Analyse de machines tournantes comportant des non-linéarités avec Cast3M

Benoit Prabel

CEA Saclay, DEN/DM2S/SEMT/DYN

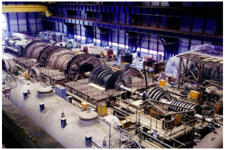
2016-10-10, GDR DYNOLIN

Plan

Motivation

Contexte

- Etude et dimensionnement de machines tournantes
- Géométrie parfois complexe



Turboalternateur EDF

Développement d'outils dans logiciel EF

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- Etude et dimensionnement de machines tournantes
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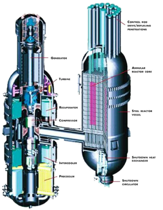

Gas Turbine Modular Helium Reactor

Développement d'outils dans logiciel EF

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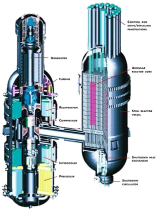
Développement d'outils dans logiciel EF



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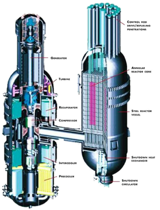

Gas Turbine Modular Helium Reactor

⇓

Développement d'outils dans logiciel EF

- Choix de Cast3M ⇒maîtrise et pérennisation

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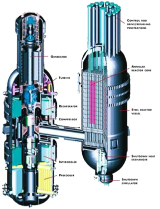

Gas Turbine Modular Helium Reactor

⇓

Développement d'outils dans logiciel EF

- Choix de Cast3M ⇒maîtrise et pérennisation
- Analyses modales (modes $\mathbb{R}$ et $\mathbb{C}$) et harmoniques (balourd ...)

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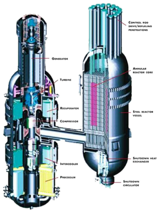

Gas Turbine Modular Helium Reactor

$\Downarrow$

Développement d'outils dans logiciel EF

- Choix de Cast3M $\Rightarrow$maîtrise et pérennisation
- Analyses modales (modes $\mathbb{R}$ et $\mathbb{C}$) et harmoniques (balourd ...)
- Modélisations : poutre/2D Fourier/3D, massif/coque, isotrope/orthotrope ...

Motivation

Présence de composants au caractère rapidement non-linéaire (paliers, squeeze-film ...)

⇒Analyses non-linéaires nécessaires !

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- Intégration temporelle

Motivation

Présence de composants au caractère rapidement non-linéaire (paliers, squeeze-film ...)

⇒Analyses non-linéaires nécessaires !

- Intégration temporelle
- Analyses dans le domaine fréquentiel (HBM)

Plan

Plan

Rotor in rotating frame

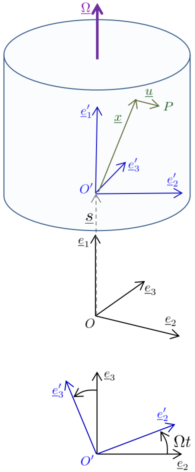
$$[M]\underline{\ddot{u}}' + [\Omega G + C_{visc}]\underline{\dot{u}}' + [K_{elas} + \Omega^2 K_{cent} + K(\sigma^0)]\underline{u}' = F'_{ext}$$

Linking rotating and non-rotating frame

$$\underbrace{\begin{pmatrix} U1 \\ U2 \\ U3 \end{pmatrix}}_{U} = \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\Omega t & -\sin\Omega t \\ 0 & \sin\Omega t & \cos\Omega t \end{bmatrix}}_{R(\Omega t)} \cdot \underbrace{\begin{pmatrix} U1' \\ U2' \\ U3' \end{pmatrix}}_{U'}$$

A Lagrange multiplier Λ is introduced :

$$\underbrace{\begin{pmatrix} 0 & -I & R \\ -I^T & 0 & 0 \\ R^T & 0 & 0 \end{pmatrix}}_{L(\Omega t)} \cdot \underbrace{\begin{bmatrix} \Lambda \\ U \\ U' \end{bmatrix}}_{q} = \underbrace{\begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}}_{0}$$



Intégration temporelle

Equations of motion

Rotor in rotating frame

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$$U = R(\Omega t) \cdot U'$$

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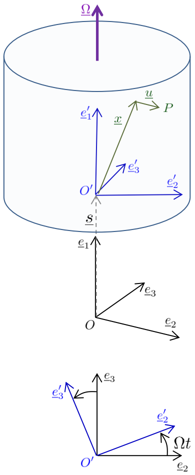
$$L(\Omega t) \cdot q = 0$$



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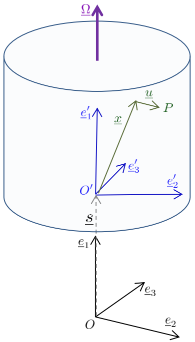

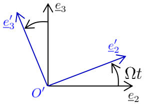

Intégration temporelle

The Newmark scheme

$$\ddot{u}_{n+1} = \frac{1}{\beta \Delta t^2}(u_{n+1} - u_n) - \frac{1}{\beta \Delta t}\dot{u}_n + (1 - \frac{1}{2\beta})\ddot{u}_n$$

$$\dot{u}_{n+1} = \frac{\gamma}{\beta \Delta t}(u_{n+1} - u_n) + (1 - \frac{\gamma}{\beta})\dot{u}_n + \Delta t(1 - \frac{\gamma}{2\beta})\ddot{u}_n$$

Schemes properties

Scheme	γ	β	$\Delta t_c / \Delta t^e$
Central difference	1/2	0	2
Fox and Goodwin	1/2	1/12	2.45
Linear acceleration	1/2	1/6	3.46
Average acceleration	1/2	1/4	∞
Modified average acceleration	$1/2 + \alpha$	$1/4(1+\alpha)^2$	∞
Runge-Kutta 4	-	-	$2\sqrt{2}$
Fu-De Vogelaere	-	-	$2\sqrt{2}$

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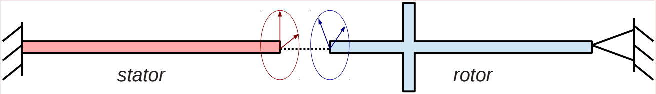
Plan

Intégration temporelle

Newmark average acceleration method (implicit)

Application to a linear 4-dof stator-rotor test problem



Equation of motion

$$Kq + C\dot{q} + M\ddot{q} = F$$

+

Kinematics constraint

$$L(\Omega t)q = 0$$

avec : $q^T = \begin{pmatrix} \lambda_y & \lambda_z & u_y & u_z & u'_y & u'_z \end{pmatrix}$

Prabel, B. Some remarks on time integration of 3D rotor-stator assembly, ECCOMAS 2016

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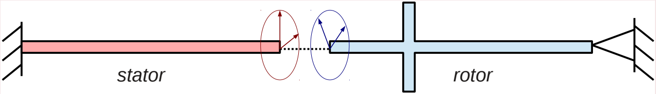

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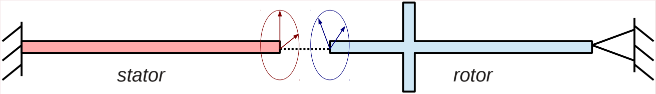

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$$K = \begin{bmatrix} 0 & 0 & 1 & 0 & -\cos\Omega t & \sin\Omega t \\ & 0 & 0 & 1 & -\sin\Omega t & -\cos\Omega t \\ & & k & 0 & 0 & 0 \\ & & & k & 0 & 0 \\ & & & & k' & 0 \\ sym & & & & & k' \end{bmatrix}$$

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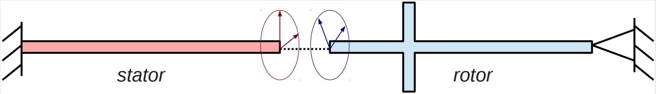

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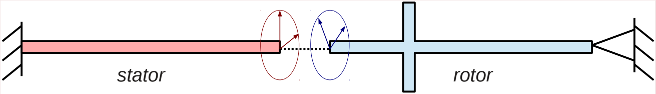

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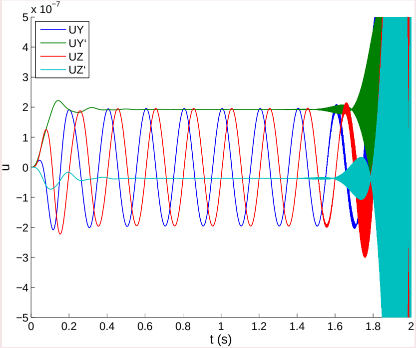
Intégration temporelle

Newmark average acceleration method (implicit)

Results of the 4-dof test problem



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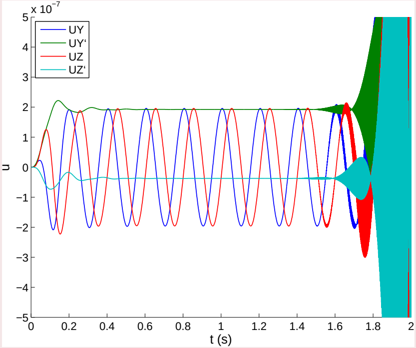

⇒instability of the "unconditionally stable scheme" !

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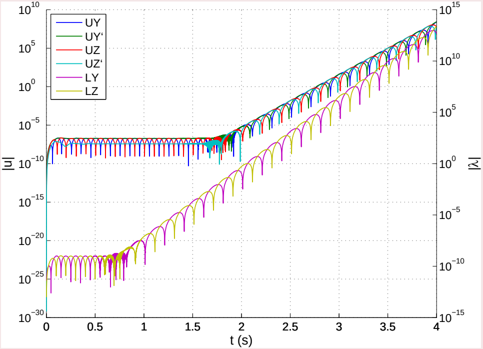

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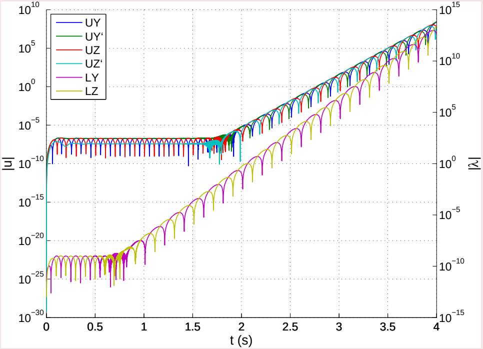

⇒instability of the "unconditionally stable scheme" !
⇒Lagrange multipliers are suspected...

Intégration temporelle

Newmark average acceleration method (implicit)

After long thinking...

Intégration temporelle

Newmark average acceleration method (implicit)

... the solution was written !

Cardona, A., & Geradin, M. (1989). Time integration of the equations of motion in mechanism analysis. Computers & structures, 33(3), 801-820

"Newmark trapezoidal rule is unconditionally unstable in the presence of constraints" (analysis based on the computation of the eigenvalues and eigenvectors of the amplification matrix A of a constrained system)

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$\Rightarrow$Proposed solution : use time integrator with controlled numerical damping.

Intégration temporelle

Implicit schemes with numerical damping

Hilber-Hughes-Taylor and α-generalized methods

$$(1-\alpha_m)M\ddot{u}_{n+1} + \alpha_m M\ddot{u}_n + (1-\alpha_f)C\dot{u}_{n+1} + \alpha_f C\dot{u}_n + (1-\alpha_f)Ku_{n+1} + \alpha_f Ku_n \\ = (1-\alpha_f)F^{ext}_{n+1} + \alpha_f F^{ext}_n$$

Intégration temporelle

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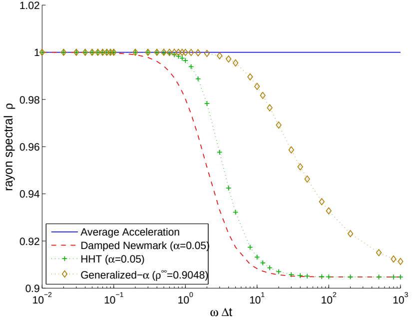
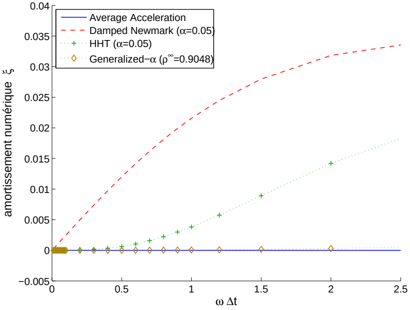
$$= (1-\alpha_f)F^{ext}_{n+1} + \alpha_f F^{ext}_n$$

Spectral radius



Numerical damping



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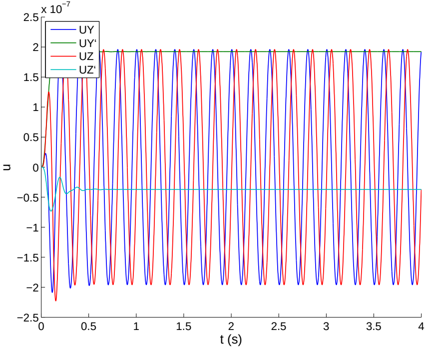


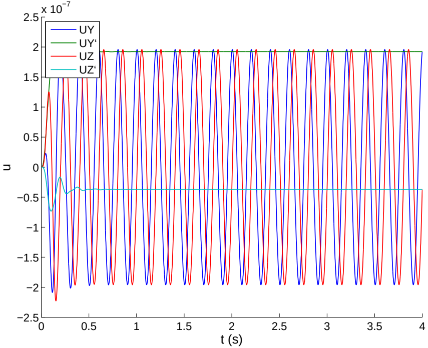
☺ No more instability

☹ Small dependance of the solution to the numerical damping

Intégration temporelle

Implicit schemes with numerical damping

Results of the 4-dof test problem



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Implicit schemes with numerical damping

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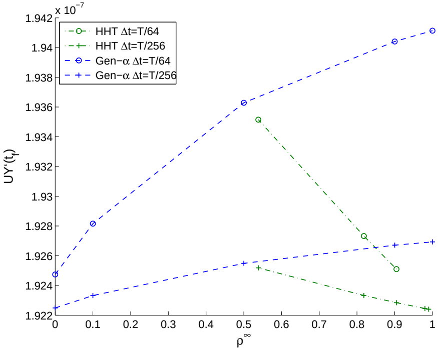

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Intégration temporelle

Implicit schemes with numerical damping

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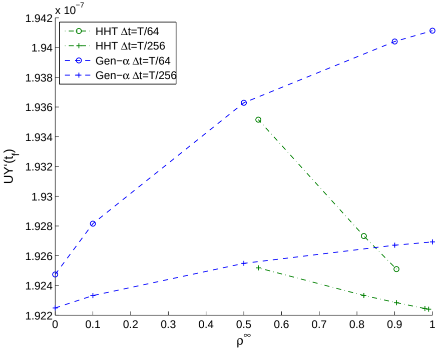

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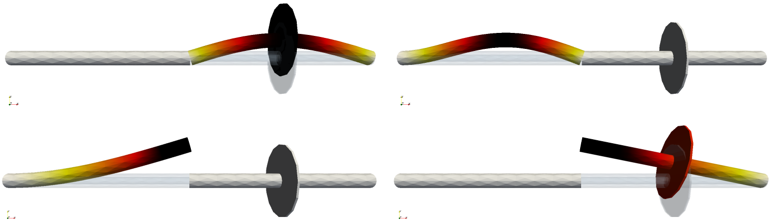
Intégration temporelle

Implicit schemes with numerical damping

Application to a linear 3D rotor-stator assembly
Mode shape constituting the CMS base :

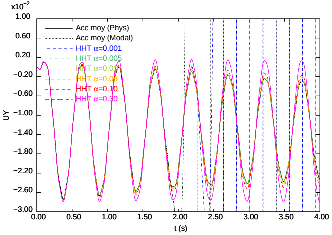


Intégration temporelle

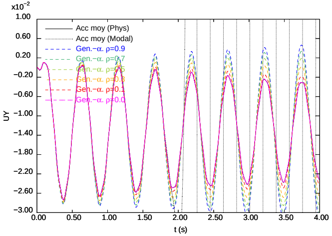
Implicit schemes with numerical damping

Application to a linear 3D rotor-stator assembly

HHT



α-generalized



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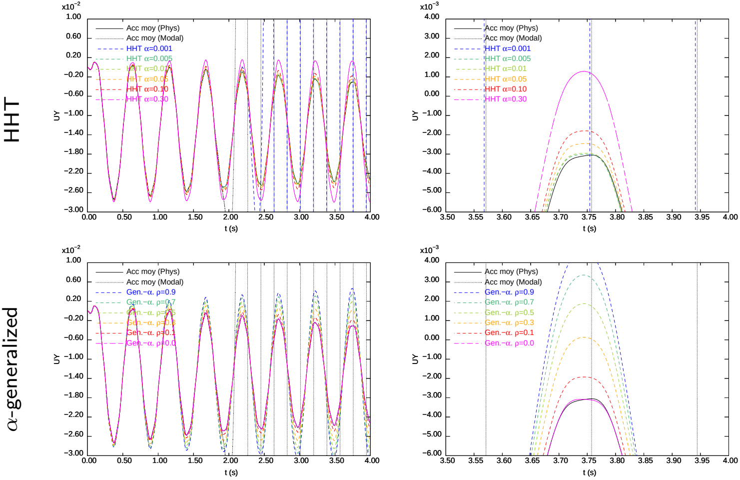

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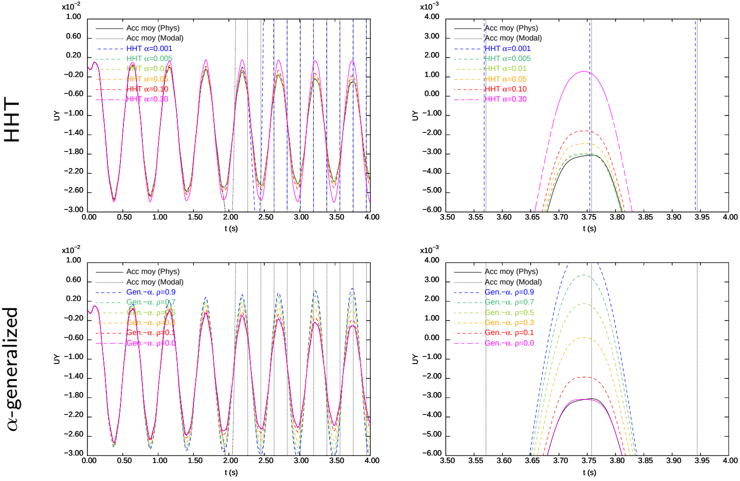

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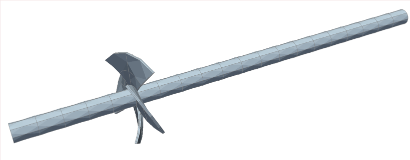
Plan

Intégration temporelle

Newmark central difference method (explicit)

Simple 3D rotor test problem



Intégration temporelle

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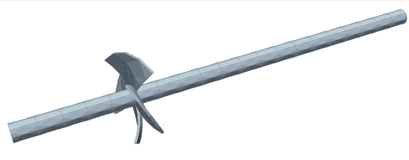

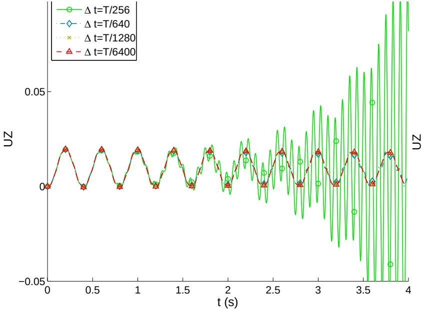

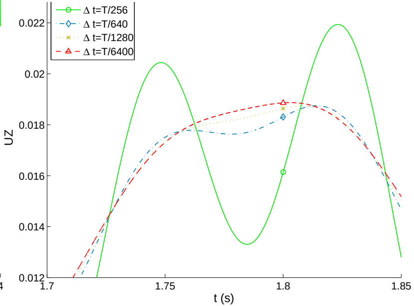

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Cause of numerical instability ?

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“Classical” simplification in explicit software

$$\left[M + \frac{\Delta t}{2} C\right] \ddot{u}_{n+1} = F^{ext}_{n+1} - K u_{n+1} - C \dot{u}_{n+\frac{1}{2}}$$

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C includes Gyroscopic effect $\Rightarrow \Delta t_c \searrow\searrow$ if neglected

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$\Rightarrow$Inversion of $\left[M + \frac{\Delta t}{2} C\right]$ required

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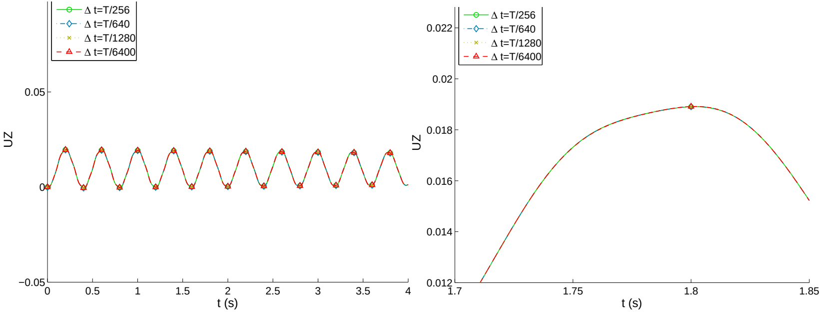

Plan

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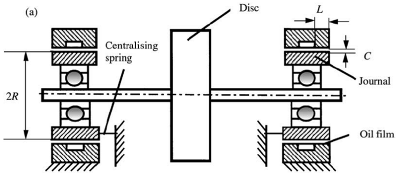
Newmark central difference method (explicit)

- Test case of Zhu et al.



Grandeur	a	b	c	d
$\alpha = \frac{m_{bearing}}{m_{disc}}$	0.1	0.267	0.1	0.1
$K = \frac{k_{spring}}{k_{shaft}}$	0.05	0.287	0.25	0.01
$\xi = \frac{c_{disc}}{2m_{disc}w_0}$	0.0005	0.0005	0.0005	0.0005
$U = \frac{F^{unbalance}}{\Omega^2 m_{disc} C}$	0.2	0.45	0.2	0.3
$B = \frac{\mu R L^3}{m_{bearing} w_0 C^3}$	0.05	0.145	0.01	0.025

and $w_0 = \sqrt{k_{shaft}/m_{disc}}$

- Unbalance force
- Nonlinear force:
- Goal = determine the response for every speed of rotation Ω

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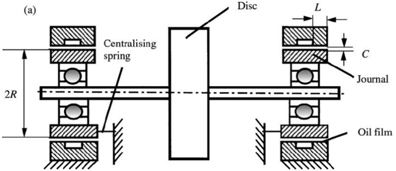

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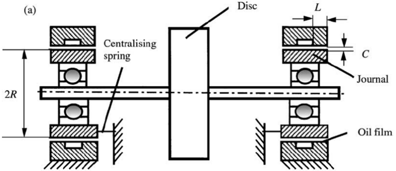

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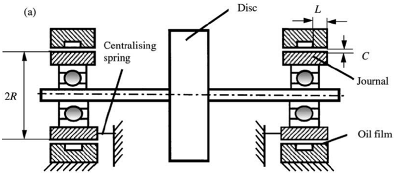

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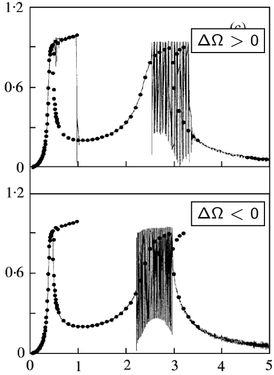

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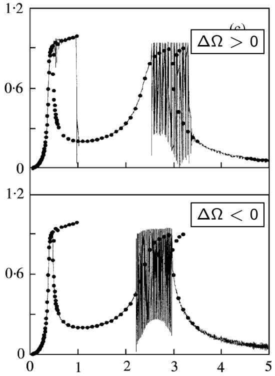
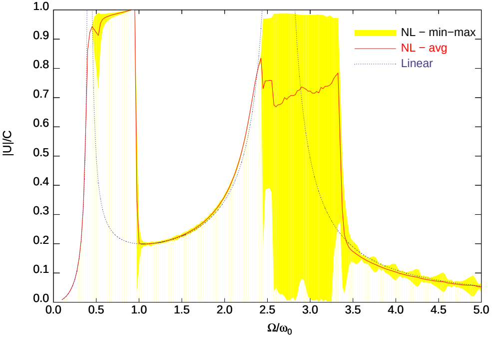


- Cast3M



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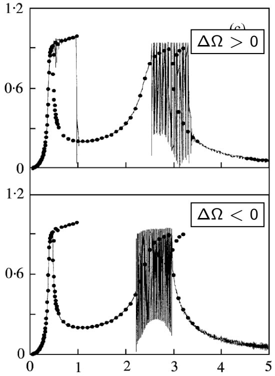

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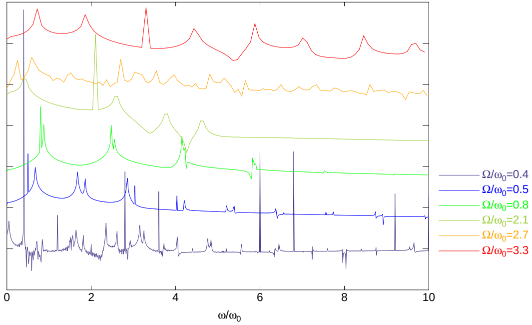

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- Implicit integration ⇒numerically damped scheme
- Explicit integration ⇒“full” scheme
- It takes often a long (computational) time to reach steady state

⇒ Choice of efficient explicit computation on modal basis (DYNE operator of Cast3M) ☺

⇒ An alternative would be the computation of the nonlinear response in frequency domain...

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Plan

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HBM

Intégration dans Cast3M

◇Pb to solve:
$-r = M\ddot{u} + C\dot{u} + Ku - f^{nl}(\dot{u}, u) - f^{ext} = 0$

⇒discretization, model definition, boundary condition, loading, eigenmodes basis, ...

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$u(t) = U_0 + \sum_{k=1..H} U_k \cos k\omega t + V_k \sin k\omega t$
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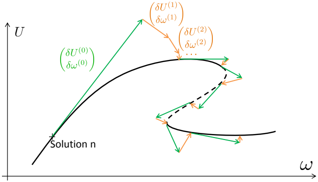

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HBM

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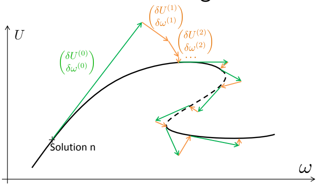

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◇Stability analysis (Floquet exponents with Hill's method):
$\left[\Delta_0 + \lambda\Delta_1 + \lambda^2\Delta_2\right] \tilde{\mathbf{U}} = 0$

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Duffing oscillator

- Duffing equation of motion:

$$\ddot{u} + 2\xi\omega_0\dot{u} + \omega_0^2 u + \mu u^3 = p\omega_0^2 \cos\omega t$$

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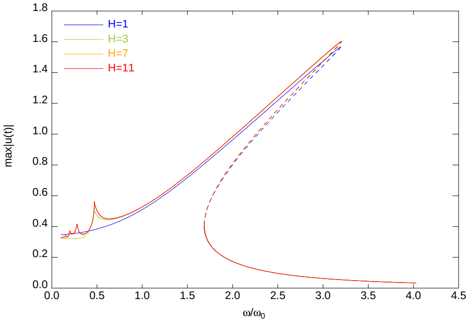

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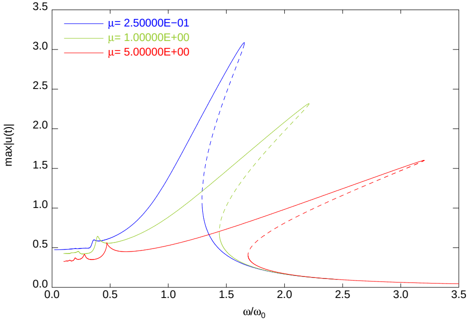

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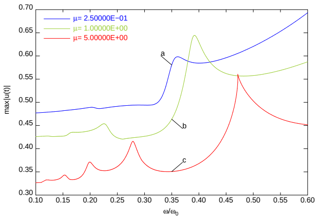

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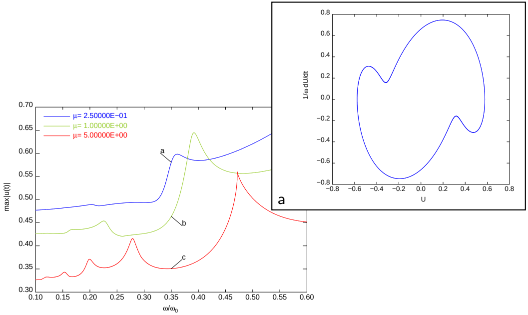

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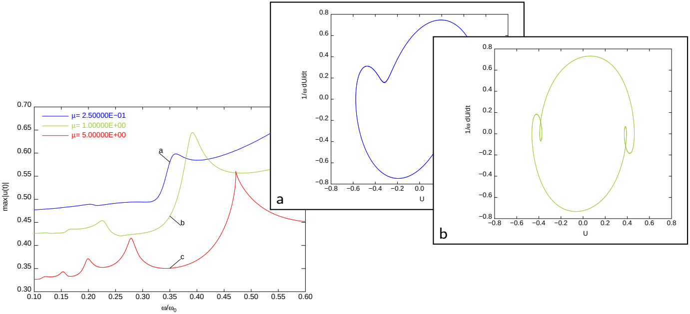

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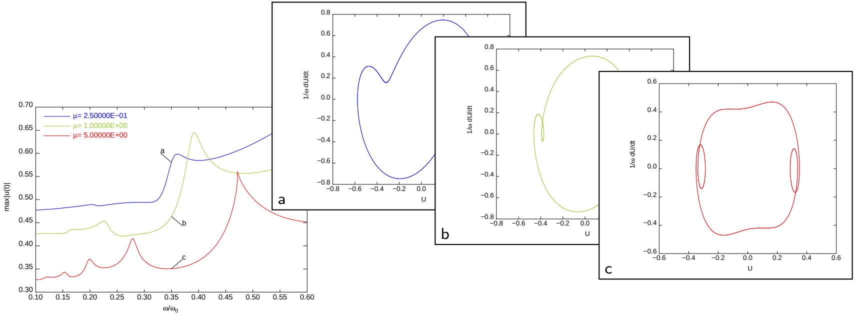

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- Results for $\omega/\omega_0 = 1.6$:

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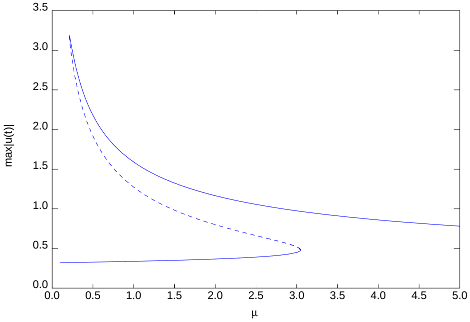
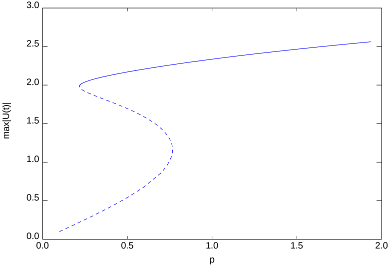


- Forced Van der Pol equation of motion:

$$\ddot{u} - \alpha\left(1 - u^2\right)\dot{u} + u = p\cos\omega t$$

Fixed values: $\alpha = 1\%$ and $\omega = 1$. Continuation with respect to p.



HBM

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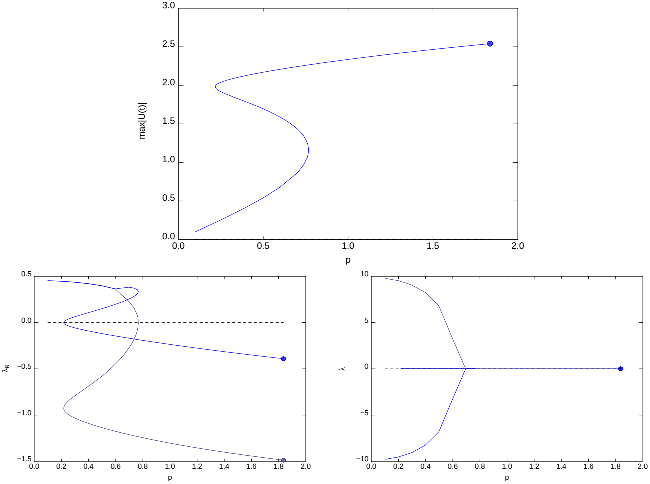

HBM

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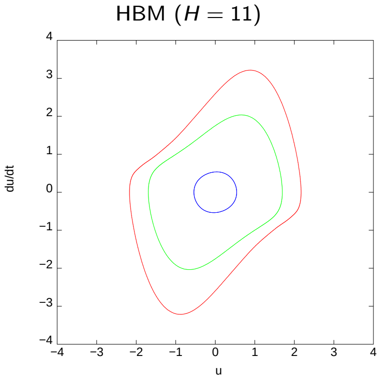
- Phase portrait for $p = 0.5$:



HBM

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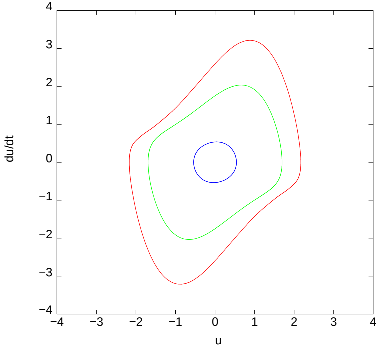
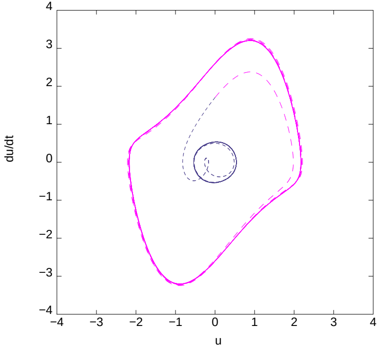
HBM ($H = 11$)



Time integration ($\Delta t > 0$ and $\Delta t < 0$)



Plan

Plan

HBM

0. Calcul de la courbe de réponse

$$\begin{pmatrix} \mathbf{U} & \omega \end{pmatrix}^T$$

1. Localisation des points de bifurcation

$$\begin{pmatrix} \mathbf{U} & \omega & \phi \end{pmatrix}^T$$

2. Suivi des points de bifurcation

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Xie, L. Baguet S., Prabel, B. and Dufour, R., Numerical Tracking of Limit Points for Direct Parametric Analysis in Nonlinear Rotordynamics, Journal of Vibration and Acoustics, 138(2)-2016
Xie, L. Baguet S., Prabel, B. and Dufour, R., Bifurcation tracking by Harmonic Balance Method for performance tuning of nonlinear dynamical systems, Mechanical Systems and Signal Processing, accepted

HBM

1. Localisation des points de bifurcation

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New unknown : eigenvector ϕ

New equation : stability equation

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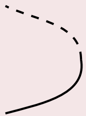
New unknown : eigenvector ϕ
New equation : stability equation

Limit Point ($\lambda = 0$)



$$\mathbf{R} = 0$$
$$\mathbf{R}_{,\mathbf{U}} \cdot \phi = 0$$
$$\phi^T \phi = 1$$

HBM

1. Localisation des points de bifurcation

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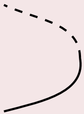
New unknown : eigenvector ϕ

New equation : stability equation

Limit Point ($\lambda = 0$)



$$\mathbf{R} = 0$$
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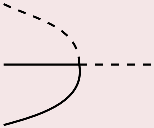
Branch Point ($\lambda = 0$)



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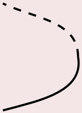

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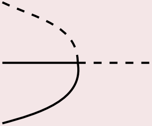
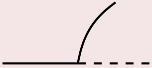


$$\mathbf{R} = 0$$
$$\mathbf{R}_{,\mathbf{U}} \cdot \phi = 0$$
$$\mathbf{R}_{,\omega} \cdot \phi = 0$$
$$\phi^T \phi = 1$$

Neimark-Sacker Point ($\lambda_{1,2} = \pm\kappa$)



$$\mathbf{R} = 0$$
$$\Delta_0 \cdot \phi_R - \kappa\Delta_1 \cdot \phi_I - \kappa^2\Delta_2 \cdot \phi_R = 0$$
$$\Delta_0 \cdot \phi_I + \kappa\Delta_1 \cdot \phi_R - \kappa^2\Delta_2 \cdot \phi_I = 0$$
$$q^T \phi_R = 1$$
$$q^T \phi_I = 0$$

2. Suivi des points de bifurcation

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New unknown : parameter α
Prediction : along ϕ
New equation (correction steps) : pseudo-arc-length equation :

$$\begin{pmatrix}\Delta \mathbf{U} & \Delta\omega & \Delta\alpha\end{pmatrix} \cdot \begin{pmatrix}\delta \mathbf{U} & \delta\omega & \delta\alpha\end{pmatrix}^T = 0$$

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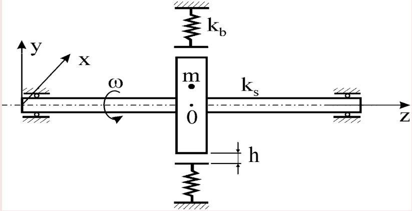
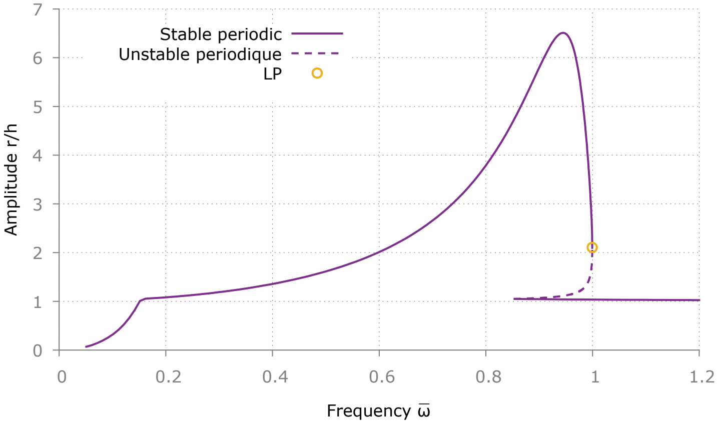
Example: Jeffcott rotor (2 dof system) submitted to contact with friction



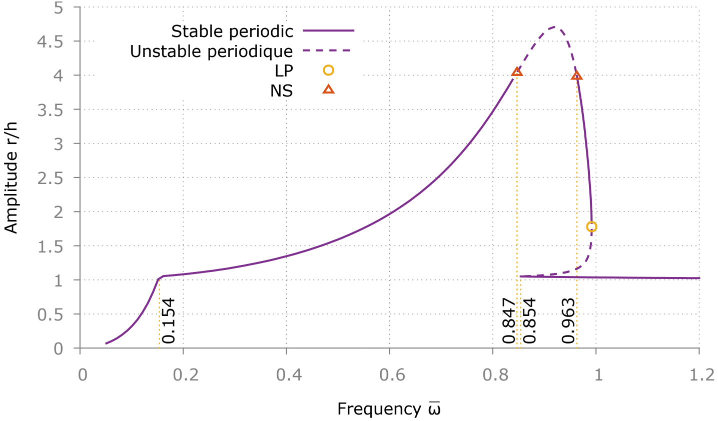
Response for $\mu = 0.05$



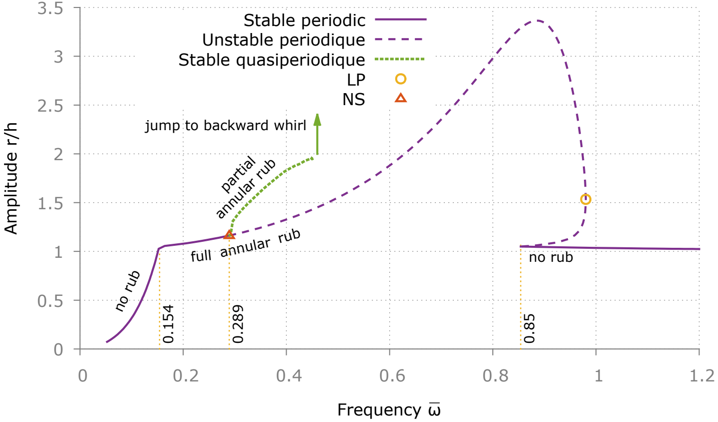
HBM

2. Suivi des points de bifurcation

Response for $\mu = 0.11$



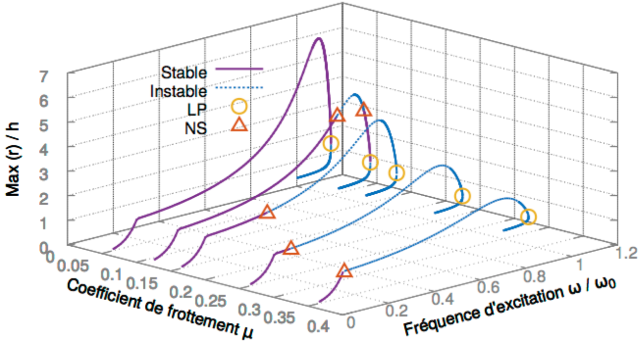
Response for $\mu = 0.20$



HBM

2. Suivi des points de bifurcation

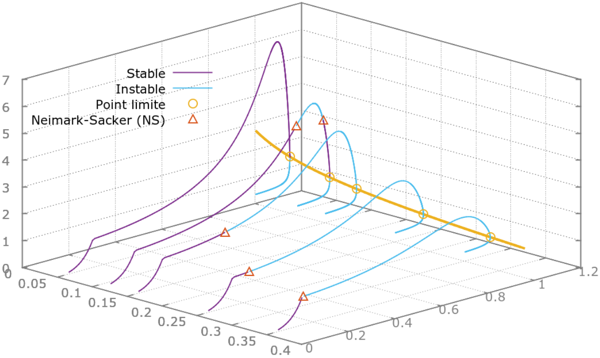
Response for different values of μ



HBM

2. Suivi des points de bifurcation

Limit Point tracking (new parameter $= \mu$)



HBM

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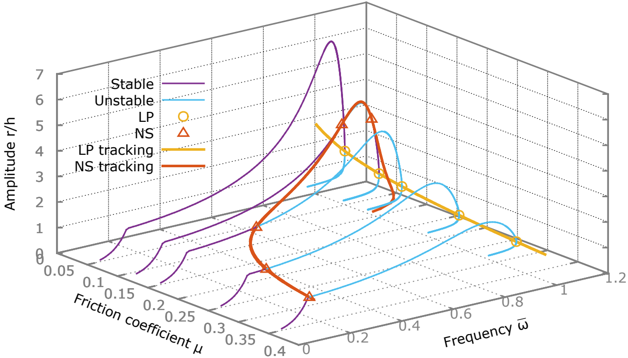
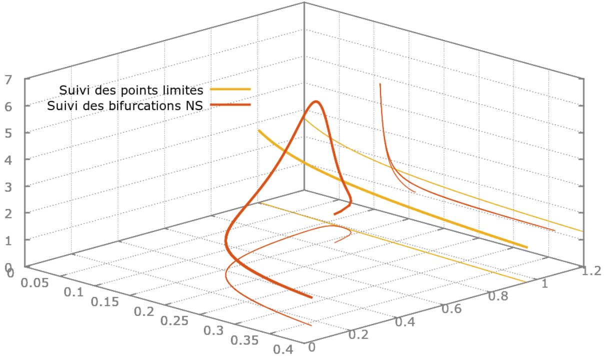
Neimark-Sacker tracking (new parameter = μ)



LP and NS tracking



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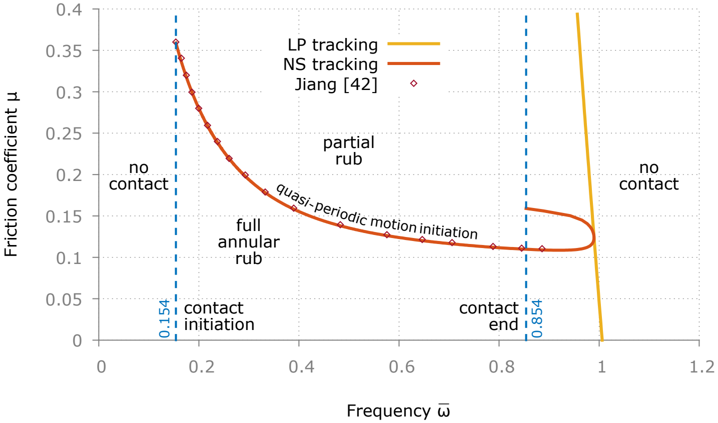

Plan

I. Motivation

II. Intégration temporelle

III. Equilibrage harmonique

IV. Conclusions et Perspectives

Conclusions et Perspectives

- 😐 HHT and α-generalized implicit schemes implemented in Cast3M (only for linear analysis)
- ☺ Efficient explicit schemes available for nonlinear rotordynamics in Cast3M

- ☺ Continuation + HBM + AFT + Stability analysis implemented in Cast3M
- ☹ limited for now to (relatively small) and forced system
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Future : Change ☹ into ☺ and explore collocation method, quasi-periodic motion computation, ...

Thank you for your attention !

Any question ?