Dual Approximate Dynamic Programming for Large Scale Hydro Valleys

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ENSTA ParisTech and ENPC ParisTech

PGMO Days 2015

Joint work with J.-C. Alais and C. Ousri, supported by the FMJH Program Gaspard Monge for Optimization.
Motivation

Electricity production management for hydro valleys

- **1 year time horizon**: compute each month the Bellman functions (“water values”)
- **stochastic framework**: rain, market prices
- **large-scale valley**: 5 dams and more

We wish to remain within the scope of Dynamic Programming.
How to avoid the curse of dimensionality?

Aggregation methods
- fast to run method
- require some homogeneity between units

Stochastic Dual Dynamic Programming (SDDP)
- efficient method for this kind of problems
- strong assumptions (convexity, linearity)

Dual Approximate Dynamic Programming (DADP)
- spatial decomposition method
- complexity almost linear in the number of dams
- approximation methods in the stochastic framework

This talk: present numerical results for large-scale hydro valleys using DADP.
Lecture outline

1. **Dams management problem**
   - Hydro valley modeling
   - Optimization problem

2. **DADP in a nutshell**
   - Spatial decomposition
   - Constraint relaxation

3. **Numerical experiments**
   - Academic examples
   - More realistic examples
1. Dams management problem
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2. DADP in a nutshell
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3. Numerical experiments
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Operating scheme

\[ u_t^i : \text{water turbinated by dam } i \text{ at time } t, \]
\[ x_t^i : \text{water volume of dam } i \text{ at time } t, \]
\[ a_t^i : \text{water inflow at dam } i \text{ at time } t, \]
\[ p_t^i : \text{water price at dam } i \text{ at time } t, \]

**Randomness:** \[ w_t^i = (a_t^i, p_t^i) \text{ , } w_t = (w_t^1, \ldots, w_t^N). \]
Dynamics and cost functions

Dam dynamics:

\[ x_{t+1}^i = f_t^i(x_t^i, u_t^i, w_t^i, z_t^i) , \]

\[ = x_t^i - u_t^i + a_t^i + z_t^i - s_t^i , \]

\[ z_{t+1}^i \text{ being the outflow of dam } i: \]

\[ z_{t+1}^i = g_t^i(x_t^i, u_t^i, w_t^i, z_t^i) , \]

\[ = u_t^i + \max \left\{ 0, x_t^i - u_t^i + a_t^i + z_t^i - \bar{x}_t^i \right\} . \]

We assume the Hazard-Decision information structure \((u_t^i \text{ is chosen once } w_t^i \text{ is observed}), \) so that \(u_t^i \leq u_t^i \leq \min \{ \bar{u}_t^i, x_t^i + a_t^i + z_t^i - \bar{x}_t^i \} . \)

Gain at time \( t < T : \)

\[ L_t^i(x_t^i, u_t^i, w_t^i, z_t^i) = p_t^i u_t^i - \epsilon(u_t^i)^2. \]

Final gain at time \( T : \)

\[ K_t^i(x_T^i) = -a T \min\{0, x_T^i - \bar{x}_T^i \}^2. \]
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The **global optimization problem** reads:

\[
\begin{align*}
\max_{(X,U,Z)} \quad & \mathbb{E}\left( \sum_{i=1}^{N} \left( \sum_{t=0}^{T-1} L_{t}^{i}(X_{t}^{i}, U_{t}^{i}, W_{t}^{i}, Z_{t}^{i}) + K_{i}(X_{T}) \right) \right),
\end{align*}
\]

subject to:

\[
\begin{align*}
X_{t+1}^{i} &= f_{t}^{i}(X_{t}^{i}, U_{t}^{i}, W_{t}^{i}, Z_{t}^{i}), \quad \forall i, \quad \forall t, \\
\sigma(U_{t}^{i}) &\subset \sigma(W_{0}, \ldots, W_{t}), \quad \forall i, \quad \forall t, \\
Z_{t+1}^{i} &= g_{t}^{i}(X_{t}^{i}, U_{t}^{i}, W_{t}^{i}, Z_{t}^{i}), \quad \forall i, \quad \forall t.
\end{align*}
\]

**Assumption.** Noises \(W_{0}, \ldots, W_{T-1}\) are independent over time.
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Price decomposition

- Dualize the coupling constraints \( Z_t^{i+1} = g_t^i(X_t^i, U_t^i, W_t^i, Z_t^i) \).
  Note that the associated multiplier \( \Lambda_t^{i+1} \) is a random variable.

- Solve the dual problem using a gradient-like algorithm.

- At iteration \( k \), the duality term:
  \[ \Lambda_t^{i+1,(k)} \cdot (Z_t^{i+1} - g_t^i(X_t^i, U_t^i, W_t^i, Z_t^i)) \]
  is the difference of two terms:
  - \( \Lambda_t^{i+1,(k)} \cdot Z_t^{i+1} \leadsto \text{dam } i+1 \),
  - \( \Lambda_t^{i+1,(k)} \cdot g_t^i(\cdots) \leadsto \text{dam } i \).

- Dam by dam decomposition for the maximization in \( (X, U, Z) \)
at \( \Lambda_t^{i+1,(k)} \) fixed.
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DADP core idea

The $i$-th subproblem writes:

$$
\max_{u^i, z^i, x^i} \mathbb{E}\left( \sum_{t=0}^{T-1} \left( L_t^i(x^i_t, u^i_t, w^i_t, z^i_t) + \Lambda_t^i(k) \cdot z^i_t \right) - \Lambda_t^{i+1} \cdot g_t^i(x^i_t, u^i_t, w^i_t, z^i_t) \right) + K^i(x^i_T),
$$

but $\Lambda_t^{i,(k)}$ depends on the whole past of noises $(w_0, \ldots, w_t)$. . .

The core idea of DADP is

- to replace the constraint $z_t^{i+1} - g_t^i(x^i_t, u^i_t, w^i_t, z^i_t) = 0$ by its conditional expectation with respect to $y_t^i$:

$$
\mathbb{E}(z_t^{i+1} - g_t^i(x^i_t, u^i_t, w^i_t, z^i_t) \mid y_t^i) = 0,
$$

- where $(y_0^i, \ldots, y_{T-1}^i)$ is a “well-chosen” information process.
DADP core idea

The $i$-th subproblem writes:

$$\max_{u_i, z_i, x_i} \mathbb{E}\left( \sum_{t=0}^{T-1} \left( L_t^i(x_t^i, u_t^i, w_t^i, z_t^i) + \Lambda_t^i(k) \cdot z_t^i \right. \right.$$

$$\left. \left. - \Lambda_{t+1,k}^i \cdot g_t^i(x_t^i, u_t^i, w_t^i, z_t^i) \right) + K^i(x_T^i) \right),$$

but $\Lambda_t^i(k)$ depends on the whole past of noises $(w_0, \ldots, w_T)$. . .

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- where $(y_0^i, \ldots, y_{T-1}^i)$ is a “well-chosen” information process.
Subproblems in DADP

DADP thus consists of a constraint relaxation.

It is easy to see that such a relaxation is equivalent to replace the multiplier $\Lambda_t^{i,(k)}$ by its conditional expectation $\mathbb{E}(\Lambda_t^{i,(k)} \mid Y_{t-1}^i)$. The expression of the $i$-th subproblem becomes:

\[
\max_{U^i, Z^i, X^i} \mathbb{E} \left( \sum_{t=0}^{T-1} \left( L_t^i(X_t^i, U_t^i, W_t^i, Z_t^i) + \mathbb{E}(\Lambda_t^{i,(k)} \mid Y_{t-1}^i) \cdot Z_t^i \right. \\
- \mathbb{E}(\Lambda_t^{i+1,(k)} \mid Y_t^i) \cdot g_t^i(X_t^i, U_t^i, W_t^i, Z_t^i) \right) \\
+ K^i(X_T^i) \right).
\]

If the process $Y_{t-1}^i$ follows a dynamical equation, DP applies.
A crude relaxation: \( Y^i_t \equiv \text{cste} \)

1. The multipliers \( \Lambda^i,(k) \) appear only in the subproblems by means of their expectations \( \mathbb{E}(\Lambda^i,(k)) \), so that each subproblem involves a 1-dimensional state variable.

2. For the gradient algorithm, the coordination task reduces to:

   \[
   \mathbb{E}(\Lambda^i,(k+1)) = \mathbb{E}(\Lambda^i,(k)) \\
   + \rho_t \mathbb{E}\left(Z^i_{t+1,(k)} - g^i_t(X^i_t, U^i_t, W^i_t, Z^i_t)\right). 
   \]

3. The constraints taken into account by DADP are in fact:

   \[
   \mathbb{E}\left(Z^i_{t+1} - g^i_t(X^i_t, U^i_t, W^i_t, Z^i_t)\right) = 0. 
   \]

The DADP solutions do not satisfy the initial constraints: need to use an heuristic method to regain admissibility.
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Three case studies

Discretization

\[ T \sim 12 \]
\[ X \sim 41 \]
\[ U \sim 6 \]
\[ W \sim 10 \]

“3Dams” Valley

“4Dams” Valley

“5Dams” Valley
## Results

<table>
<thead>
<tr>
<th>Valley</th>
<th>3Dams</th>
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<tbody>
<tr>
<td>DP CPU time</td>
<td>5'</td>
<td>1700'</td>
<td>677000'</td>
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<td>DP value</td>
<td>2482.0</td>
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**Table:** Results obtained by DP

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<td>2401.3</td>
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<td>Gap with DP</td>
<td>−3.2%</td>
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<td>Dual value</td>
<td>2687.5</td>
<td>3995.8</td>
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**Table:** Results obtained by DADP “Expectation”

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2Results obtained using a 16 core 32 threads Intel®Core i7 based computer.
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Three valleys

Discretization

$T \sim 12, \ W \sim 10$

fine grids for $X$ and $U$

Vicdessos Valley  Dordogne Valley  Stooopt Valley
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<td>SDDP&lt;sub&gt;d&lt;/sub&gt; CPU time</td>
<td>29500'</td>
<td></td>
<td>106000'</td>
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<tr>
<td>SDDP&lt;sub&gt;d&lt;/sub&gt; value</td>
<td>2228.5</td>
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CPU time comparison

CPU time (logarithmic scale)

# dams

Time

DP
SDDP
DADP

P. Carpentier & J.-P. Chancelier
DADP applied to large scale hydro valleys
PGMO Days 22 / 23
Conclusions and perspectives

Conclusions for DADP

- Fast numerical convergence of the method.
- Near-optimal results even when using a “crude” relaxation.
- Method that can be used for very large valleys

General perspectives

- Apply to more complex topologies (smart grids).
- Connection with other decomposition methods.
- Theoretical study.
P. Carpentier et G. Cohen.  
*Décomposition-coordination en optimisation déterministe et stochastique.*  

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*Résolution de grands problèmes en optimisation stochastique dynamique.*  

J.-C. Alais.  
*Risque et optimisation pour le management d’énergies.*  
Thèse de doctorat, Université Paris-Est, 2013.

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*Contributions aux méthodes de décomposition en optimisation stochastique.*  

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*Price decomposition in large-scale stochastic optimal control.*  