Complicated dynamics exhibited by thin shells displaying numerous internal resonances: application to the steelpan

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Summary. Nonlinear vibrations of a steelpan, a musical instrument from the Caribbean islands, are experimentally and theoretically studied. Modal analysis reveals the existence of numerous modes displaying internal resonances, enabling the structure to transfer energy from low or high frequency modes. The complicated dynamics in forced vibrations is precisely measured by following the different harmonics of the responses. Energy transfers are explained in light of high order couplings, and simple models displaying 1:2:2 and 1:2:4 internal resonance are fitted to the experiments. The measurements reveals that mode couplings are activated for very low amplitudes of order 1/25 times the thickness, and that numerous modes are rapidly excited, giving rise to frequency responses with complex shape. This ease in transferring energy is a key feature for explaining the peculiar tone of the steelpan. The identified 1:2:4 model is finally used to recover the time oscillations of an impacted note in normal playing condition, resulting in a perfect agreement for the first four harmonics.

Core of the study

Figure 1: Photograph of a steel band, of a particular steelpan in normal playing conditions and sketch of the musical notes pattern on the upper surface of the instrument

Steelpans are musical percussion instruments coming from the island of Trinidad and Tobago in the Caribbean. They are traditionnally played in orchestras called steelbands. These orchestras are composed of several steelpans covering a range of several octaves. Each steelpan is made of an oil barrel that is subjected to several stages of metal forming that stretch and bend the structure. The top of the barrel is pressed, hammered, punched and burnt in order to obtain a sort of main bowl within which convex domes are formed, each one corresponding to a particular musical note (Fig. 1).

A modal analysis reveals that the mode shapes of the instruments are localized on the convex domes, each one being associated to a set of modes which natural frequencies are precisely tuned according to harmonic relationships to obtain musical notes [3]. The first (fundamental) mode of the domes is tuned to the frequency of the corresponding musical note. Then, the second mode is tuned to twice the frequency of the fundamental one, giving rise to 1:2 relationship. Due to localization and to the octave tuning of successive notes, it is generally observed that the second mode is degenerate so that a 1:2:2 internal resonance is present. The third mode is tuned either at the third or at the fourth of the fundamental frequency, so that 1:2:2:3 or 1:2:2:4 internal resonances are possible. Finally, higher modes are also found to be tuned, hence the presence of modes at six times and/or eight times the fundamental frequency are also generally found. In normal playing where a note is stroke with a stick, vibrations amplitudes are such that geometric nonlinearities cannot be neglected. This property is recognized as a key feature for explaining the peculiar tone of the steelpan [1]. This nonlinearities, combined with the numerous possible internal resonances, activates energy transfer to higher modes. In playing conditions, energy transfers up to a dozen of mode are usually observed.

The aim of the present work is to study energy exchanges and activation of internal resonances in the nonlinear dynamics exhibited by the steelpan, and more generally on systems that show 1:2:2:4 internal resonances. Forced vibrations and frequency response curves are used to identify and localize instabilities and mode coupling. For that purpose, the different harmonics of the responses are measured with a lock-in amplifier, during controlled step-by-step, forward and backward sweeps around the resonances of several eigenfrequencies. Frequency responses show complicated dynamics and activation of mode coupling from low to high-frequency as well as from high to low frequencies. The two following three degrees of freedom models are theoretically studied:

$$\begin{cases} \ddot{q}_1 + 2\xi_1\omega_1\dot{q}_1 + \omega_1^2 q_1 + \alpha_1 q_1 q_2 + \alpha_2 q_1 q_3 = \delta_1 F_1 \cos(\Omega t), \\ \ddot{q}_2 + 2\xi_2\omega_2\dot{q}_2 + \omega_2^2 q_2 + \alpha_3 q_1^2 = \delta_2 F_2 \cos(\Omega t), \\ \ddot{q}_3 + 2\xi_3\omega_3\dot{q}_3 + \omega_3^2 q_3 + \alpha_4 q_1^2 = \delta_2 F_3 \cos(\Omega t) \end{cases}$$

$$\begin{cases} \ddot{q}_1 + 2\xi_1\omega_1\dot{q}_1 + \omega_1^2 q_1 + \alpha_1 q_1 q_2 = \delta_1 F_1 \cos(\Omega t), \\ \ddot{q}_2 + 2\xi_2\omega_2\dot{q}_2 + \omega_2^2 q_2 + \alpha_2 q_1^2 + \alpha_3 q_2 q_3 = \delta_2 F_2 \cos(\Omega t), \\ \ddot{q}_3 + 2\xi_3\omega_3\dot{q}_3 + \omega_3^2 q_3 + \alpha_4 q_2^2 = \delta_3 F_3 \cos(\Omega t), \end{cases}$$

where $\delta_i \in \{0, 1\}$ are Kronecker symbols used to distinguish the oscillators which are forced (if oscillator i is forced, $\delta_j = 0 \; \forall j \neq i$) [2]. Those models are obtained by normal form reductions, by keeping in the oscillators all nonlinear resonant terms responsible of a 1:2:2 internal resonance (model on the left) and a 1:2:4 internal resonance (model on the right). Original analytical results are obtained by the multiple scale method. It is shown that the nonlinear dynamics of the oscillators in 1:2:4 internal resonance may be interpreted as a cascade of two 1:2 internal resonances, whereas the one of the 1:2:2 case is equivalent to two nested 1:2 internal resonances. Instability regions in the amplitude/frequency domains are exhibited, leading to the appearance of coupled periodic solutions as well as quasiperiodic solutions obtained after Neimark-Sacker bifurcations.

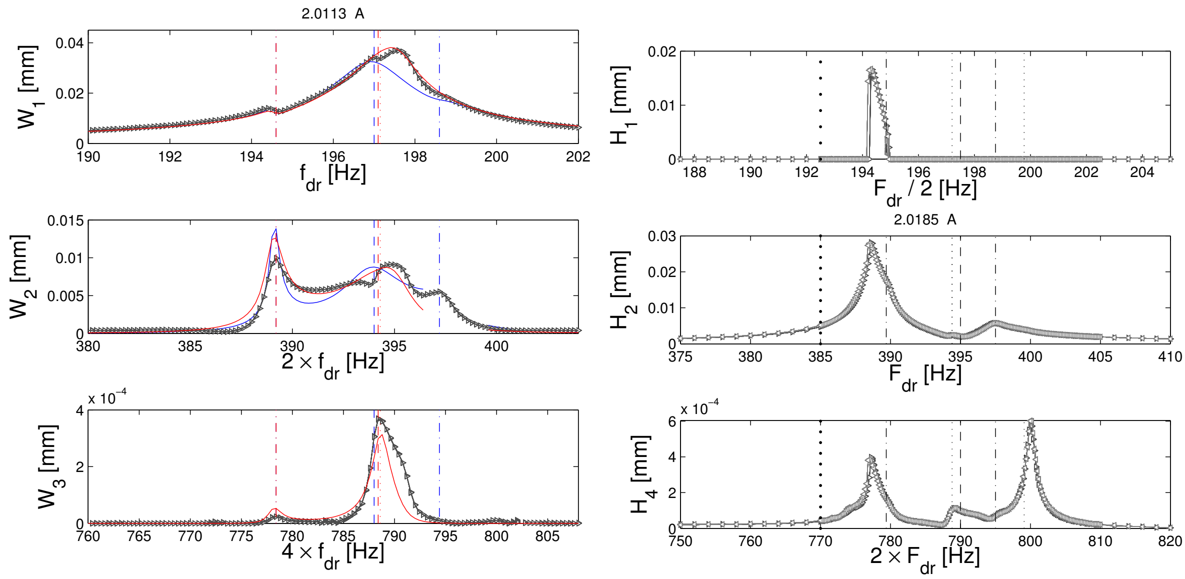

Figure 2: (left) Resonance curve of the G3 note of the steelpan excited in the vicinity of its **fundamental** frequency. The results from the 1:2:2 (blue curve) and 1:2:4 (red curve) three dof. models fitted to experimental results ('▷' and ◁') are also shown. First (w_1), second (w_2) and fourth (w_3) harmonics of the signal. (right) Experimental resonance curve of the G3 note of the steelpan excited in the vicinity of its **second** natural frequency. Subharmonic (H_1), fundamental (H_2) and second harmonics (H_4) of the signal.

Figure 2 gives a snapshot of the results in the case of the G3 note of the steelpan for which the natural frequencies are tuned so that $\omega_3 \simeq \omega_2 \simeq 2\omega_1$, $\omega_4 \simeq \omega_5 \simeq 4\omega_1$, $\omega_6 \simeq 6\omega_1$ and $\omega_7 \simeq 8\omega_1$ (where the ω_i are the natural frequencies of the note), thus displaying a 1:2:2:4:4:6:8 internal resonance. If the fundamental mode is directly driven by a resonant driving so that the driving frequency is $\Omega \simeq \omega_1$, an energy transfer to modes 2, 3 (oscillating at 2Ω) and 4 (oscillating at 4Ω) is observed on the left column of Fig 2. If the second and third modes are driven with $\Omega \simeq \omega_2 \simeq \omega_3$, energy transfers to the other modes are also observed, with, among others, a subharmonic resonant excitation of the fundamental mode, that oscillates at half the driving frequency ($\Omega/2$). The fittings of the theoretical models are also displayed, showing a correct agreement, even if those models should be composed of at least four degrees of freedom instead of three to be closer to the experiments.

Conclusion

A complicated dynamics was exhibited on a particular vibrating shell structure with tuned eigenfrequencies. Numerous experiments in forced vibrations have been performed, showing energy transfers explained by up to 1:2:2:4:6:8 internal resonance. To explained those results, two three degrees of freedom models have been analytically explored, showing that the dynamics can be qualitatively explained by cascades and mixes of 1:2 internal resonances. A fitting of the models to the experiments in forced vibrations was also useful to synthesize a free vibration response of the structure very close to the experiments (not shown here).

References

[1] A. Achong (2003) Mode locking on the nonlinear notes of the steelpan. *Journal of Sound and Vibration*, **266**, 193-197.

[2] M. Monteil, C. Tousé, O. Thomas and S. Benacchio (2013) Nonlinear forced vibrations of thin structures with tuned eigenfrequencies: the cases of 1:2:4 and 1:2:2 internal resonances. *Nonlinear dynamics*, online

[3] T. D. Rossing, U. J. Hansen and D. S. Hampton (2000) Vibrational mode shapes in Caribbean steelpans. I. Tenor and double second. *Journal of the Acoustical Society of America*, **108**(2) 803-812.