

Mixing Dynamic Programming and Spatial Decomposition Methods

Application to the decentralized management of urban microgrids

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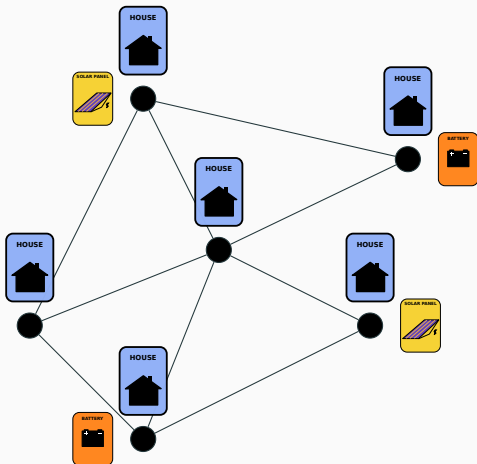
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Motivation

We consider a *peer-to-peer* urban microgrid where houses exchange energy

We formulate it as a **large-scale stochastic** optimization problem



Question: **how to manage it in an (sub)optimal manner?**

Motivation

We will see that, for **large** district microgrid, e.g.

- 48 buildings
- 16 batteries
- 30 edges network

methods **mixing spatial and temporal decomposition** (price and resource) give better results than the **standard SDDP** algorithm

- in terms of CPU time: **×3 faster**
- in terms of cost gap: **1.5% better**

Network	SDDP CPU time	Price CPU time	SDDP cost value	Price cost value
48-Nodes	453'	128'	3550 ± 2.3	3490 ± 2.3

Lecture outline

Optimization upper and lower bounds by decomposition

Nodal decomposition of a network optimization problem

Numerical results on urban microgrids of increasing size

Conclusion

Optimization upper and lower bounds by decomposition

Abstract problem

We consider the following **optimization problem**

$$\begin{aligned} V^\# &= \inf_{u^1, \dots, u^N} \sum_{i=1}^N J^i(u^i) \\ \text{s.t. } & u^i \in \mathbb{U}_{\text{ad}}^i, \quad i = 1, \dots, N \\ & \underbrace{(\Theta^1(u^1), \dots, \Theta^N(u^N))}_{\text{coupling constraint}} \in S \end{aligned}$$

with

- $u^i \in \mathbb{U}^i$ be a local decision variable
- $J^i : \mathbb{U}^i \rightarrow \mathbb{R}$, $i \in \llbracket 1, N \rrbracket$ be a local objective
- \mathbb{U}_{ad}^i be a subset of \mathbb{U}^i
- $\Theta^i : \mathbb{U}^i \rightarrow \mathcal{C}^i$ be a local constraint mapping
- S be a subset of $\mathcal{C} = \mathcal{C}^1 \times \dots \times \mathcal{C}^N$

We denote by S° the **polar cone** of S

$$S^\circ = \{p \in \mathcal{C}^* \mid \langle p, r \rangle \leq 0 \quad \forall r \in S\}$$

Price and resource value functions

For each $i \in \llbracket 1, N \rrbracket$, we define

- the **local price value function**

$$\underline{V}^i[p^i] = \inf_{u^i} J^i(u^i) + \langle p^i, \Theta^i(u^i) \rangle, \quad \forall p^i \in (\mathcal{C}^i)^*$$

- the **local resource value function**

$$\overline{V}^i[r^i] = \inf_{u^i} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i, \quad \forall r^i \in \mathcal{C}^i$$

Theorem

For any

- **admissible price** $p = (p^1, \dots, p^N) \in S^\circ$
- **admissible resource** $r = (r^1, \dots, r^N) \in S$

$$\sum_{i=1}^N \underline{V}^i[p^i] \leq V^\# \leq \sum_{i=1}^N \overline{V}^i[r^i]$$

Application to multistage stochastic optimization

We consider a **large scale stochastic optimal control problem**

$$V_0^\#(x_0) = \inf_{\mathbf{X}, \mathbf{U}} \mathbb{E} \left[\sum_{i=1}^N \sum_{i=1}^N \sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) \right]$$

s.t. $\mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1})$, $\mathbf{X}_0^i = x_0^i$
 $\sigma(\mathbf{U}_t^i) \subset \sigma(\mathbf{W}_0, \dots, \mathbf{W}_t)$
 $(\Theta_t^1(\mathbf{X}_t^1, \mathbf{U}_t^1), \dots, \Theta_t^N(\mathbf{X}_t^N, \mathbf{U}_t^N)) \in S_t S_t$

with

- $\mathbf{W} = (\mathbf{W}_1, \dots, \mathbf{W}_T)$: **global** white noise process
- $\mathbf{X}^i = (\mathbf{X}_0^i, \dots, \mathbf{X}_T^i)$: i -th **local** state process
- $\mathbf{U}^i = (\mathbf{U}_0^i, \dots, \mathbf{U}_{T-1}^i)$: i -th **local** control process
- $g_t^i: \mathbb{X}_t^i \times \mathbb{U}_t^i \times \mathbb{W}_t \rightarrow \mathbb{X}_{t+1}^i$: i -th **local** dynamics
- $\Theta_t^i: \mathbb{X}_t^i \times \mathbb{U}_t^i \rightarrow \mathcal{C}^i$: i -th **local** coupling function
- $L_t^i: \mathbb{X}_t^i \times \mathbb{U}_t^i \times \mathbb{W}_t \rightarrow \mathbb{R}$: i -th **local** instantaneous cost
- $K^i: \mathbb{X}_T^i \rightarrow \mathbb{R}$: i -th **local** final cost

Bounds for the global problem by spatial decomposition

Theorem

For any *admissible price process* $\mathbf{P} = (\mathbf{P}^1, \dots, \mathbf{P}^N) \in S^\circ$

and any *admissible resource process* $\mathbf{R} = (\mathbf{R}^1, \dots, \mathbf{R}^N) \in S$

$$\sum_{i=1}^N \underline{V}_0^i[\mathbf{P}^i](x_0^i) \leq V_0^\sharp(x_0) \leq \sum_{i=1}^N \overline{V}_0^i[\mathbf{R}^i](x_0^i)$$

with, for each $i \in \llbracket 1, N \rrbracket$, the *price local value function*

$$\begin{aligned} \underline{V}_0^i[\mathbf{P}^i](x_0^i) &= \inf_{\mathbf{x}^i, \mathbf{u}^i} \mathbb{E} \left[\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + \langle \mathbf{P}_t^i, \Theta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i) \rangle + K^i(\mathbf{X}_T^i) \right] \\ \text{s.t. } \mathbf{X}_{t+1}^i &= g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \quad \mathbf{X}_0^i = x_0^i \\ \sigma(\mathbf{U}_t^i) &\subset \sigma(\mathbf{W}_0, \dots, \mathbf{W}_t) \end{aligned}$$

and the *resource local value function*

$$\begin{aligned} \overline{V}_0^i[\mathbf{R}^i](x_0^i) &= \inf_{\mathbf{x}^i, \mathbf{u}^i} \mathbb{E} \left[\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}) + K^i(\mathbf{X}_T^i) \right] \\ \text{s.t. } \mathbf{X}_{t+1}^i &= g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}), \quad \mathbf{X}_0^i = x_0^i \\ \sigma(\mathbf{U}_t^i) &\subset \sigma(\mathbf{W}_0, \dots, \mathbf{W}_t) \\ \Theta_t^i(x_t^i, \mathbf{U}_t^i) &= \mathbf{R}_t^i \end{aligned}$$

Mix of spatial and temporal decompositions

1. To obtain bounds of the optimal value $V_0^\#(x_0)$, we have performed **two spatial decompositions** to compute

- a collection $\{\underline{V}_0^i[\mathbf{P}^i](x_0^i)\}_{i \in \llbracket 1, N \rrbracket}$ of price local value functions
- a collection $\{\overline{V}_0^i[\mathbf{R}^i](x_0^i)\}_{i \in \llbracket 1, N \rrbracket}$ of resource local value functions

The computation of these local values can be performed in **parallel**

2. To compute each local value, we perform **temporal decomposition** based on **Dynamic Programming** (DP). For each i , we will obtain

- a sequence $\{\underline{V}_t^i[\mathbf{P}^i]\}_{t \in \llbracket 0, T \rrbracket}$ of price local value functions
- a sequence $\{\overline{V}_t^i[\mathbf{R}^i]\}_{t \in \llbracket 0, T \rrbracket}$ of resource local value functions

The computation of these local values functions is done **sequentially**

To be able to use Dynamic Programming, we need to **carefully choose the coordination processes \mathbf{P} and \mathbf{R}** so that the dimension of the state in the local price and resource sub-problems remains tractable for DP.

Computing local value functions by time decomposition

We only deal with **deterministic price and resource processes**

No additional state variables in the sub-problems

Price decomposition

- Fix a **deterministic** admissible price $p = (p^1, \dots, p^N) \in S^o$
- For each i , compute $\underline{V}_0^i[p^i](x_0^i)$ by **Dynamic Programming** (state x_t^i)

$$\begin{aligned}\underline{V}_t^i[p^i](x_t^i) &= \inf_{u_t^i} \mathbb{E}[L_t(x_t^i, u_t^i, \mathbf{W}_{t+1}) \\ &\quad + \langle p_t^i, \Theta_t^i(x_t^i, u_t^i) \rangle \\ &\quad + \underline{V}_{t+1}^i[p^i](g_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}))]\end{aligned}$$

- Return the sequence $\{\underline{V}_t^i[p^i]\}_{i \in \llbracket 1, M \rrbracket}$

Resource decomposition

- Fix a **deterministic** resource $r = (r^1, \dots, r^N) \in S$
- For each i , compute $\bar{V}_0^i[r^i](x_0^i)$ by **Dynamic Programming** (state x_t^i)

$$\begin{aligned}\bar{V}_t^i[r^i](x_t^i) &= \inf_{u_t^i} \mathbb{E}[L_t(x_t^i, u_t^i, \mathbf{W}_{t+1}) \\ &\quad + \bar{V}_{t+1}^i[r^i](g_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}))] \\ &\quad \text{s.t. } \Theta_t^i(x_t^i, u_t^i) = r_t^i\end{aligned}$$

- Return the sequence $\{\bar{V}_t^i[r^i]\}_{i \in \llbracket 1, M \rrbracket}$

Bounds improvement by optimization

For any **admissible deterministic price process** $p \in S^o$ and **resource process** $r \in S$, we have computed lower and upper bounds for the problem optimal value:

$$\sum_{i=1}^N \underline{V}_0^i[p^i](x_0^i) \leq V_0^\#(x_0) \leq \sum_{i=1}^N \overline{V}_0^i[r^i](x_0^i)$$

To obtain **tighter bounds**, we **maximise** the lower bound w.r.t. the price and **minimise** the upper bound w.r.t. the resource:

$$\sup_{p \in S^o} \sum_{i=1}^N \underline{V}_0^i[p^i](x_0^i) \leq V_0^\#(x_0) \leq \inf_{r \in S} \sum_{i=1}^N \overline{V}_0^i[r^i](x_0^i)$$

Being deterministic variables, the **optimal price and resource processes** \hat{p} and \hat{r} can be computed using **gradient-like algorithms**.

Admissible control policies

Once the “best” value functions $\underline{V}_t^i[\hat{p}^i]$ and $\bar{V}_t^i[\hat{r}^i]$ have been obtained, it becomes possible to devise **global admissible policies**

- **global price policy** $\{\underline{\pi}_t\}_{t \in [0, T-1]}$

$$\begin{aligned} \underline{\pi}_t(x_t^1, \dots, x_t^N) \in \arg \min_{u_t^1, \dots, u_t^N} \mathbb{E} \left[\sum_{i=1}^N L_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}) + \sum_{i=1}^N \underline{V}_{t+1}^i[\hat{p}^i](\mathbf{x}_{t+1}^i) \right] \\ \text{s.t. } \mathbf{x}_{t+1}^i = g_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}), \quad \forall i \in [1, N] \\ (\Theta_t(x_t^1, u_t^1), \dots, \Theta_t(x_t^N, u_t^N)) \in S_t \end{aligned}$$

- **global resource policy** $\{\bar{\pi}_t\}_{t \in [0, T-1]}$

$$\begin{aligned} \bar{\pi}_t(x_t^1, \dots, x_t^N) \in \arg \min_{u_t^1, \dots, u_t^N} \mathbb{E} \left[\sum_{i=1}^N L_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}) + \sum_{i=1}^N \bar{V}_{t+1}^i[\hat{r}^i](\mathbf{x}_{t+1}^i) \right] \\ \text{s.t. } \mathbf{x}_{t+1}^i = g_t^i(x_t^i, u_t^i, \mathbf{W}_{t+1}), \quad \forall i \in [1, N] \\ (\Theta_t(x_t^1, u_t^1), \dots, \Theta_t(x_t^N, u_t^N)) \in S_t \end{aligned}$$

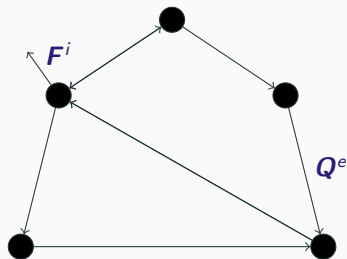
Progress status

- First, we have obtained **lower** and **upper** bounds for a global optimization problem with coupling constraints thanks to two **spatial decomposition** schemes
 - price decomposition
 - resource decomposition
- Second, we have computed the lower and upper bounds by dynamic programming (**temporal decomposition**)
- Using the price and resource Bellman value functions, we have devised two **online policies** for the **global** system
- Now, we apply these decomposition schemes to a **large-scale network problem**

Nodal decomposition of a network optimization problem

Network and flows

Directed graph $G = (\mathcal{V}, \mathcal{E})$



- Q_t^e flow through edge e ,
- F_t^i flow imported at node i

Let A be the *node-edge* incidence matrix

Each node corresponds to a building with its own devices (battery, solar panel...)

At each time $t \in \llbracket 0, T - 1 \rrbracket$, the **Kirchhoff current law** couples node and edge flows

$$A Q_t + F_t = 0$$

Optimization problem at a given node

At each node $i \in \mathcal{V}$, given a node flow process F^i , we minimize the cost

$$J_{\mathcal{V}}^i(F^i) = \min_{\mathbf{x}^i, \mathbf{u}^i} \mathbb{E} \left[\sum_{t=0}^{T-1} L_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i) + K^i(\mathbf{X}_T^i) \right]$$

subject to, for all $t \in \llbracket 0, T-1 \rrbracket$

i) nodal dynamics constraints (for battery and hot water tank)

$$\mathbf{X}_{t+1}^i = g_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, \mathbf{W}_{t+1}^i)$$

ii) non-anticipativity constraints (future remains unknown)

$$\sigma(\mathbf{U}_t^i) \subset \sigma(\mathbf{W}_0, \dots, \mathbf{W}_{t+1})$$

iii) nodal load balance equations (production + import = demand)

$$\Delta_t^i(\mathbf{X}_t^i, \mathbf{U}_t^i, F_t^i, \mathbf{W}_{t+1}^i) = 0$$

Some differences with the previous stochastic optimization problem

- **Hasard-Decision** setting
- **Local noise** \mathbf{W}_t^i in the formulation of problem at node i ,
- **Global noise** $\mathbf{W}_t = (\mathbf{W}_{t+1}^1, \dots, \mathbf{W}_{t+1}^N)$ in the non-anticipativity constraint

Transportation cost and global optimization problem

We define the **network cost** as the sum over time and edge of the costs of flows Q_t^e through the edges of the network

$$J_{\mathcal{E}}(\mathbf{Q}) = \mathbb{E} \left(\sum_{t=0}^{T-1} \sum_{e \in \mathcal{E}} l_t^e(Q_t^e) \right)$$

This transportation cost is **additive** in space and in time!

The global **optimization problem** is obtained by gathering all costs

$$\begin{aligned} V_0^\# &= \min_{\mathbf{F}, \mathbf{Q}} \sum_{i \in \mathcal{V}} J_{\mathcal{V}}^i(\mathbf{F}^i) + J_{\mathcal{E}}(\mathbf{Q}) \\ &\text{s.t. } A\mathbf{Q} + \mathbf{F} = 0 \end{aligned}$$

Price and resource decompositions

- **Price** problem:

$$\begin{aligned}\underline{V}_0[p] &= \min_{F, Q} \sum_{i \in \mathcal{V}} J_{\mathcal{V}}^i(F^i) + J_{\mathcal{E}}(Q) + \langle p, A Q + F \rangle \\ &= \sum_{i \in \mathcal{V}} \left(\min_{F_i} J_{\mathcal{V}}^i(F^i) + \langle p^i, F^i \rangle \right) + \left(\min_Q J_{\mathcal{E}}(Q) + \langle A^T p, Q \rangle \right)\end{aligned}$$

- **Resource** problem:

$$\begin{aligned}\overline{V}_0[r] &= \min_{F, Q} \sum_{i \in \mathcal{V}} J_{\mathcal{V}}^i(F^i) + J_{\mathcal{E}}(Q) \quad \text{s.t.} \quad A r + F = 0, \quad Q = r \\ &= \sum_{i \in \mathcal{V}} \left(\min_{F_i} J_{\mathcal{V}}^i(F^i) \quad \text{s.t.} \quad F^i = -(A r)^i \right) + \left(\min_Q J_{\mathcal{E}}(Q) \quad \text{s.t.} \quad Q = r \right)\end{aligned}$$

Objective

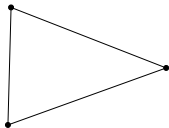
Find **deterministic** processes \hat{p} and \hat{r} with a **gap as small as possible**

$$\max_p \underline{V}_0[p] \leq V_0^\# \leq \min_r \overline{V}_0[r]$$

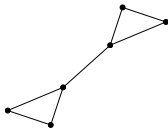
Numerical results on urban microgrids of increasing size

Different urban configurations

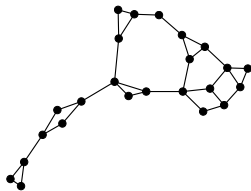
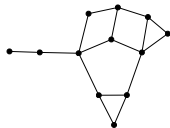
3-Nodes



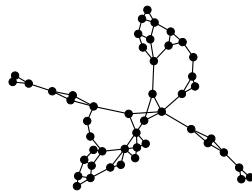
6-Nodes



12-Nodes



24-Nodes



48-Nodes

Problem settings

- One day horizon with a 15mn time step: $T = 96$
- Weather corresponds to a sunny day in Paris (*June 28th, 2015*)
- We mix three kinds of buildings
 1. battery + electrical hot water tank
 2. solar panel + electrical hot water tank
 3. electrical hot water tankand suppose that all consumers are commoners sharing their devices

Algorithms inventory

Price decomposition

Price decomposition: maximization w.r.t. a **deterministic price** p

- Each nodal subproblem solved by SDDP (quick convergence)
- Maximisation w.r.t. p by Quasi-Newton (BFGS) method

$$p^{(k+1)} = p^{(k)} + \rho^{(k)} H^{(k)} \nabla \underline{V}(p^{(k)})$$

- Oracle $\nabla \underline{V}(p)$ estimated by Monte Carlo ($N^{scen} = 1,000$)

Resource decomposition

Similar to price decomposition

Global SDDP

We use the SDDP algorithm as **reference method**

- Noises $\mathbf{W}_t^1, \dots, \mathbf{W}_t^N$ are independent node by node:
total support size is $|\text{supp}(\mathbf{W}_t^i)|^N$. **We need to resample the noise!**
- Convergence once gap between UB and LB is lower than 1%

Exact upper and lower bounds on the global problem

	Network	3-Nodes	6-Nodes	12-Nodes	24-Nodes	48-Nodes
State dim.	$ \mathcal{X} $	4	8	16	32	64
Global SDDP	time	1'	3'	10'	79'	453'
Global SDDP	LB	225.2	455.9	889.7	1752.8	3310.3
Price	time	6'	14'	29'	41'	128'
Price	LB	213.7	447.3	896.7	1787.0	3396.4
Resource	time	3'	7'	22'	49'	91'
Resource	UB	253.9	527.3	1053.7	2105.4	4016.6

For the **48-Nodes** microgrid,

- price decomposition gives a **better exact lower bound** than global SDDP

$$\begin{aligned} \underline{V}_0[sddp] &\leq \underline{V}_0[price] \leq V^\# \leq \bar{V}_0[resource] \\ 3310.3 &\leq 3396.4 \leq V^\# \leq 4016.6 \end{aligned}$$

- price decomposition is more than **3 times faster** than global SDDP

Policy evaluation by Monte Carlo simulation (1,000 scenarios)

	3-Nodes	6-Nodes	12-Nodes	24-Nodes	48-Nodes
SDDP policy	226 ± 0.6	471 ± 0.8	936 ± 1.1	1859 ± 1.6	3550 ± 2.3
Price policy	228 ± 0.6	464 ± 0.8	923 ± 1.2	1839 ± 1.6	3490 ± 2.3
Gap	+0.9 %	-1.5%	-1.4%	-1.1%	-1.7%
Resource policy	229 ± 0.6	471 ± 0.8	931 ± 1.1	1856 ± 1.6	3503 ± 2.2
Gap	+1.3 %	0.0%	-0.5%	-0.2%	-1.2%

All the cost values above are **statistical upper bounds** of $V^\#$

For the **48-Nodes** microgrid,

- price policy **beats** global SDDP policy and resource policy

$$V^\# \leq C[\text{price}] \leq C[\text{resource}] \leq C[\text{sddp}]$$

$$V^\# \leq 3490 \leq 3503 \leq 3550$$

- the **exact upper bound** given by resource decomposition is **not so tight**

$$\underline{V}_0[\text{price}] \leq V^\# \leq C[\text{price}] \leq \bar{V}_0[\text{resource}]$$

$$3396.4 \leq V^\# \leq 3490 \leq 4016.6$$

gap
<3%
>18%

Conclusion

Conclusion

- We have two algorithms that **decompose spatially and temporally** a large-scale optimization problem under coupling constraints
- On this case study, **price decomposition beats global SDDP** for large instances (≥ 24 nodes)
 - in time (more than twice faster)
 - in precision (more than 1% better)
- **Can we obtain tighter bounds?** (*especially for resource decomposition...*)
If we select properly price P and resource R processes among the class of **Markovian** processes (instead of deterministic ones) we can obtain “better” nodal value functions (with an extended local state)

Further details in

François Pacaud (2018). “Decentralized Optimization Methods for Efficient Energy Management under Stochasticity”. PhD Thesis, Université Paris Est.