APPLICATION OF PREDICTION METHODS TO NONLINEAR PERCUSSION INSTRUMENTS

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Abstract: Most of the vibrations and sounds produced by nonlinear percussion instruments, such as cymbals and gongs, exhibit the typical properties of chaotic dynamical systems. More specifically, for large amplitude of vibrations, nonlinear effects becomes preponderant and can lead to chaotic behaviour. In the perspective of analyzing and generating such time series, advantage must be taken of the recent advances in non-linear signal processing. As spectral linear methods are not able to compute time series having broadband Fourier spectra, it seems natural to proceed to the reconstruction of a pseudo-phase space, using time-delayed coordinates, and to compute an approximation of the reconstructed dynamics.

A prediction method based on the modeling of the local neighbourhood-to-neighbourhood evolution in the reconstructed phase space has been used. An algorithm which allows, from a short learning time series, to generate long duration signals, has been written. The synthetic time series are expected to present the same dynamical properties (fractal dimension and Lyapunov exponents) as the original. The method is then applied to experimental time series obtained from experiments on forced oscillations of a cymbal, and a freely oscillating gong.

INTRODUCTION

Nonlinear signal processing methods have received particular attention and developments since the early eighties; general surveys that present the background and applications are now available [1, 2]. These methods are based on the reconstruction theorem of Takens and Mañé [3, 4]. This theorem states that, assuming the signal observed arise from a deterministic dynamical system, a trajectory in a pseudo-phase space –homeomorphic to the original one– can be reconstructed. Once this reconstruction has been achieved, and once the attractor has been embedded in a time-delayed reconstructed space, one is able to compute orbits invariants, such as fractal dimensions (geometrical invariants) [5] or Lyapunov exponents (dynamical invariant) [6]. Moreover, it has become possible to approximate the reconstructed dynamics, taking advantage of the evolution of small neighbourhoods onto the attractor. This allows to compute models for the observed dynamics, and to use it for predictions [7, 8, 9] or for noise reduction purpose [10, 11]. In the context of prediction of chaotic time series, these techniques are particularly suitable for signals arising from a low-dimensional dynamical system which presents a dense and broadband Fourier spectra, since all the models proposed in the context of linear signal processing fail to reproduce the complicate behaviour of chaotic time series [12, 13].

The aim of this study is to apply the methods proposed in the prediction of chaotic time series to experimental signals recorded on cymbals and gongs. The behaviour of these two percussion instruments can be strongly non-linear. Previous studies led on those intruments have shown that, when forced harmonically, they exhibit a chaotic behaviour assessed by a positive Lyapunov exponent [14]. Dimension calculations also have shown that it appears to be low-dimensional chaos [15]. In this manner, a signal-based attempt to generate time series whose behaviour would be similar to the observed ones must rely on the ideas of the prediction of chaotic time series. It can also been understood as a first step towards the synthesis from the reconstructed phase space. A kind of *inverse problem* for the reconstruction has indeed to be solved: given a phase plane, which signal can be generated from it, and how far is it from the original one ?

This paper will present first the algorithm used to compute a synthetic, long duration time series, from a short one. The selected method consists in modeling the local neighbourhood-to-neighbourhood evolution in the reconstructed phase space. This method, referred to as *local*, has the advantage to build a model adapted to the local variations of the trajectory in the reconstructed phase space. This is particularly suitable in the case of reconstructed time series which present sharp variations. The negative point is that only a piecewise model is found. On the other hand, *global methods* are also available. The idea in this case is to use all the data points to compute a global dynamical model decomposed on a given basis of functions. Although impressive results on standard dynamical systems are exhibited [16], the field of application seems to be narrower [9]. The algorithm will then be applied to experimental data. Two different cases will be treated. the first signal is the acceleration at one point of a cymbal, which is driven sinusoidally. The second signal is the acceleration of one point of a freely oscillating gong. These two different examples will reveal the success and the problems encountered when processing experimental signals.

ALGORITHM

A *nearest-neighbours* strategy has been choosen to predict the future state $s(N+1)$ from a given time series $\{s(k)\}_{k=1...N}$, referred to as the *learning data set.* Iterating the predictions give rise to a long time series, and scaling laws demonstrated in [7, 11] shows that iterative procedure give better results than direct predictions. Assuming that the learning data set arises from a chaotic process, the first step of the algorithm will be to reconstruct the phase space using time-delayed coordinates. Given an embedding dimension d_E and a time-delay T, the state vector is : $\mathbf{y}(k) = [s(k), s(k-T), ..., s(k-(d_E-1)T)]$ (general considerations for choosing T and d_E can be found for example in [1, 2]).

One major problem coming from the chaotic nature of the signal is related to the sensitivity to initial conditions : two nearby trajectories diverge exponentially fast on the attractor. In the sense of the usual error prediction, defined by the difference between the predicted trajectory and the original one, one can conclude that only *short-term* predictions are possible. However, a better criteria for the validity of the predicted time series is the conservation of the phase plane and thus of the orbits invariants. An example in the next section will show how the sensivity to initial conditions affects the trajectory but not the attractor.

To predict one step ahead in the reconstructed phase space, one assumes the existence of a local map $\mathbf{F}$, valable only in a neighbourhood of a point $\mathbf{y}(k)$, and such that : $\mathbf{y}(k+1) = \mathbf{F}(\mathbf{y}(k))$. As the approximation is *local*, one can approximate $\mathbf{F}$ by a linear polynomial:

$$\mathbf{F}(\mathbf{y}(k)) = \mathbf{B} + \mathbf{A}\mathbf{y}(k)$$

or develop further, using higher-order terms. In this study, a quadratic fit for $\mathbf{F}$ has also been tested: $\mathbf{F}(\mathbf{y}(k)) = \mathbf{B} + \mathbf{A}\mathbf{y}(k) + \frac{1}{2}\mathbf{y}(k)^t\mathbf{C}\mathbf{y}(k)$. The main task of the algorithm is to compute the matrices $\mathbf{A}$, $\mathbf{B}$ and $\mathbf{C}$ using least-squares on the neighbourhood of $\mathbf{y}(k)$. Denoting $\{\mathbf{y}(k_i)\}_{i=1...N_v}$ the N_v nearest neighbours of $\mathbf{y}(k)$, the matrices $\mathbf{A}$, $\mathbf{B}$ are computed, in the linear case, by minimizing:

$$e_{rms} = \sum_{i=1..N_v} ||\mathbf{y}(k_i+1) - \mathbf{B} - \mathbf{A}\mathbf{y}(k)||_2^2$$

Here, we have to mention that two different dimensions are used: the embedding dimension d_E is used only for the search of the N_v nearest neighbours. One should take d_E large enough to avoid choosing false neighbours. Then all the equations are projected on a d_L-dimension subspace ($d_L \leq d_E$) where d_L is the *local dimension*, i.e. the dimension of the manifold on which the trajectory is supposed to live. Hence, all the matrix inversions are performed for d_L dimensions.

Once the local map is approximated, one has finally to calculate the predicted sample $\hat{s}(k+1)$ as first coordinate of $\mathbf{F}(\mathbf{y}(k))$. The procedure is then iterated to compute a predicted signal.

RESULTS ON COMPUTER-GENERATED TIME SERIES

The algorithm has been tested on many standard dynamical systems: discrete maps (Henon and Ikeda maps) and continuous-time systems (Lorenz and Rössler attractors). The sensitivity to initial conditions is illustrated in figure 1, which shows the inability to predict with a bounded error beyond a time which is inversely proportional to the largest Lyapunov exponent of the system. In this figure, two predicted time series are shown, one computed using a linear fit, the second one with a quadratic fit. The predicted trajectories draw aside from the original one but stay on the attractor. Thus the phase plane and the orbits invariants are unchanged [17]. Lyapunov exponents for the signal predicted with the linear fit have been computed, using an algorithm described in [14]. The Lyapunov spectrum gives a measure of the rate of divergence/convergence of the trajectories in the phase space. For the Lorenz system, these are: $\lambda_1 = 1.51$, $\lambda_2 = 0$, $\lambda_3 = -22.5$. From the predicted time series, they have been found equal to: $\lambda_1 = 1.57$, $\lambda_2 = -0.01$, $\lambda_3 = -19.34$.

Figure 2 shows how the normalized quadratic error grows as the number of iterated prediction increases. The normalized error is: $e = \frac{\sigma_\delta}{\sigma_{mean}}$, where $\sigma_\delta^2 = \frac{1}{P}\sum_{j=1}^{P}(s_j(u) - \hat{s}_j(u))^2$. P is a number of different initial conditions used to calculate the learning data set s_j (here

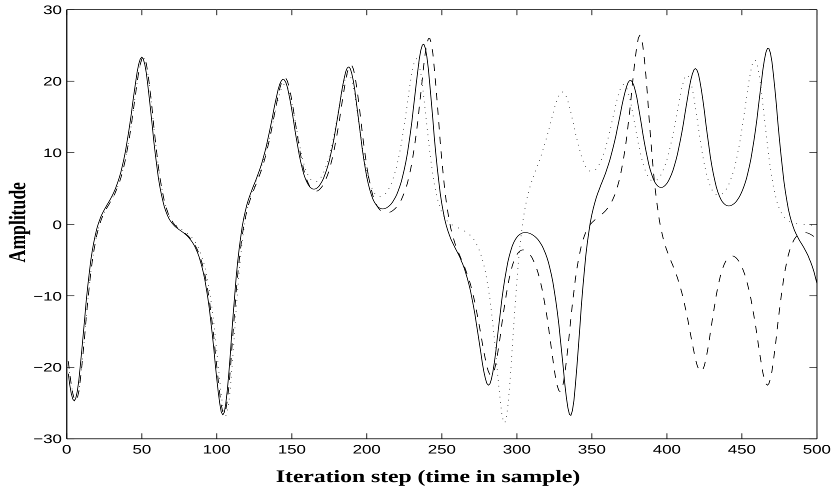

Fig. 1: Prediction for a time series calculated from the Lorenz system. Original (—), prediction with a linear fit (...), and with a quadratic fit (- - -), illustrating the sensitivity to initial conditions.

P has been taken equal to 20). $\sigma^2_{mean} = \frac{1}{P}\sum_{j=1}^{P}(s_j(u) - \overline{s})^2$, where $\overline{s}$ is the mean of the realizations at time u. Figure 2 clearly points out that the error grows at a rate dictated by the largest Lyapunov exponent.

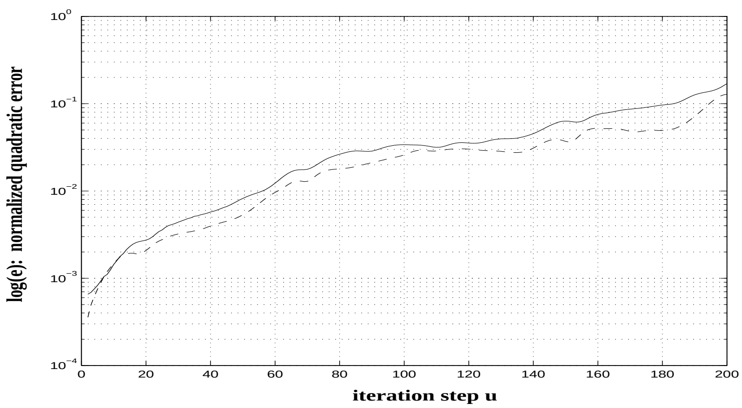

Fig. 2: Normalized quadratic error as a function of the iteration step. A linear prediction (solid line) and a quadratic prediction (dashed line) has been used. the error grows at a rate dictated by the largest Lyapunov exponent: the slope of the curve is indeed equal to 1.45.

For the choice of the parameters of the algorithm, general considerations are available in the literature to avoid pitfalls. However, one is generally interested in having a learning data set as short as possible. In this manner, attention must be paid to the dimension used, since increasing d_E and d_L leads to a reconstructed attractor which is not enough populated to perform the matrix inversions involved in the algorithm in good conditions. In figures 1 and 2, a learning data set of 20000 points has been used, although in these theoretical cases, about 5000 points is generally enough to perform a good prediction. d_E was set to 4 and

d_L to 3, standard value for T for the Lorenz system is $T = 0.1\ s$.

One problem linked to the size of the neighbourhood used to compute the local approximation is that, for some points $\mathbf{y}(k)$ on the attractor, the N_v nearest neighbours do not span the d_L-dimension subspace. This phenomenon leads to a bad conditioning of the local matrices and thus to a predicted value which diverges [17]. To solve this problem, the algorithm increases N_v until the d_L dimensions are populated. On the other hand, one has to keep N_v as small as possible to ensure that the Taylor development is valid. Thus only a linear fit will be considered in the next section, mainly because it needs less neighbours.

APPLICATION

Forced vibrations of a cymbal

The first signal used as an illustration of the abovepresented method is the vibration of a cymbal, recorded by means of an accelerometer glued at the edge. The cymbal is excited at its center by a shaker. When increasing the amplitude of the forcing, transitions from a linear to a chaotic regime are observed. Similar experiments have been led in the past by Fletcher *et al.* [18, 19], a more complete description of the apparatus and the measured signals can be found in [15, 14]. The selected signal is for an excitation frequency $F_{exc} = 412$ Hz, a value which is apart from an eigenfrequency of the cymbal. A transition from quasiperiodicity, involving internal resonances, to chaos has been observed [14]. The algorithm has been applied first to a portion of signal in the quasiperiodic state, and secondly to another portion in the chaotic regime, where the Fourier spectrum is dense and broadband. In each case, 20000 points have been selected, which corresponds to less than half a second, since the signals are recorded on a DAT-tape, with sampling frequency $F_s = 48000$ Hz.

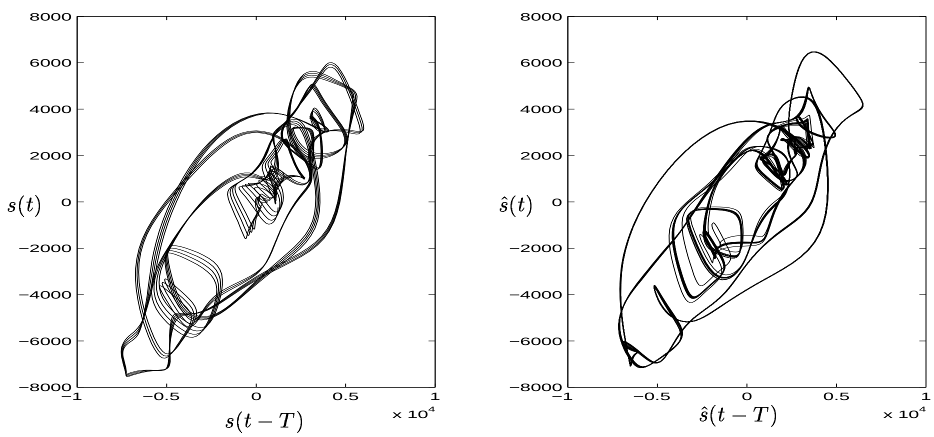

Fig. 3: Comparison between the phase plane of the original signal (left) for the cymbal excited at $F_{exc} =$ 412 Hz in the quasiperiodic regime, and the predicted signal (right). The parameters for the prediction are: $d_E = 7$, $d_L = 2$, $N_v = 12$.

A comparison between the original phase portrait (left) and the predicted one (right) (figure 3) shows that the computed signal is globally very similar in shape to the experimental trajectory. Moreover, the frequency content of the calculated signal is impressively close to the original one. Perceptively, the ear cannot make a distinction between the two signals.

A successfull computation has been also realized in the chaotic regime. Figure 4 shows the initial part of the original and the predicted signals. The comparison of spectra can be made in Figure 5, where one can see that the dense spectra of the original signal is well recovered. However, predicted spectrum tends to be more dense than the original, which results to a slight difference in the ear.

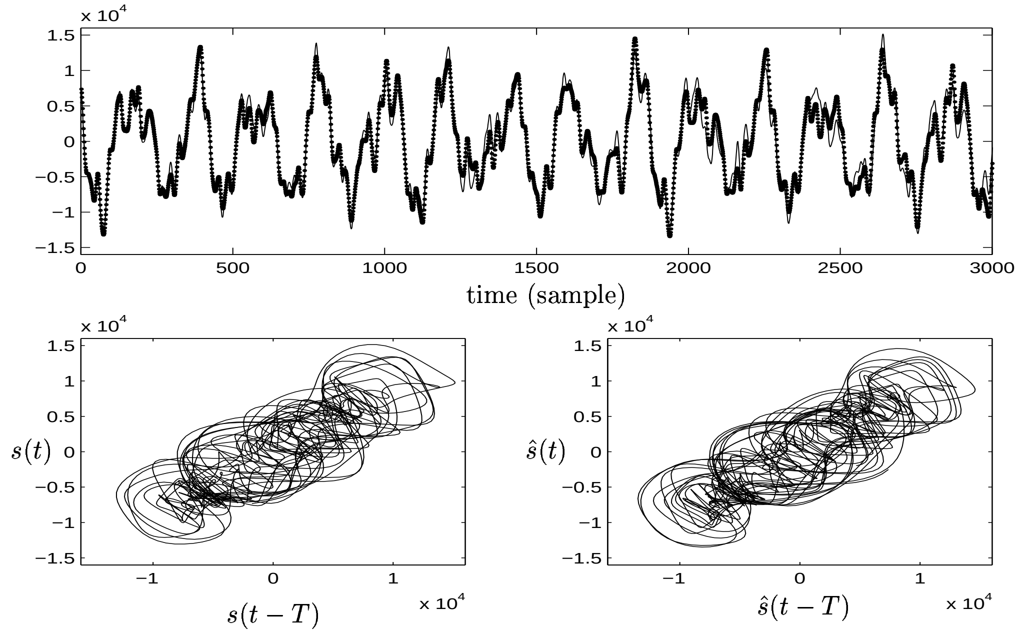

Fig. 4: Comparison between the original signal (solid line) and the predicted one (marked by points on the temporal shape). The reconstruction are also represented : original (left), predicted(right). The prediction has been realized with $T = 12$; $d_E = 10$ and $d_L = 2$.

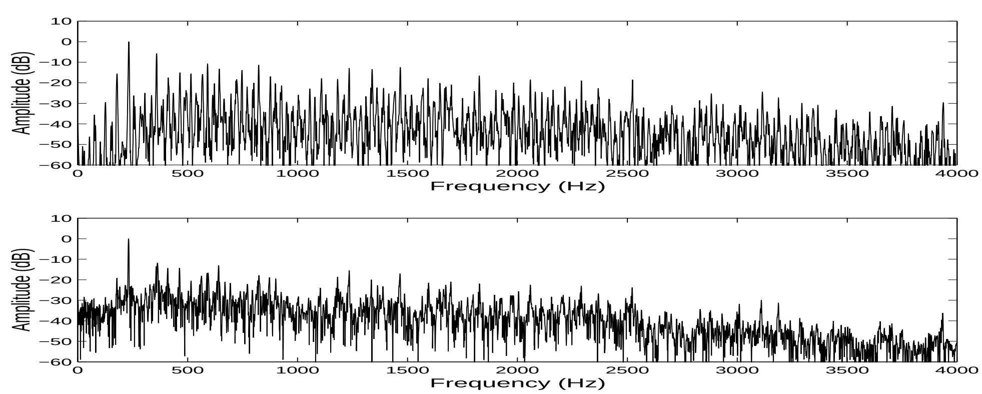

Fig. 5: Comparison of the spectra in the chaotic regime. The predicted spectrum (bottom) is more dense than the original, resulting in a slight difference in the ear.

To resume, it has been possible to generate a long time series from a short recorded one (les than half a second has been utilized). The global properties of the computed signal are close to those of the original one. This is in good accordance with the previous studies showing that a low-dimensional chaos is at work in the case of the forced vibrations. Although numerical problems can appear, the behaviour of the algorithm is globally satisfactory.

Free oscillations of a gong

A second case was considered with the free oscillations of a gong (chinese tam-tam), struck by a mallet at time $t = 0$. The signal is recorded with an accelerometer glued at the middle of the gong. Predictions have been made for a number of selected portions of the signal. The main problem in this case is that the frequency-content of the signal varies much with time. The upward cascade of energy from the low frequency to the high frequency domain, before the global decay of the sound, is characteristic of the sound produced by those instruments. In the signal considered, the frequency content is limited to the band 0-600 Hz just after the impact, and then energy is found all over the band 0-1800 Hz. Although the predicted signal is very close to the original (see Figure 6), the difference in the ear is very pronounced. This mainly follows from the underlying hypothesis of the algorithm which is not completely fulfilled in this case: the predicted signal behaves like a stationnary chaotic trajectory who lives on a strange attractor, extrapolated from a short portion of the original signal. Important psychoacoustic features of the sound (enrichment of the spectrum, followed by a global decay) are thus not rendered; although the tone quality is recognizable.

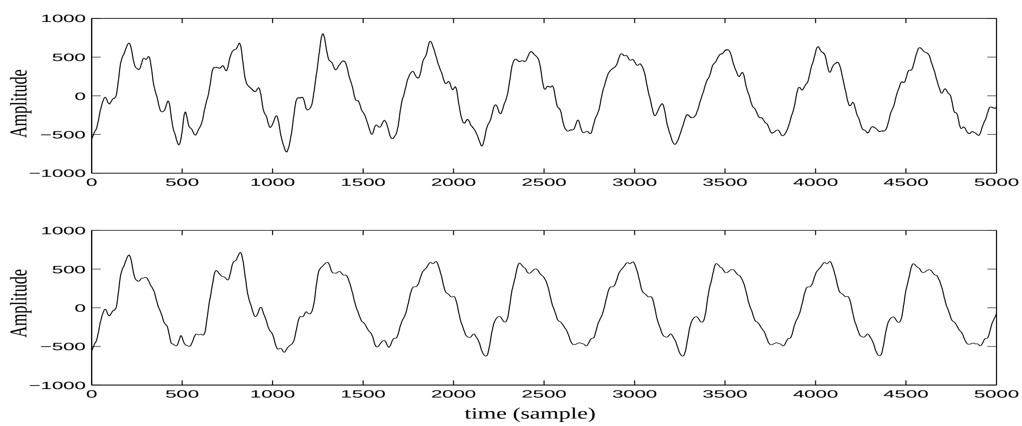

Fig. 6: Comparison between the original signal of the gong (up) and the synthetic one (bottom).

CONCLUSION

A signal-based attempt to generate synthetic time series having the same properties than experimental ones recorded on cymbals and gongs have been presented. An algorithm, based on non-linear signal processing methods, has been written, which seems to be the only way to produce synthetic signals with a dense and broadband Fourier spectrum. Successfull computations have been made on a cymbal harmonically forced at its center. This shows that the "inverse" problem stated in the introduction can be correctly solved; and this reinforced the assumptions on low-dimensional chaos for the signal at hand. A more difficult case has also been treated: a freely oscillating gong, which has time-varying dynamical properties. This case clearly points out that the algorithm encounter difficulties to generate realistic sounds when the underlying hypothesis of low-dimensional chaos are partially fulfilled, although the global shape of the synthetic signal is very close to the original. Hence, a more precise study of the link between the trajectory in reconstructed phase space and the signals predicted has to be conducted to carry out applications toward musical synthesis.

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