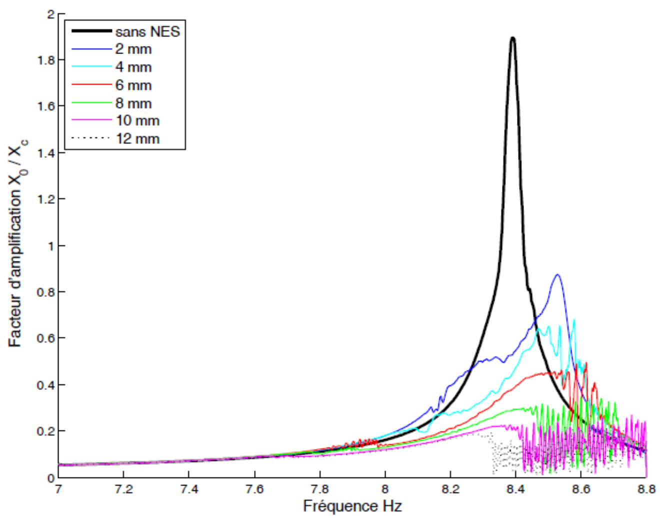
Resonance with the in-phase mode

The French Community is Very Active

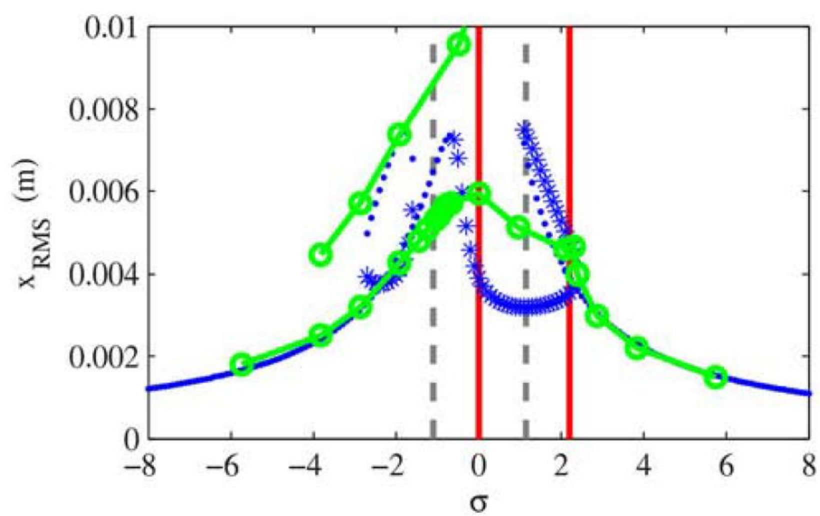
Multimodal in acoustics: Bellet et al. (ECM)



One resonance: Vaurigaud et al. (ENTPE)



Adverse dynamics: Gourc et al. (ISAE)



Summary for the NES

➕

The NES can absorb broadband disturbances.

The NES can also “cut” a resonance peak.

➖

Sensitive to the forcing amplitude (threshold).

Adverse dynamics (bifurcations, detached resonances)

The essential nonlinearity complicates the practical realization (no stiffness at rest)

Outline of the First Part

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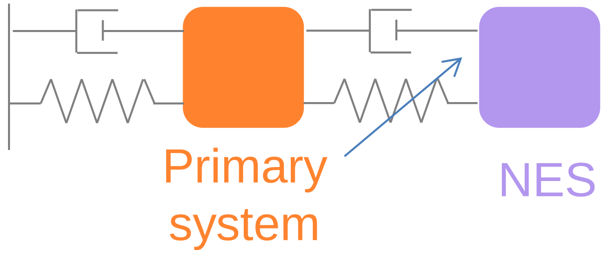
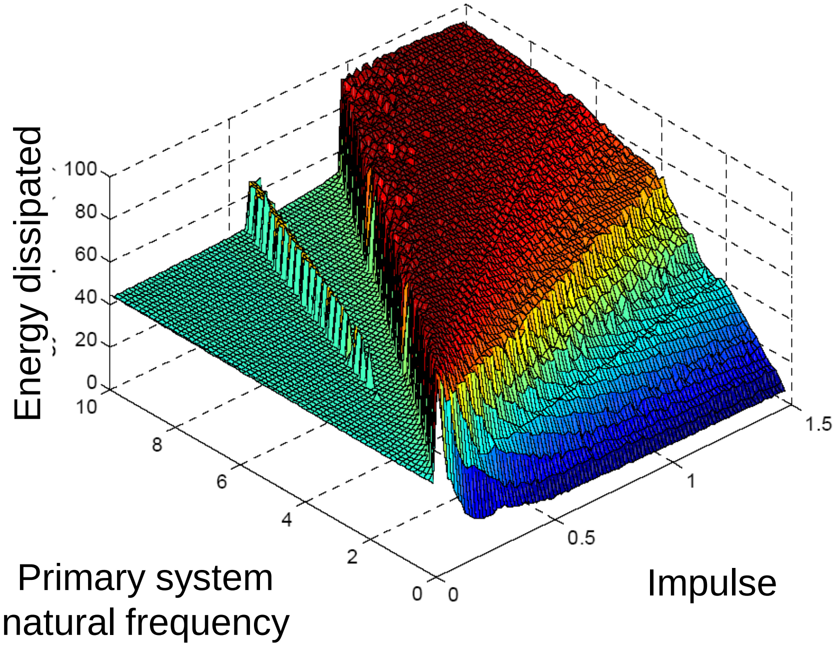
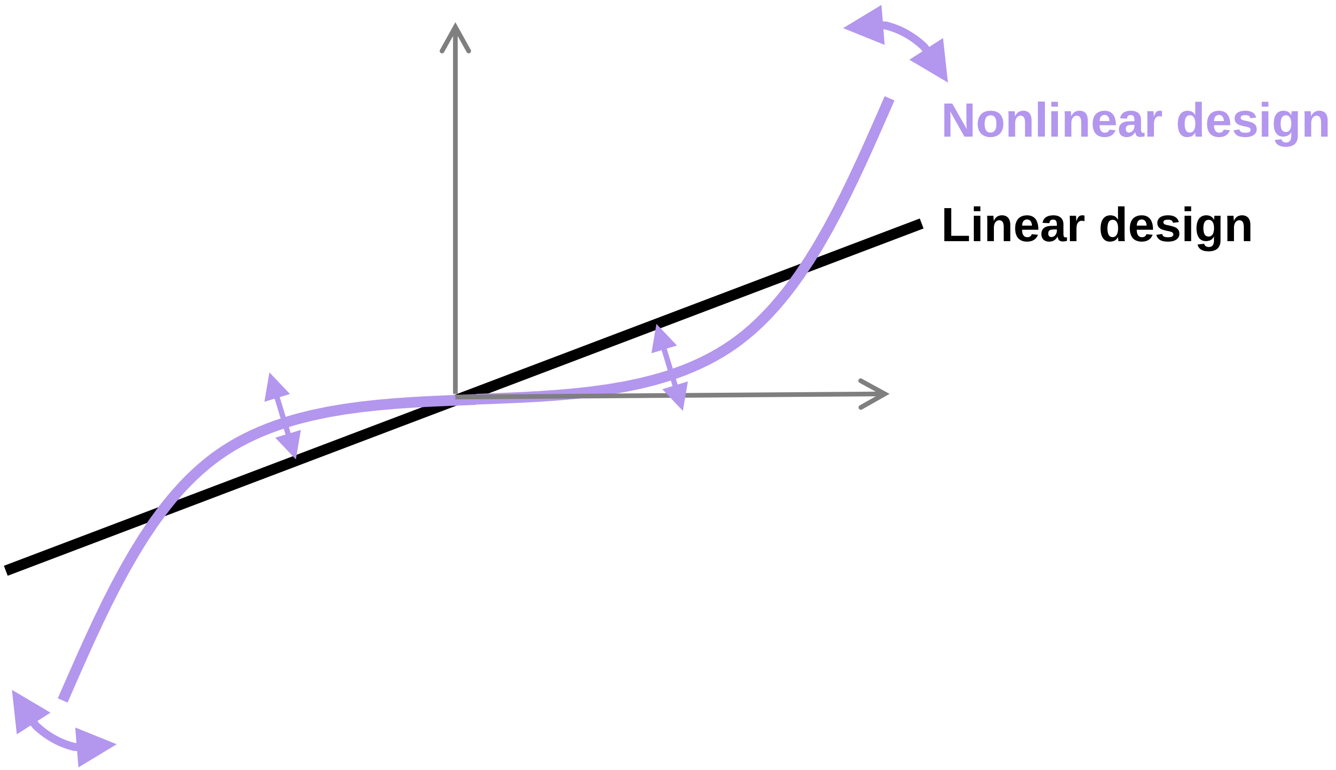
Basic Idea of the Nonlinear Tuned Vibration Absorber





Can we design a nonlinear absorber that is effective for a larger range of forcing amplitudes ?

Nonlinear Designs Are More Flexible



Exploit this additional flexibility !

- Do not assume a priori a mathematical function for the nonlinearity of the absorber.

We Synthesize the Nonlinear Restoring Force

$$m_1\ddot{x}_1 + c_1\dot{x}_1 + k_1x_1 + k_{nl1}{x_1}^3 + c_2(\dot{x}_1 - \dot{x}_2) + g(x_1 - x_2) = Fcos\omega t$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) - g(x_1 - x_2) = 0$$

$$\tau = \sqrt{\frac{k_1}{m_1}}t, \epsilon = \frac{m_2}{m_1}, \lambda = \frac{\omega_{n2}}{\omega_{n1}}, \tilde{\alpha}_3 = \frac{3k_{nl1}}{4k_1}, f = \frac{F}{k_1}, \gamma = \frac{\omega}{\omega_{n1}}$$

$$q_1(t) = \frac{x_1(t)}{f},\ q_2(t) = \frac{x_1(t) - x_2(t)}{f}$$

Transformed Equations of Motion (Exact)

$$q_1'' + 2\mu_1 q_1' + q_1 + \frac{4}{3}\tilde{\alpha}_3 f^2 {q_1}^3 + 2\mu_2 \epsilon\lambda q_2' + \lambda^2 \epsilon q_2 + \cdots$$

$$\frac{\epsilon}{m_2 {\omega_{n1}}^2} \sum_{k=2}^{\infty} \frac{f^{k-1}}{k!} \frac{d^k g}{dr^k}\bigg|_{r=0} {q_2}^k = \cos\gamma t$$

Taylor series expansion of the absorber's restoring force

$$q_2'' + 2\mu_1 q_1' + q_1 + \frac{4}{3}\tilde{\alpha}_3 f^2 {q_1}^3 + 2\mu_2 (\epsilon + 1)\lambda q_2' + \lambda^2 (\epsilon + 1) q_2 + \cdots$$

$$\frac{\epsilon + 1}{m_2 {\omega_{n1}}^2} \sum_{k=2}^{\infty} \frac{f^{k-1}}{k!} \frac{d^k g}{dr^k}\bigg|_{r=0} {q_2}^k = \cos\gamma t$$

Taylor series expansion of the absorber's restoring force

Focus on the Linear Terms: No Dependence on *f*

$$q_1'' + 2\mu_1 q_1' + q_1 + \frac{4}{3}\tilde{\alpha}_3 f^2 {q_1}^3 + 2\mu_2 \epsilon \lambda {q_2}' + \lambda^2 \epsilon q_2 + \cdots \frac{\epsilon}{m_2 {\omega_{n1}}^2} \sum_{k=2}^{\infty} \frac{f^{k-1}}{k!} \frac{d^k g}{dr^k}\bigg|_{r=0} {q_2}^k = \cos \gamma t$$

$$q_2'' + 2\mu_1 q_1' + q_1 + \frac{4}{3}\tilde{\alpha}_3 f^2 {q_1}^3 + 2\mu_2 (\epsilon + 1) \lambda {q_2}' + \lambda^2 (\epsilon + 1) q_2 + \cdots \frac{\epsilon + 1}{m_2 {\omega_{n1}}^2} \sum_{k=2}^{\infty} \frac{f^{k-1}}{k!} \frac{d^k g}{dr^k}\bigg|_{r=0} {q_2}^k = \cos \gamma t$$

Rule #1: the NLTVA should possess a linear spring to perform effectively at low forcing amplitudes (LTVA-like behavior).

Focus on the Nonlinear Terms: Dependence on *f*

$$q_1'' + 2\mu_1 q_1' + q_1 + \frac{4}{3}\tilde{\alpha}_3 f^2 {q_1}^3 + 2\mu_2 \epsilon \lambda q_2' + \lambda^2 \epsilon q_2 + \cdots$$

$$\frac{\epsilon}{m_2 {\omega_{n1}}^2} \sum_{k=2}^{\infty} \frac{f^{k-1}}{k!} \frac{d^k g}{dr^k}\bigg|_{r=0} {q_2}^k = \cos \gamma t$$

$$q_2'' + 2\mu_1 q_1' + q_1 + \frac{4}{3}\tilde{\alpha}_3 f^2 {q_1}^3 + 2\mu_2 (\epsilon + 1) \lambda q_2' + \lambda^2 (\epsilon + 1) q_2 + \cdots$$

$$\frac{\epsilon + 1}{m_2 {\omega_{n1}}^2} \sum_{k=2}^{\infty} \frac{f^{k-1}}{k!} \frac{d^k g}{dr^k}\bigg|_{r=0} {q_2}^k = \cos \gamma t$$

Coefficients of terms of order *k* depends on f^{k-1}

Proposed Tuning Rule: "Mirror" Rule

$$q_1'' + 2\mu_1 q_1' + q_1 + \frac{4}{3}\tilde{\alpha}_3 f^{2} {q_1}^3 + 2\mu_2\epsilon\lambda q_2' + \lambda^2\epsilon q_2 + \cdots$$

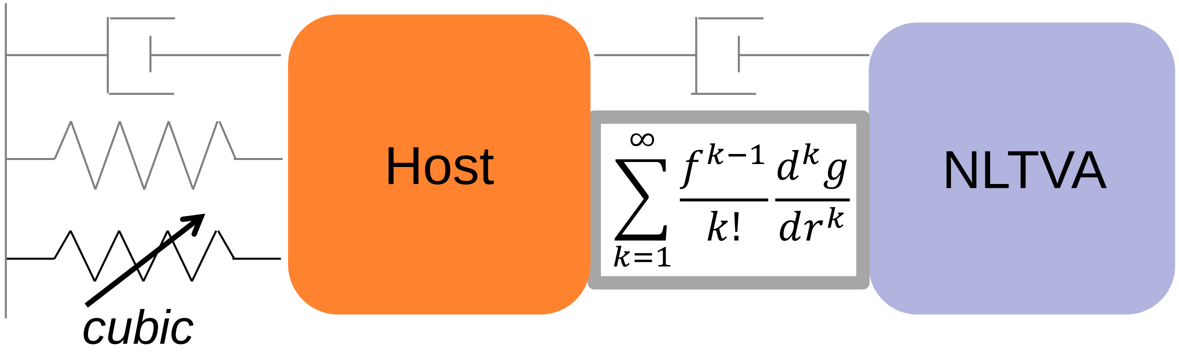
$$\frac{\epsilon}{m_2 {\omega_{n1}}^2}\sum_{k=2}^{\infty}\frac{f^{k-1}}{k!}\frac{d^k g}{dr^k}\bigg|_{r=0} {q_2}^k = \cos\gamma t$$

$$q_2'' + 2\mu_1 q_1' + q_1 + \frac{4}{3}\tilde{\alpha}_3 f^{2} {q_1}^3 + 2\mu_2(\epsilon+1)\lambda q_2' + \lambda^2(\epsilon+1) q_2 + \cdots$$

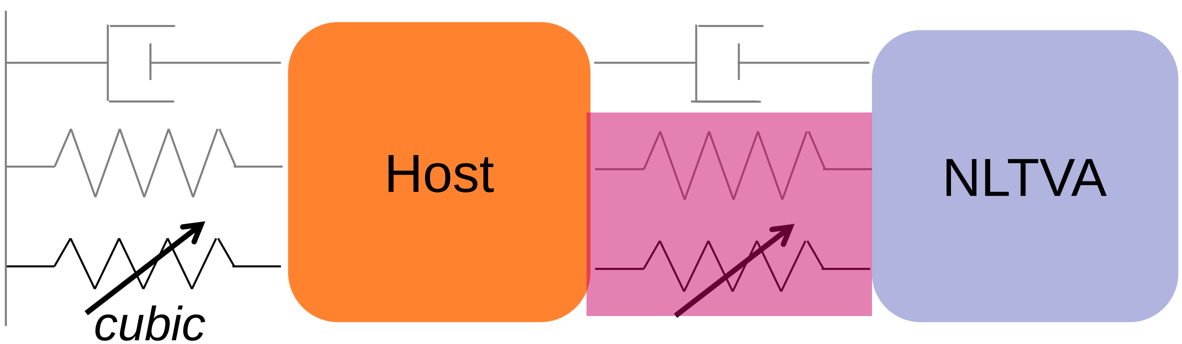
$$\frac{\epsilon+1}{m_2 {\omega_{n1}}^2}\sum_{k=2}^{\infty}\frac{f^{k-1}}{k!}\frac{d^k g}{dr^k}\bigg|_{r=0} {q_2}^k = \cos\gamma t$$

Rule #2: the NLTVA has a variation with forcing amplitude similar to that of the host system if its restoring force has the same mathematical form as that of the primary system (k=3).

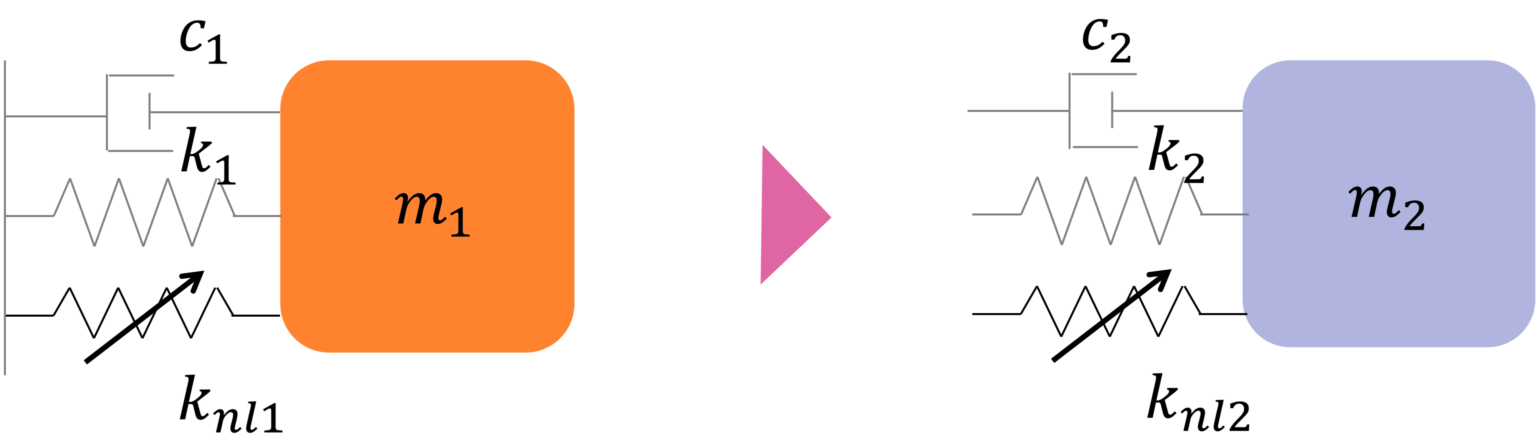
A Cubic NLTVA Should Be Coupled to a Cubic Host



Rules #1,2 → k=1 & 3



Analytic Design Formulas for Nonlinear Equal Peaks

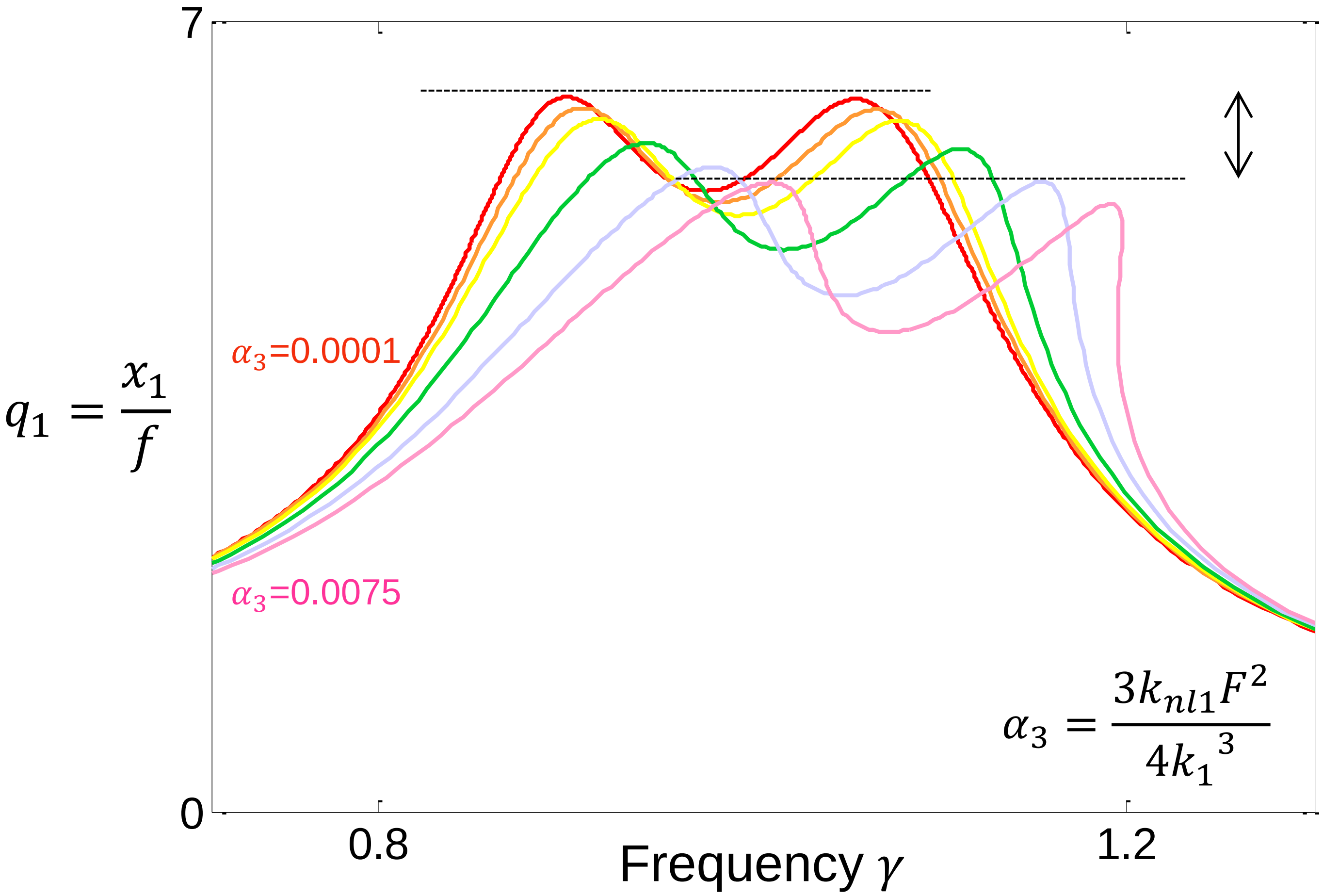


$$m_2 = \epsilon m_1, \quad c_2 = 2\sqrt{\frac{3}{8}\frac{\epsilon^3 k_1 m_1}{(1+\epsilon)^3}}$$

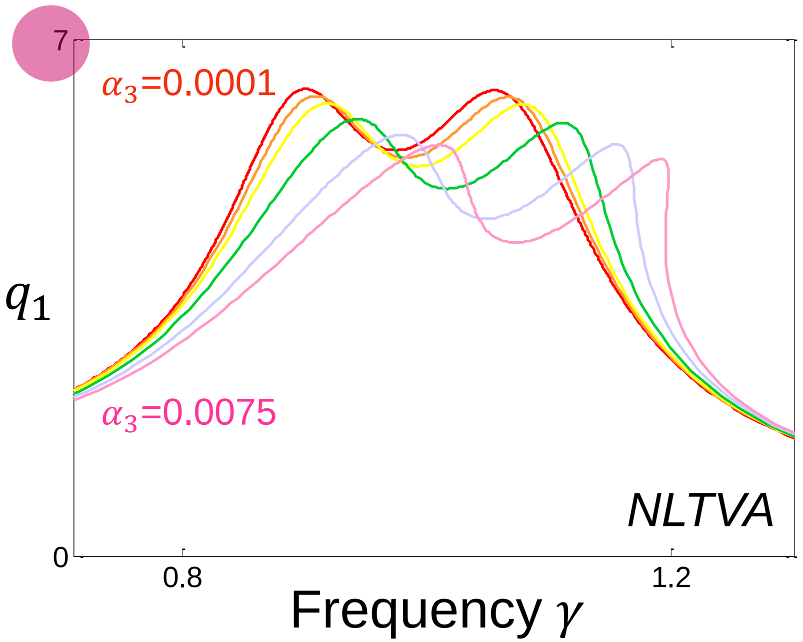
$$k_2 = \frac{\epsilon k_1}{(1+\epsilon)^2}, \quad k_{nl2} = \frac{2\epsilon^2 k_{nl1}}{1+4\epsilon}$$

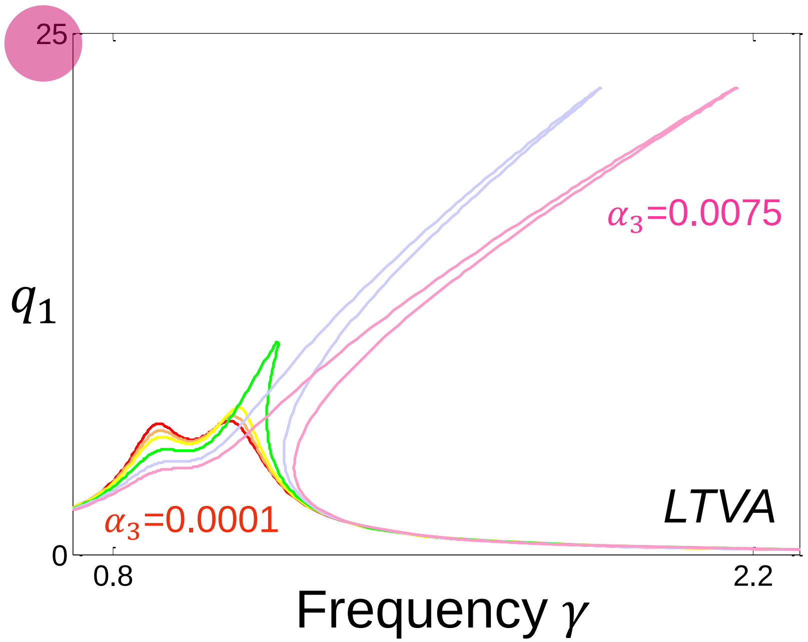
Nonlinear generalization of
Den Hartog's equal-peak method

The NLTVA Performs According to Plan

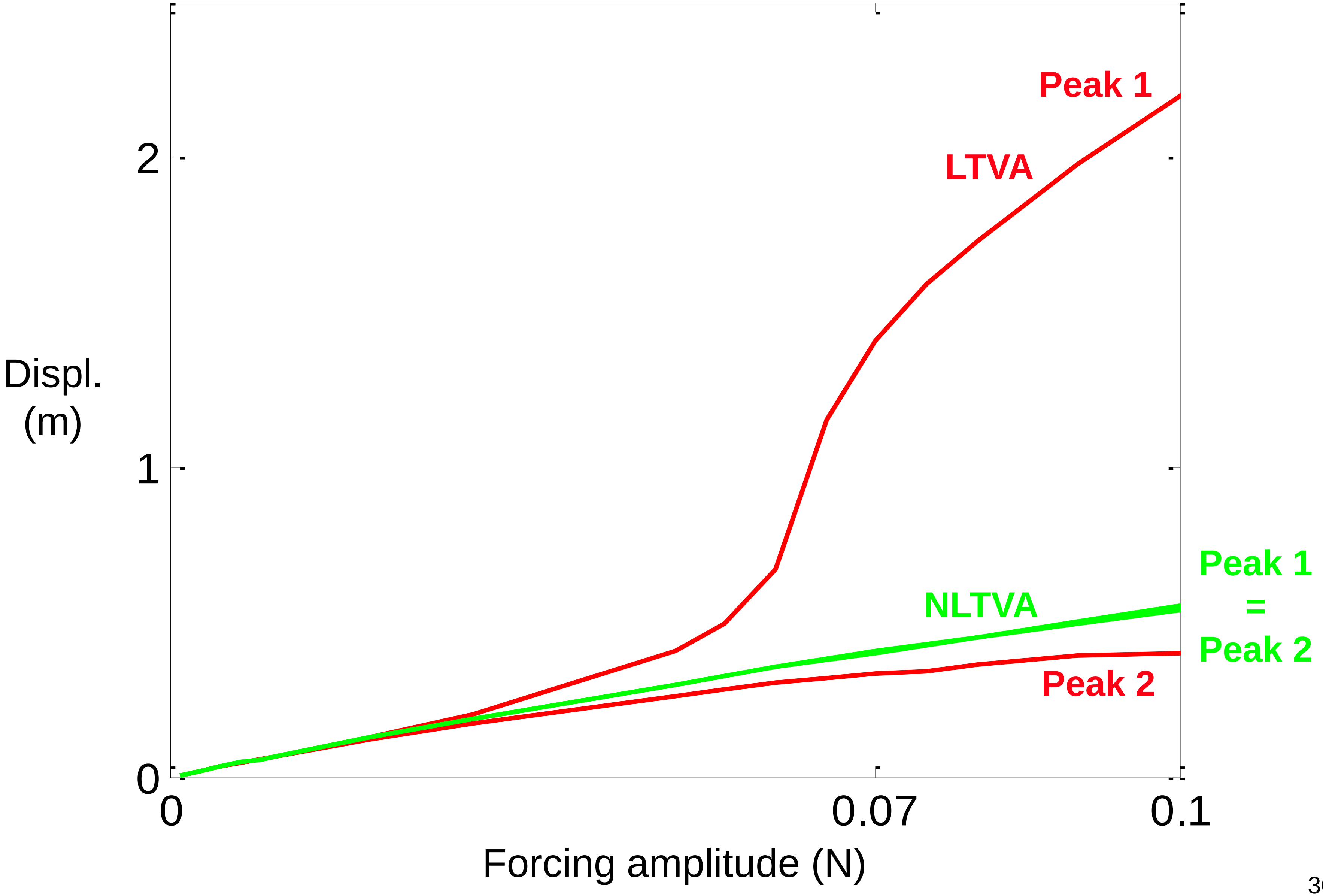


LTVA Completely Detuned for the Same Regimes

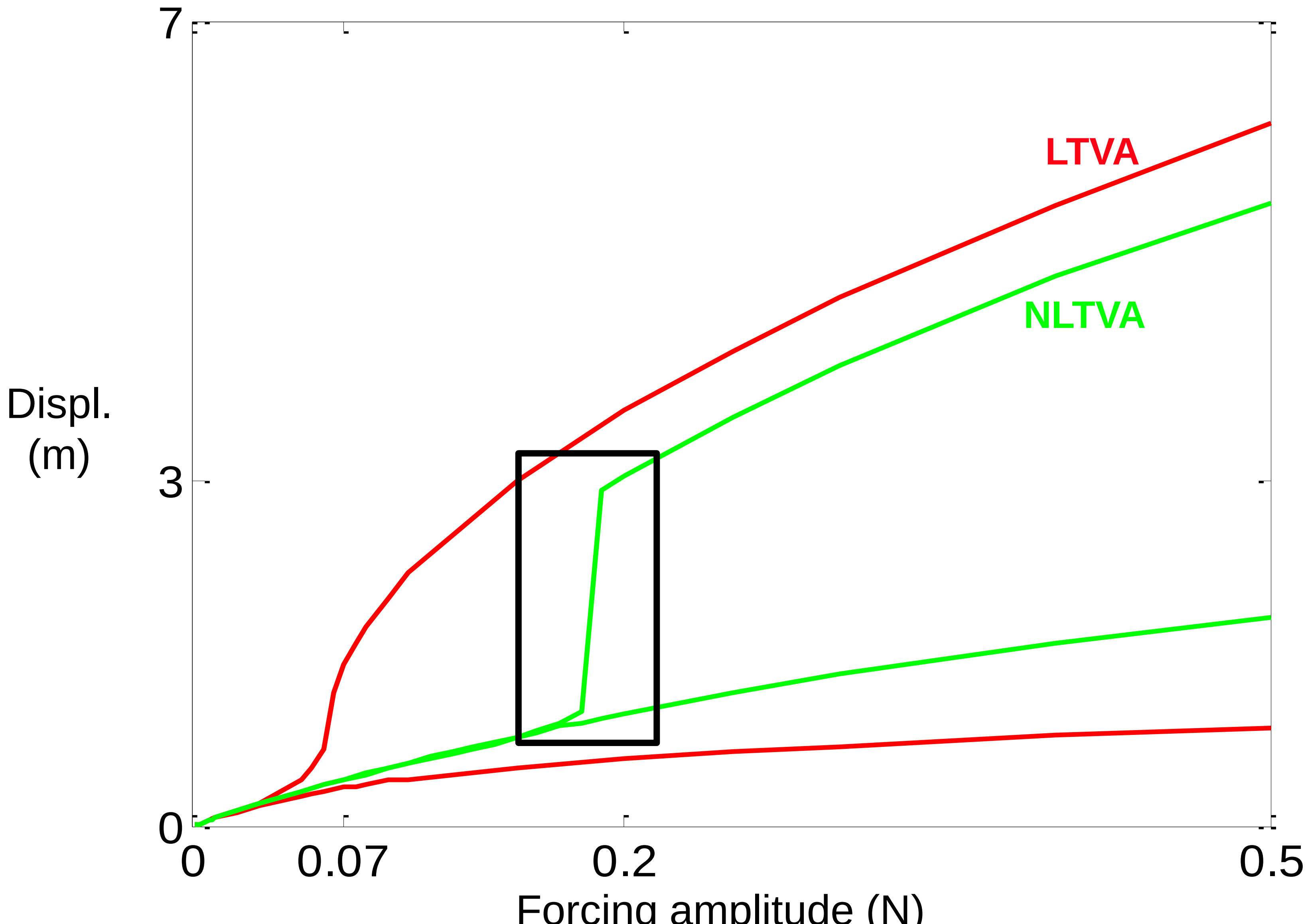




The NLTVA Always Outperforms the LTVA

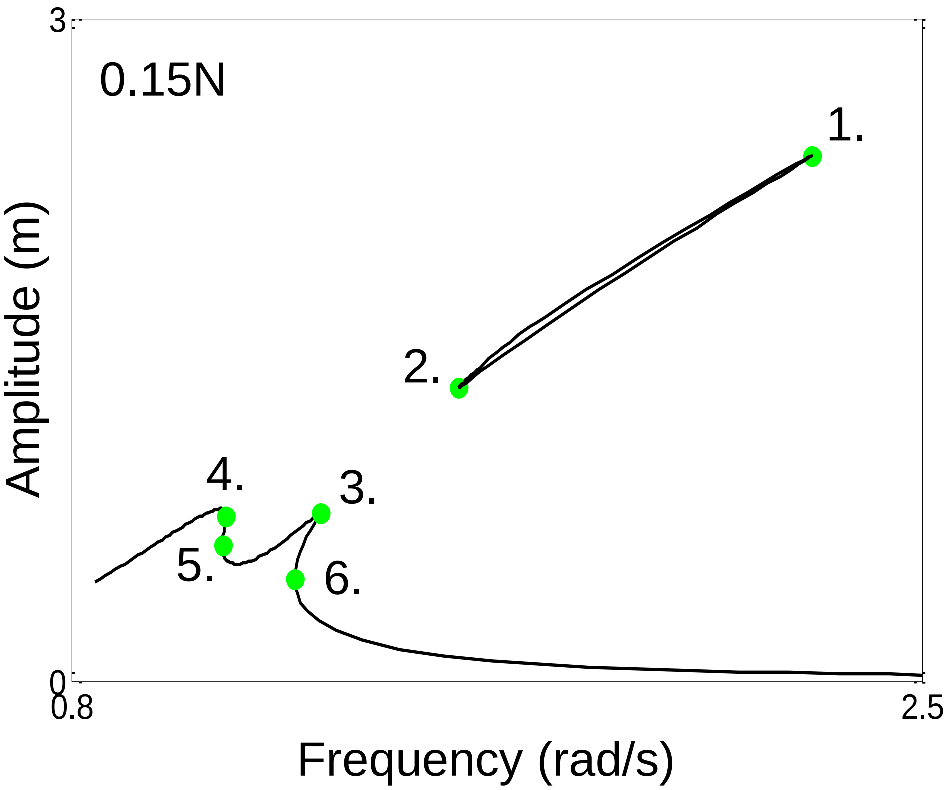


NLTVA Detuning for Larger Amplitudes

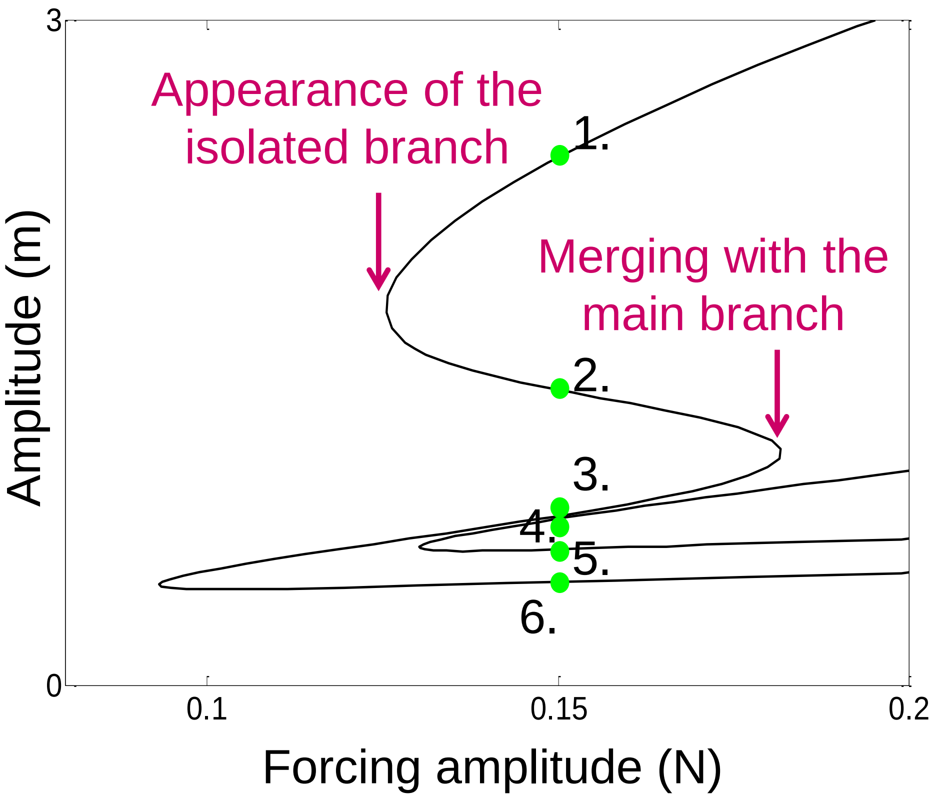


Detuning Due to an Isolated Branch of Solutions

Frequency response of the host structure



Bifurcation tracking: limit points locus



Summary for the NLTVA

+ The NLTVA is much more robust with respect to forcing amplitudes.
+ The freedom offered by nonlinearity is fully exploited.
+ The NLTVA exhibits a linear-like behavior (up to a certain point).

− Adverse dynamics (bifurcations, detached resonances)
− How to realize the tailored nonlinearity in practice (topology optimization and piezo shunting) ?

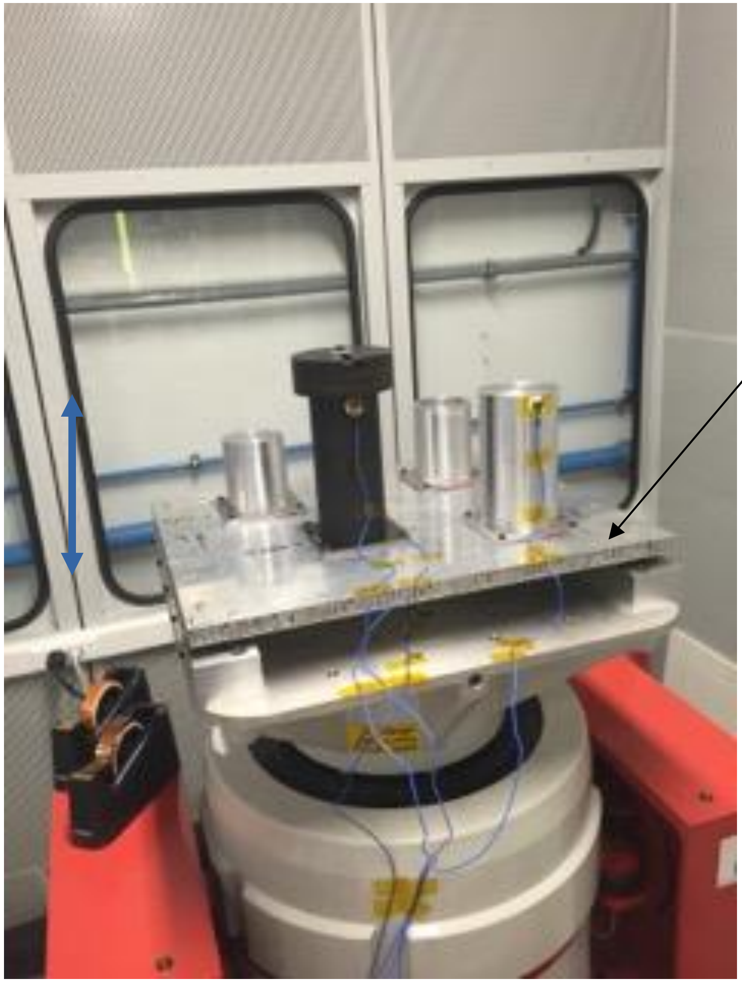
Outline for the Second Part

1. Mitigation of an uncertain resonance

2. Mitigation of limit cycle oscillations

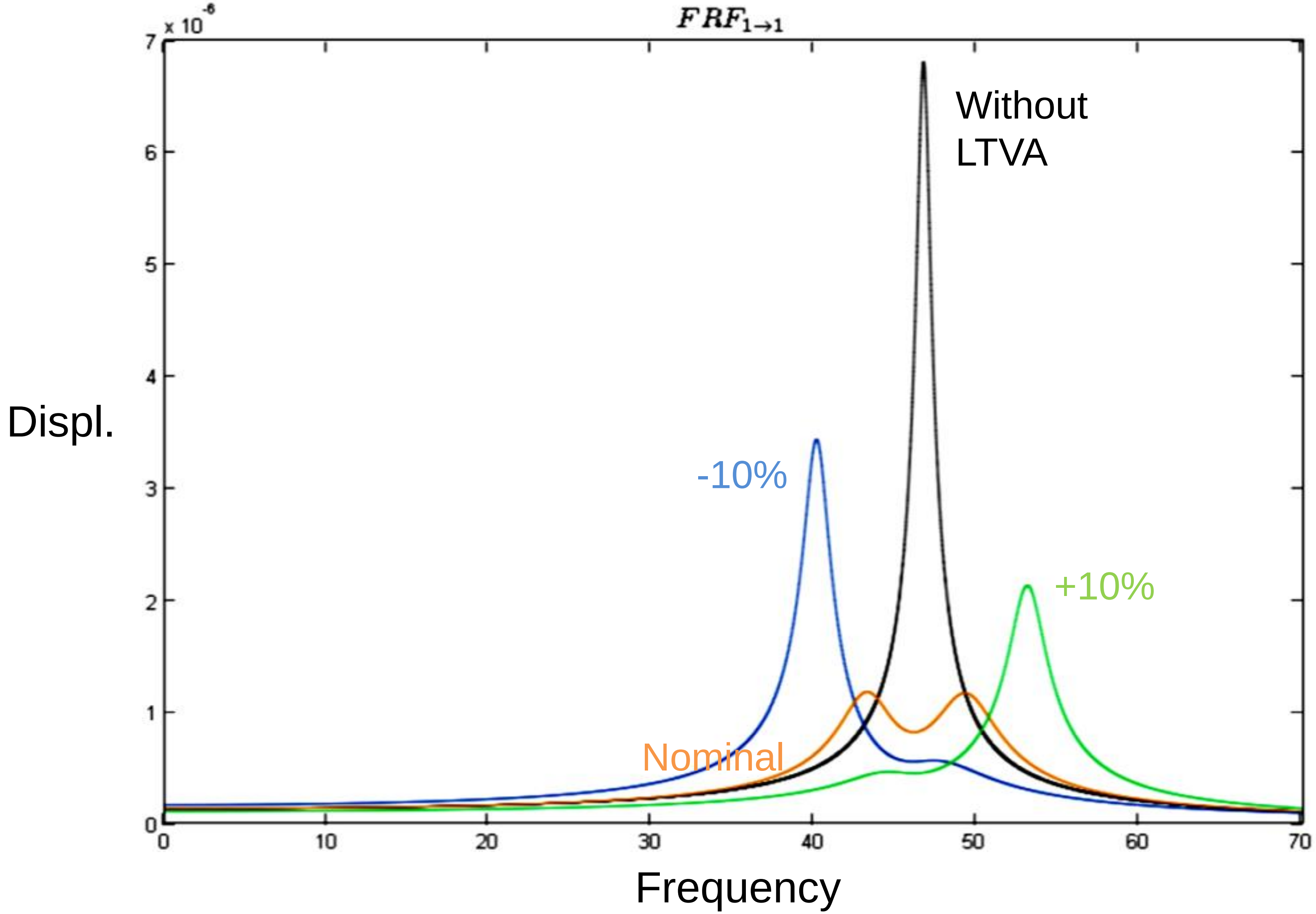
Satellite Panel with Uncertain Characteristics

Objective: passive control of the first bending mode



Aluminium sandwich panel: 50 Hz ± 5 Hz

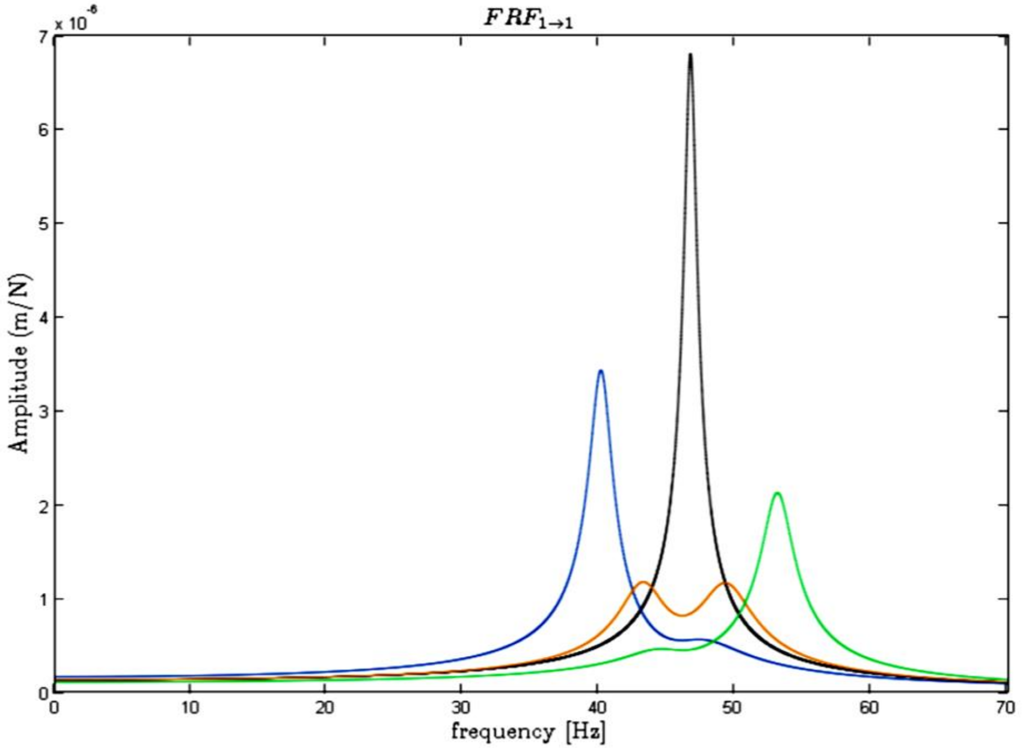
The LTVA Is Detuned For an Uncertain Resonance



Let's First Approach the Problem Numerically

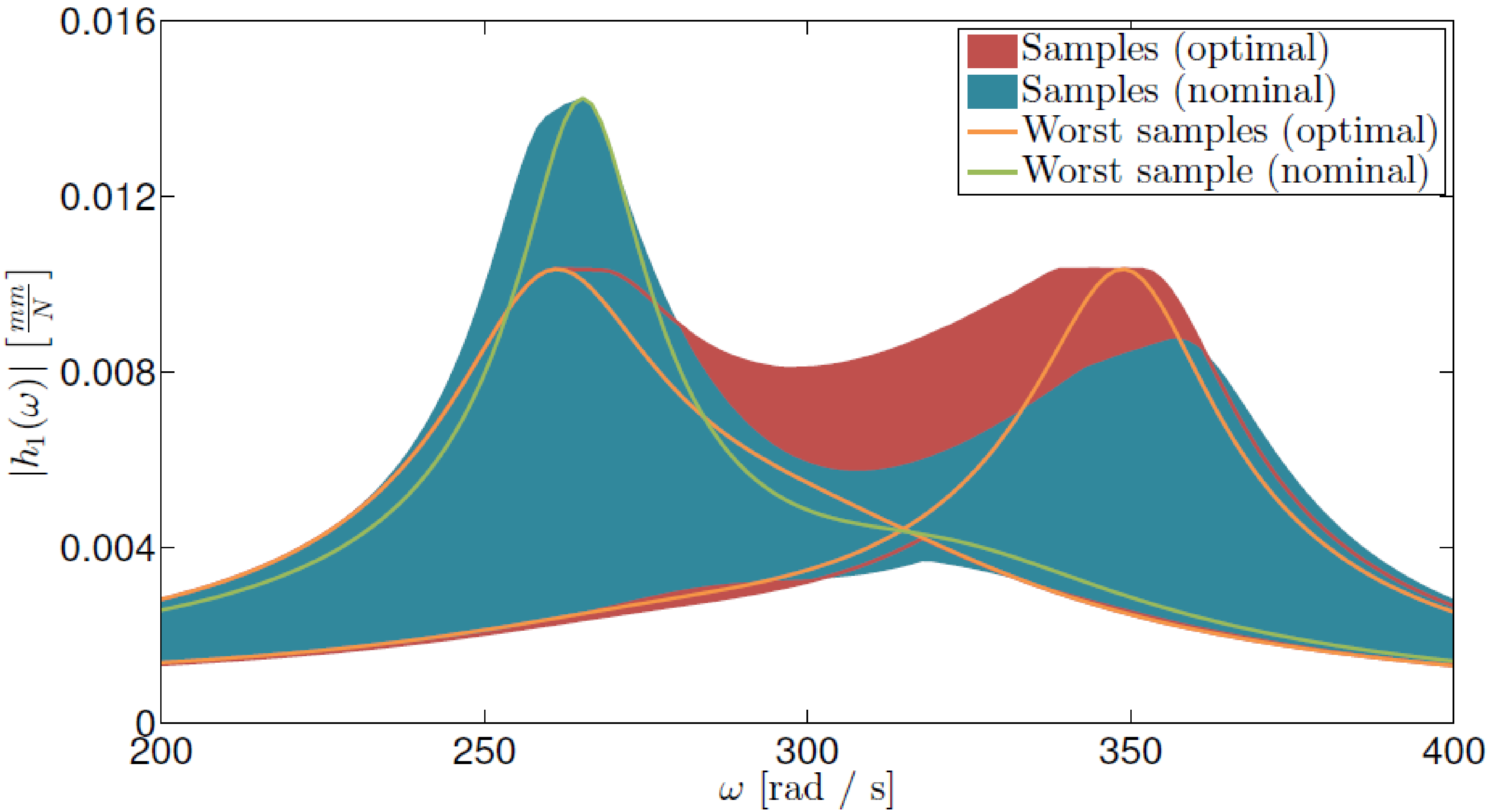
Worst-case formulation of the problem:

$$[c_2^*, k_2^*] = \arg\left[\min_{c_2, k_2 \in \mathbb{R}^+}\left(\max_{[c_1, k_1] \in \Delta} \left|h_1(\omega | m_1, c_1, k_1, m_2, c_2, k_2)\right|_\infty\right)\right]$$

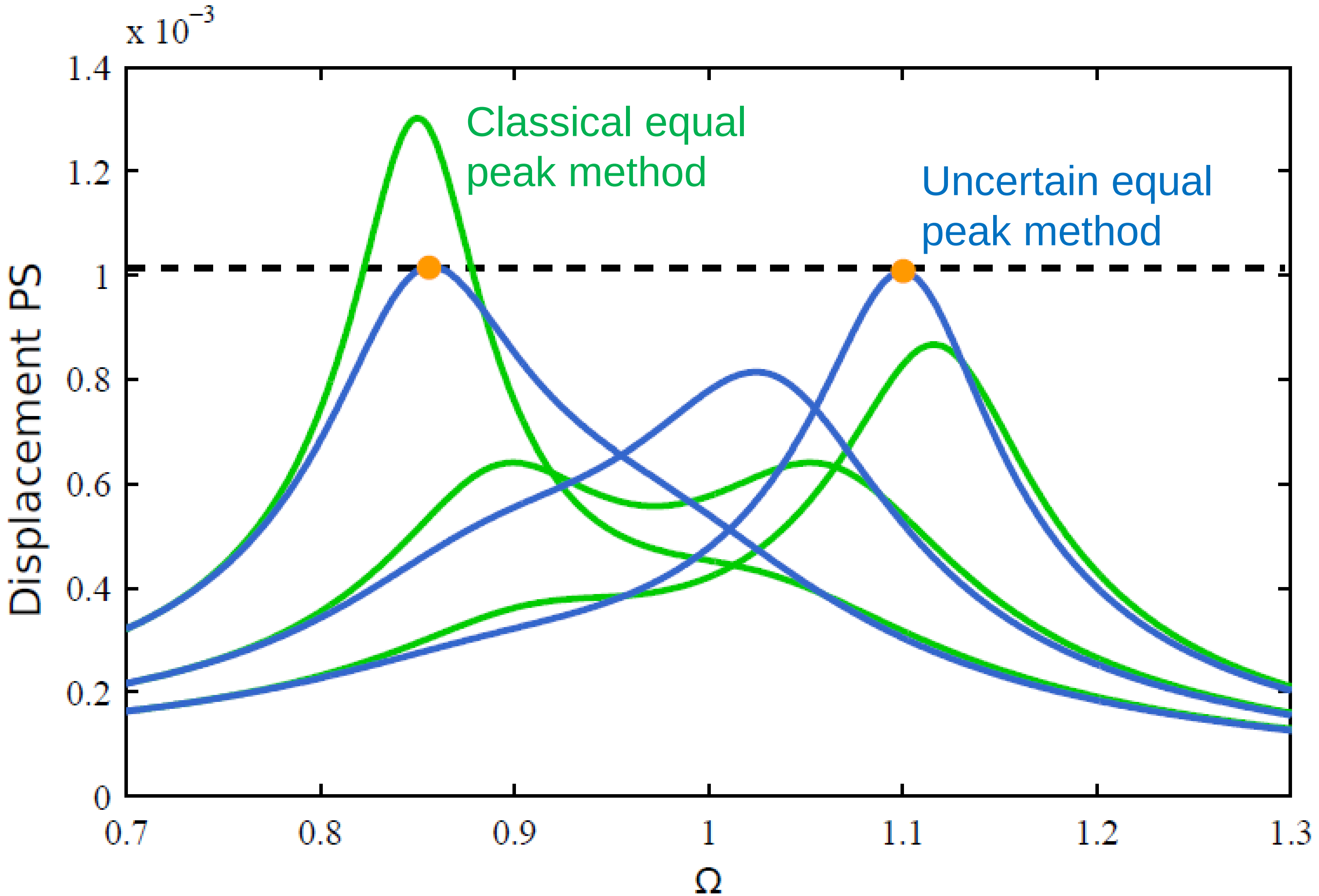


Worst Case Sample (Nominal and Optimal Cases)

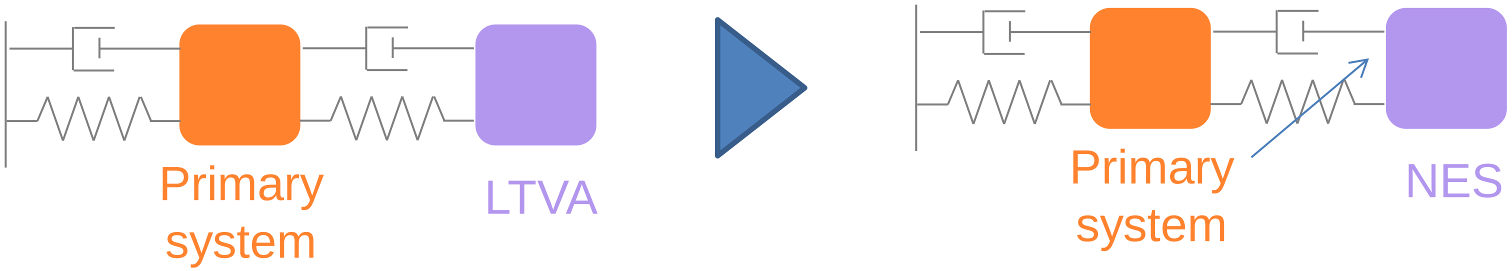
We also have equal peaks in the uncertain case !



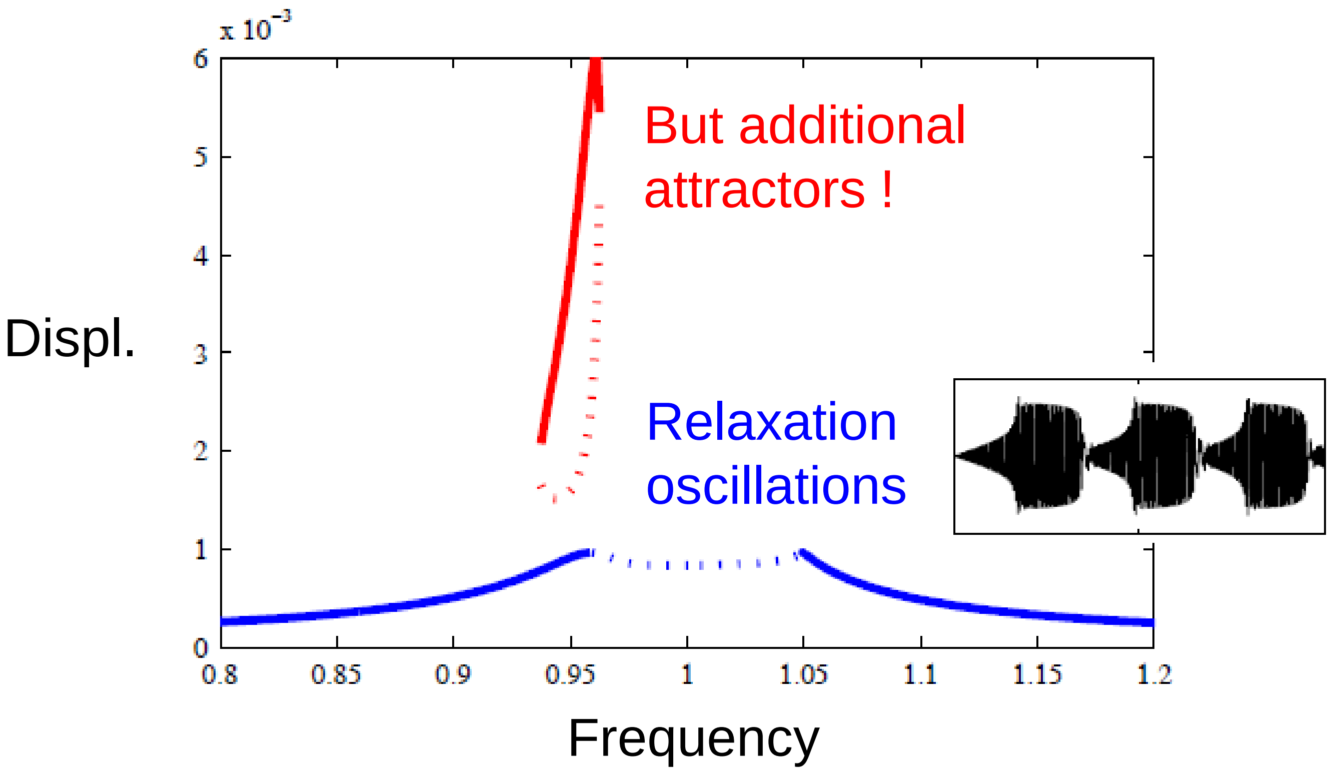
30% Improvement Brought by the Uncertain EPM



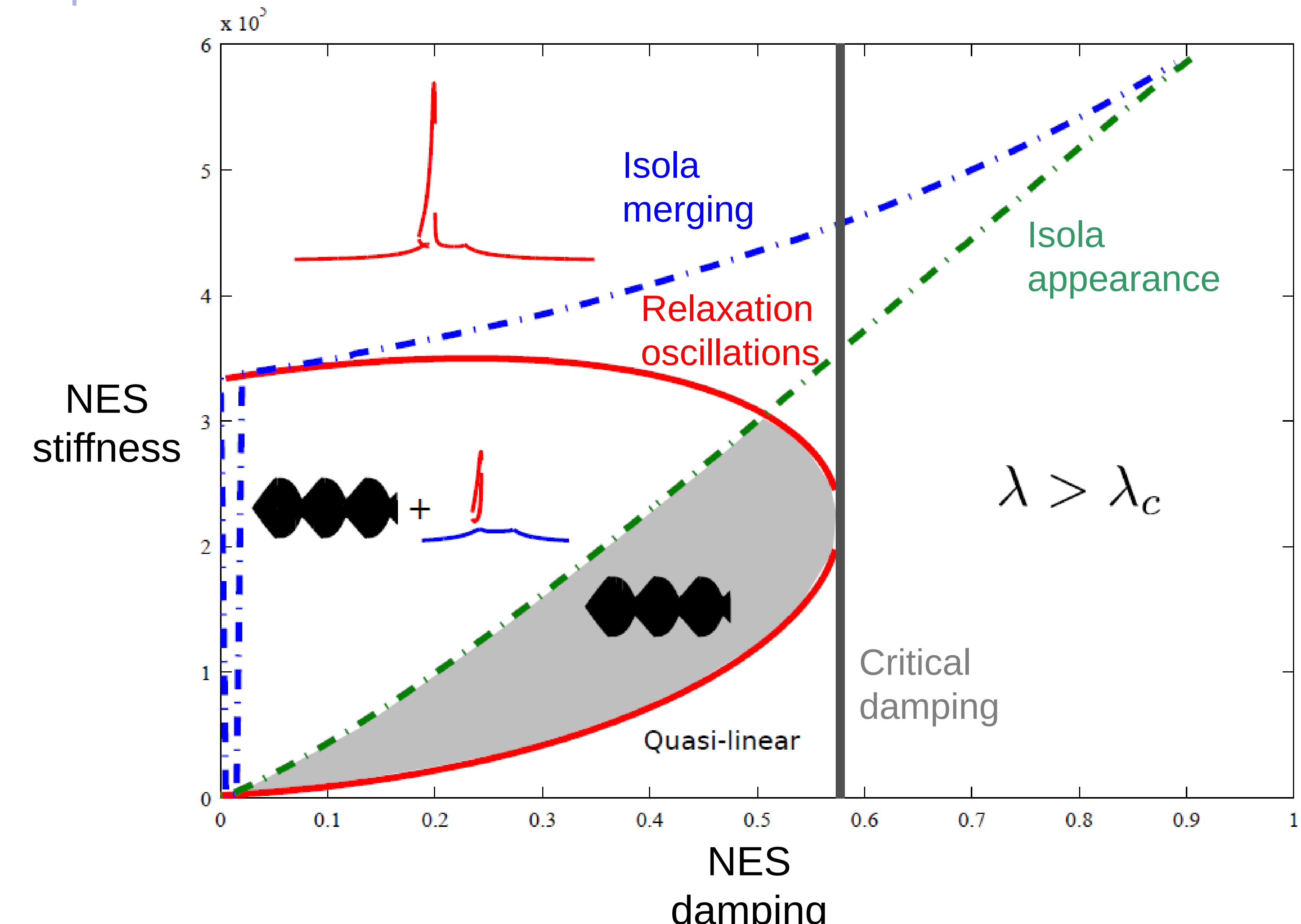
Let's Move to the Nonlinear Absorber



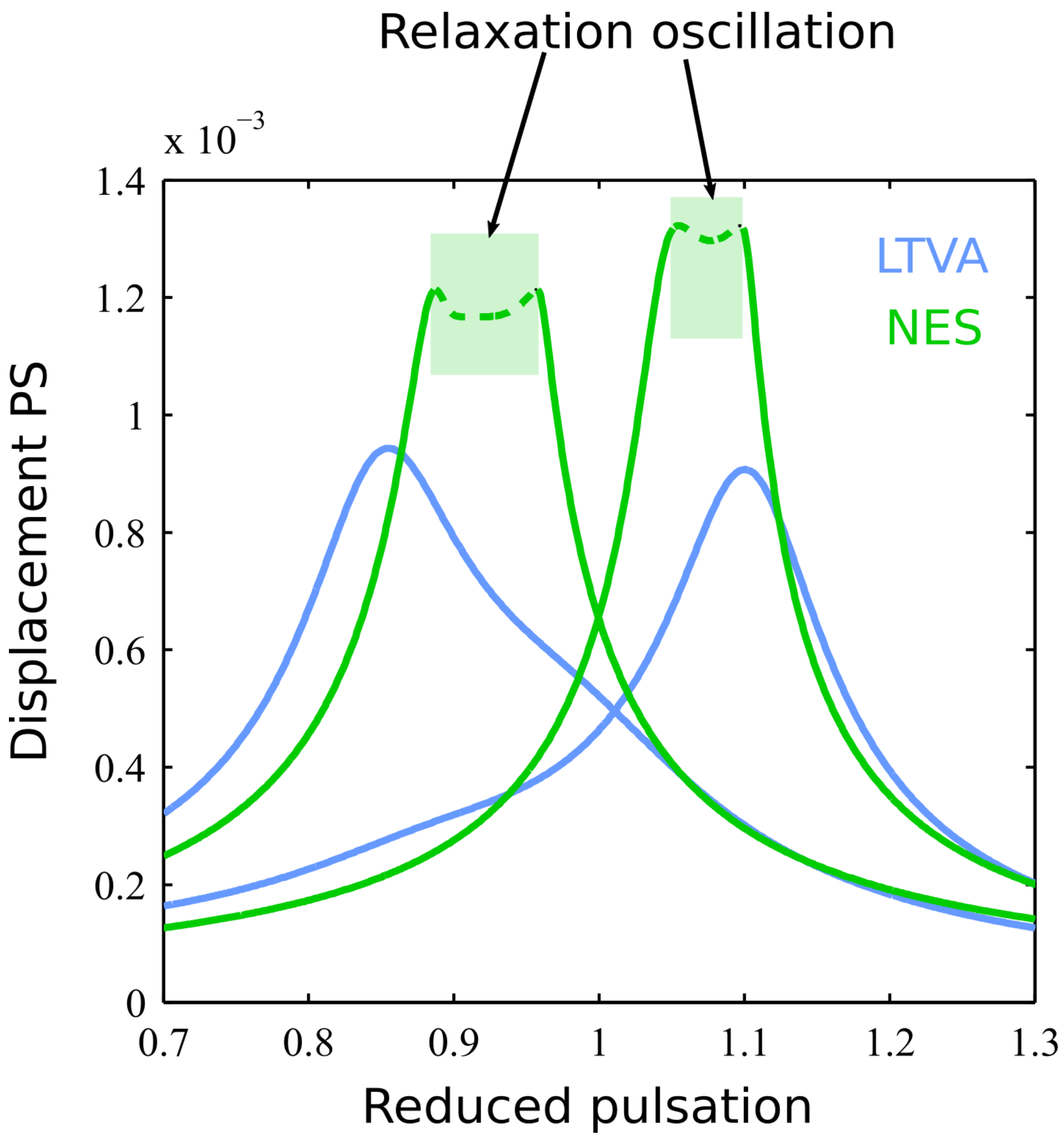
Completely different mechanism for resonance mitigation:



The Tuning Graph (Multiple Scales and HB)



LTVA Has Better Performance than NES (and NLTVA)



Outline for the Second Part

1. Mitigation of an uncertain resonance

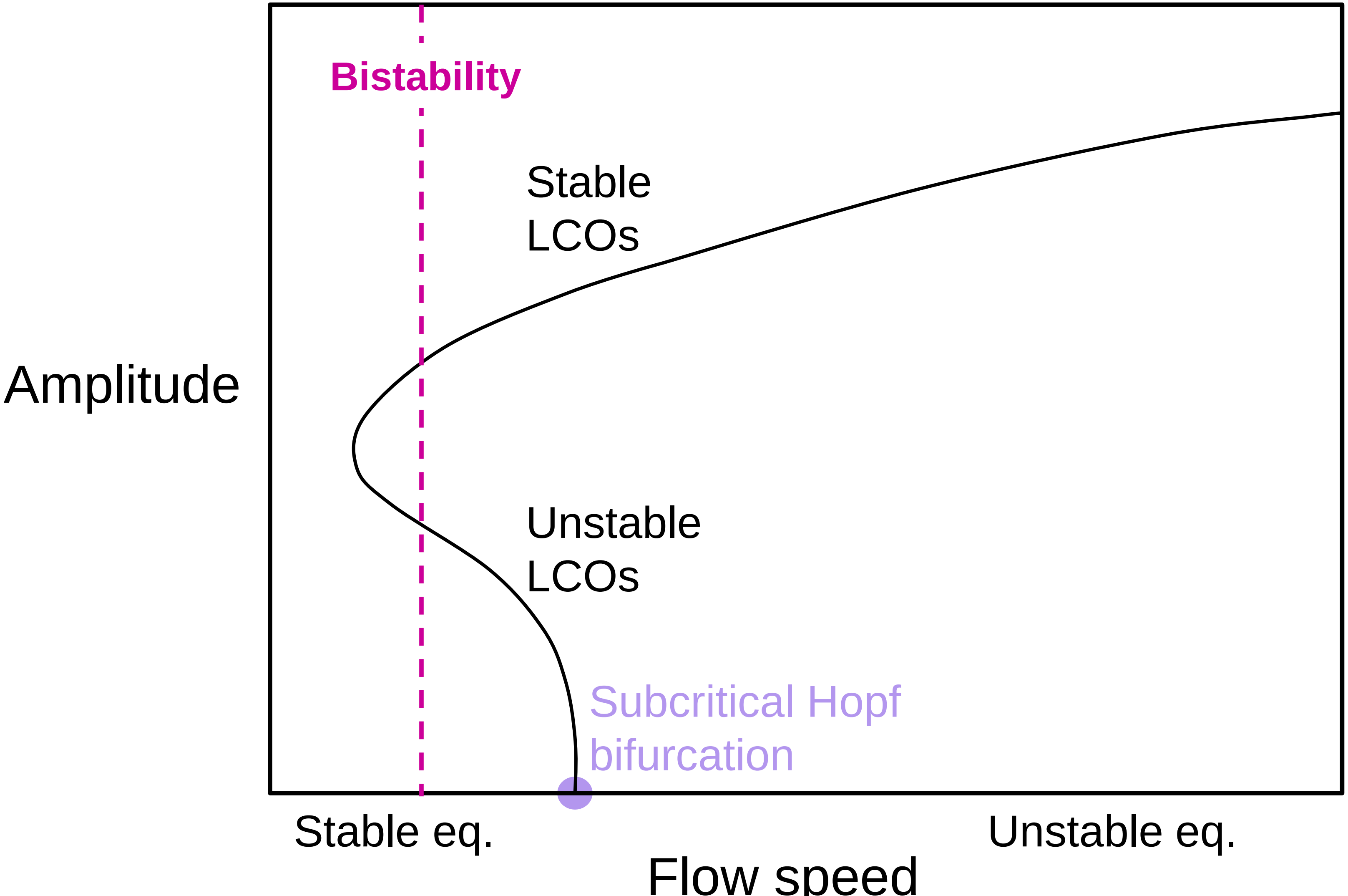
2. Mitigation of limit cycle oscillations

Suppression of Limit Cycle Oscillations

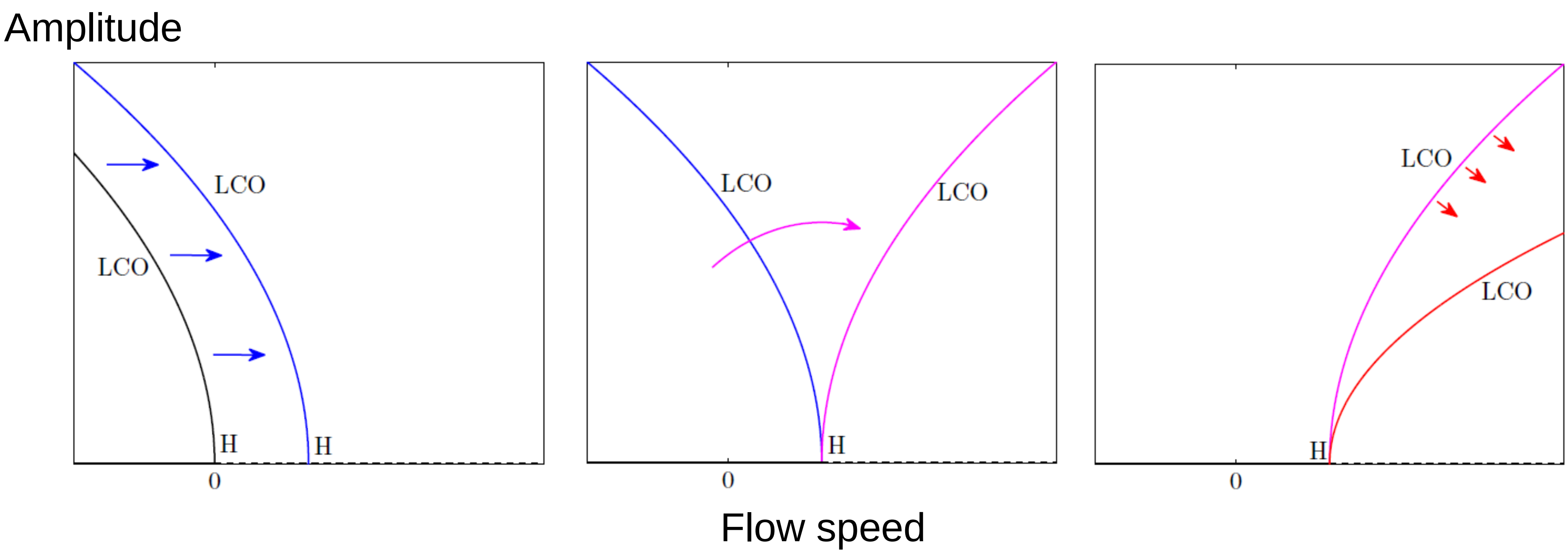
Automotive disc brakes, machine tools, drill-string systems.

F-16 aircraft

A Particularly Dangerous Situation



How Can We Design a Nonlinear Passive Absorber ?



1. Enlarge stable region
   - Design the linear spring/damper
2. Avoid bistability
   - Design the nonlinear spring
3. Reduce LCO amplitude
   - No freedom left

Van der Pol-Duffing Oscillator

A paradigmatic model for self-excited oscillations:

Fluid-structure interaction

Structural nonlinearity

$$m_1 q_1'' + c_1 \left(q_1^2 - 1\right) q_1' + k_1 q_1 + k_{nl1} q_1^3 = 0$$

We attach a NLTVA where t*he nonlinearity is not specified a priori*:

Coupling terms

$$m_1 q_1'' + c_1 \left(q_1^2 - 1\right) q_1' + k_1 q_1 + k_{nl1} q_1^3 + c_2 \left(q_1' - q_2'\right) + k_2 \left(q_1 - q_2\right) + g \left(q_1 - q_2\right) = 0$$

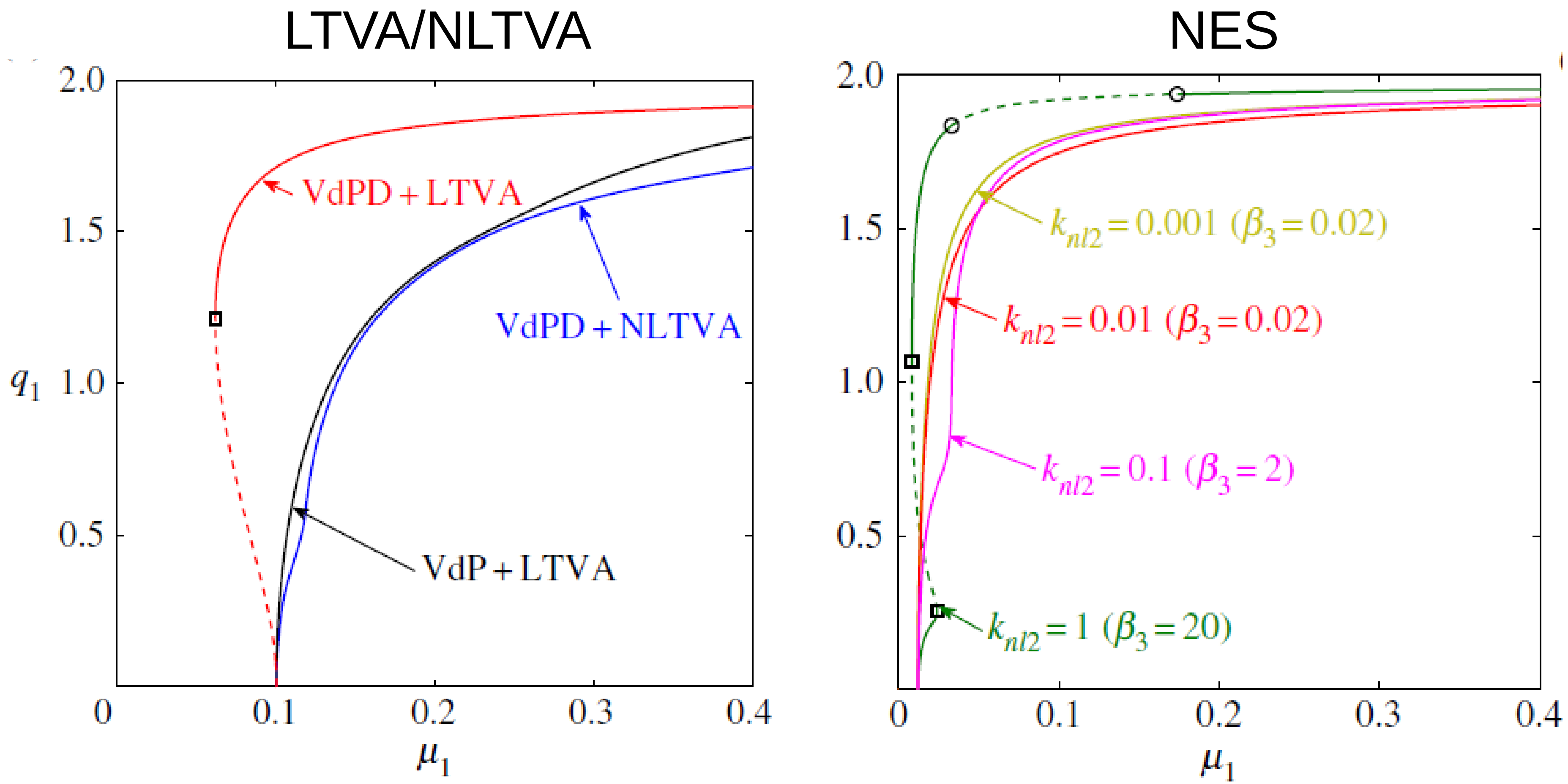
$$m_2 q_2'' + c_2 \left(q_2' - q_1'\right) + k_2 \left(q_2 - q_1\right) - g \left(q_1 - q_2\right) = 0$$

Additional equation

Proposed Tuning Rule for the NLTVA

Optimal stiffness	$$\frac{1}{\sqrt{1+\varepsilon}}$$	STABILITY ANALYSIS
Optimal damping	$$\frac{1}{2}\sqrt{\frac{\varepsilon}{1+\varepsilon}}$$	STABILITY ANALYSIS
Optimal exponent	Same as host system	PRINCIPLE OF SIMILARITY
Optimal NL coeff.	$$\frac{\varepsilon}{(1+\varepsilon)^2}k_{nl1}$$	BIFURCATION ANALYSIS

The NLTVA Outperforms the LTVA (and the NES)



In Summary

There is no optimal absorber, application-dependent.

Hard to beat the LTVA for a narrow-band excitation applied to a linear system.

Nonlinear absorbers should be considered for multimodal damping or for a nonlinear primary system. But:

- Adverse dynamics (quasiperiodic regimes and detached resonances) should be managed properly.
- The practical realization of the desired nonlinearity can be a challenge.

Some References

Robust LTVA:

Performance comparison between a nonlinear energy sink and a linear tuned vibration absorber for broadband control, E. Gourc, L. Dell'Elce, G. Kerschen, G. Michon, G. Aridon, A. Hot, 34th International Modal Analysis Conference, Orlando, USA (2016).

NES:

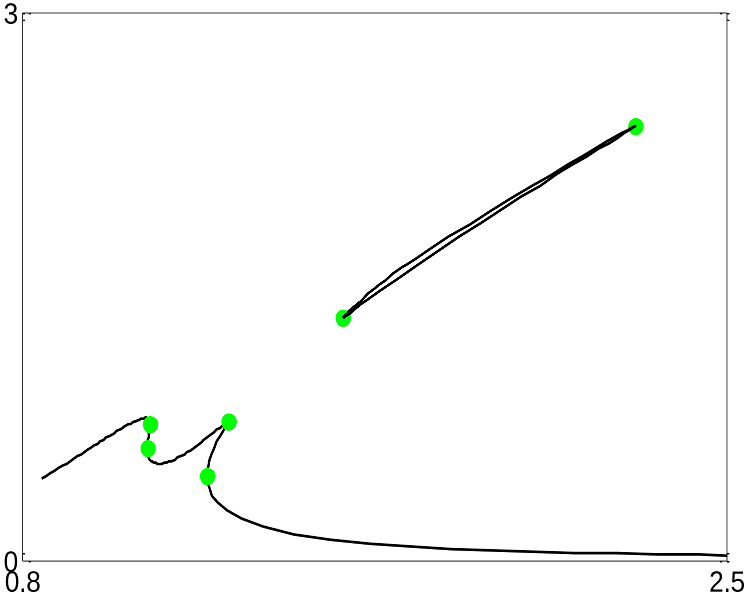
Nonlinear Targeted Energy Transfer in Mechanical and Structural Systems, A.F. Vakakis, O. Gendelman, L.A. Bergman, D.M. McFarland, G. Kerschen, Y.S. Lee, Springer, 2009.

NLTVA:

Suppression of limit cycle oscillations using the nonlinear tuned vibration absorber, G. Habib, G. Kerschen, Proceedings of the Royal Society of London A 471, 20140976 (2015).

Nonlinear generalization of Den Hartog's equal peak method, G. Habib, T. Detroux, R. Viguié, G. Kerschen, Mechanical Systems and Signal Processing 52-53 (2015), 17-28.

Thank you for your attention.



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NES: A.F. Vakakis, L.A. Bergman,
NLTVA: G. Habib, T. Detroux, C. Grappasonni
Robust LTVA: L. Dell'Elce, E. Gourc, G. Michon, G. Aridon, A. Hot