Mixing Dynamic Programming and Spatial Decomposition Methods

Application to the decentralized management of urban microgrids

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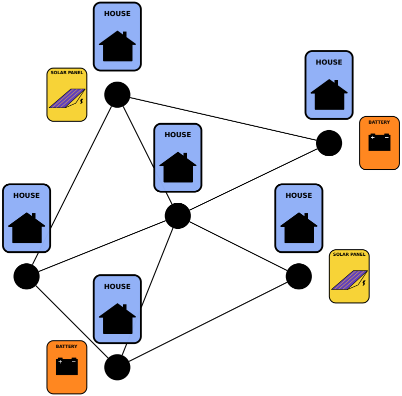
Motivation

We consider a *peer-to-peer* microgrid where houses exchange energy, and we formulate it as a large-scale stochastic optimization problem



How to manage it in an (sub)optimal manner?

Motivation

We will see that, for a large district microgid, e.g.

- 48 buildings
- 16 batteries
- 71 edges network

methods mixing temporal decomposition (dynamic programming) and spatial decomposition (price or resource allocation) give better results than the standard SDDP algorithm

- in terms of CPU time: $\times 3$ faster

SDDP CPU time: 453'	Decomp CPU time: 128'

- in terms of cost gap: 1.5% better

SDDP policy cost: 3550	Decomp policy cost: 3490

Lecture outline

1. Tools for mixing spatial and temporal decomposition methods
   - Upper and lower bounds using spatial decomposition
   - Temporal decomposition using dynamic programming
   - The case of deterministic coordination processes

2. Application to the management of urban microgrids
   - Nodal decomposition of a network optimization problem
   - Numerical results on urban microgrids of increasing size

An abstract optimization problem

We consider the following optimization problem

$$V_0^\sharp = \inf_{u^1 \in \mathbb{U}^1_{\mathrm{ad}}, \cdots, u^N \in \mathbb{U}^N_{\mathrm{ad}}} \sum_{i=1}^{N} J^i(u^i)$$
$$\text{s.t.} \quad \underbrace{\left(\Theta^1(u^1), \cdots, \Theta^N(u^N)\right) \in S}_{\text{coupling constraint}}$$

with

- $u^i \in \mathbb{U}^i$ be a local decision variable
- $J^i : \mathbb{U}^i \to \mathbb{R}, \; i \in [\![1, N]\!]$ be a local objective
- $\mathbb{U}^i_{\mathrm{ad}}$ be a subset of $\mathbb{U}^i$
- $\Theta^i : \mathbb{U}^i \to \mathcal{C}^i$ be a local constraint mapping
- S be a subset of $\mathcal{C} = \mathcal{C}^1 \times \cdots \times \mathcal{C}^N$

We denote by S^o the polar cone of S

$$S^o = \left\{p \in \mathcal{C}^\star \mid \langle p \,, r \rangle \leq 0 \quad \forall r \in S\right\}$$

Price and resource value functions

For each $i \in [\![1, N]\!]$,

- for any price $p^i \in (\mathcal{C}^i)^\star$, we define the local price value

$$\underline{V}_0^i[p^i] = \inf_{u^i \in \mathbb{U}^i_{\mathrm{ad}}} J^i(u^i) + \langle p^i , \Theta^i(u^i) \rangle$$

- for any resource $r^i \in \mathcal{C}^i$, we define the local resource value

$$\overline{V}_0^i[r^i] = \inf_{u^i \in \mathbb{U}^i_{\mathrm{ad}}} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i$$

Theorem 1 (Upper and lower bounds for optimal value)

For any

- *admissible price* $p = (p^1, \cdots, p^N) \in S^o$
- *admissible resource* $r = (r^1, \cdots, r^N) \in S$

$$\sum_{i=1}^{N} \underline{V}_0^i[p^i] \;\leq\; V_0^\sharp \;\leq\; \sum_{i=1}^{N} \overline{V}_0^i[r^i]$$

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The case of multistage stochastic optimization

Assume that the local price value

$$\underline{V}_0^i[p^i] = \inf_{u^i} J^i(u^i) + \left\langle p^i \, , \Theta^i(u^i) \right\rangle \; ,$$

corresponds to a stochastic optimal control problem

$$\underline{V}_0^i[\boldsymbol{P}^i](x_0^i) = \inf_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\left(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + \left\langle \boldsymbol{P}_t^i \, , \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) \right\rangle + K^i(\boldsymbol{X}_T^i) \right)$$

$$\text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) \; , \quad \boldsymbol{X}_0^i = x_0^i$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_t)$$

This local problem can be solved by Dynamic Programming (DP) under restrictive assumptions:

- the noise process $\boldsymbol{W}$ is a white noise process
- the price process $\boldsymbol{P}$ follows a dynamics in small dimension

DP leads to a collection $\left\{ \underline{V}_t^i[\boldsymbol{P}^i] \right\}_{t \in [\![0, T]\!]}$ of local price value functions

The case of multistage stochastic optimization II

Similar considerations hold true for the local resource value

$$\overline{V}_0^i[r^i] = \inf_{u^i} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i$$

considered as a stochastic optimal control problem

$$\begin{aligned}\overline{V}_0^i[\boldsymbol{R}^i](x_0^i) = \inf_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\Big(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + K^i(\boldsymbol{X}_T^i) \Big) \\ \text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) \ , \ \ \boldsymbol{X}_0^i = x_0^i \\ \sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_t) \\ \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) = \boldsymbol{R}_t^i\end{aligned}$$

Mix of spatial and temporal decompositions I

For any admissible price process $\boldsymbol{P} \in S^o$ and any admissible resource process $\boldsymbol{R} \in S$, we have bounds of the optimal value $V_0^\sharp$

$$\sum_{i=1}^{N} \underline{V}_0^i[\boldsymbol{P}^i] \;\leq\; V_0^\sharp \;\leq\; \sum_{i=1}^{N} \overline{V}_0^i[\boldsymbol{R}^i]$$

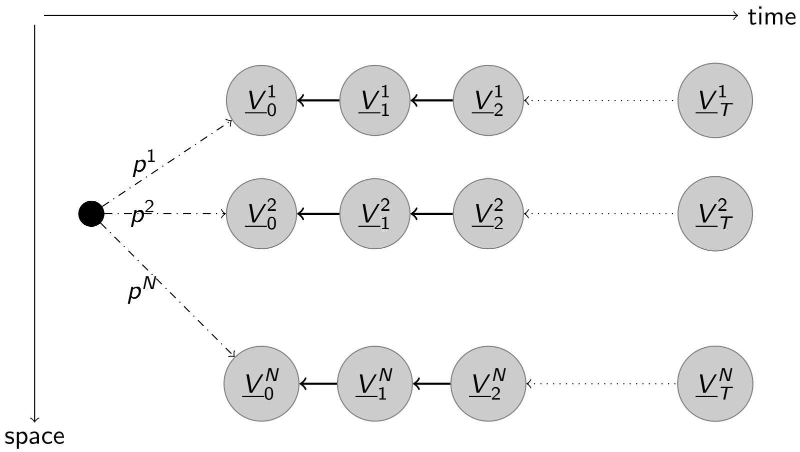
1. To obtain the bounds, we perform spatial decompositions giving
   - a collection $\left\{\underline{V}_0^i[\boldsymbol{P}^i](x_0^i)\right\}_{i\in[\![1,N]\!]}$ of price local values
   - a collection $\left\{\overline{V}_0^i[\boldsymbol{R}^i](x_0^i)\right\}_{i\in[\![1,N]\!]}$ of resource local values

   The computation of these local values can be performed in parallel

2. To compute each local value, we perform temporal decomposition based on Dynamic Programming (**DP**). For each i, we obtain
   - a sequence $\left\{\underline{V}_t^i[\boldsymbol{P}^i]\right\}_{t\in[\![0,T]\!]}$ of price local value functions
   - a sequence $\left\{\overline{V}_t^i[\boldsymbol{R}^i]\right\}_{t\in[\![0,T]\!]}$ of resource local value functions

   The computation of these local values functions is done sequentially

Mix of spatial and temporal decompositions II



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The case of deterministic price and resource processes I

We assume that $\boldsymbol{W}$ is a white noise process, and we restrict ourselves to **deterministic** admissible processes $p \in S^o$ and $r \in S$

- The local value functions $\underline{V}_t^i[p^i]$ and $\overline{V}_t^i[r^i]$ are easy to compute because they only depend on the local state variable x^i
- It is easy to obtain tighter bounds by maximizing the lower bound w.r.t. prices and minimizing the upper bound w.r.t. resources

$$\sup_{p \in S^o} \sum_{i=1}^{N} \underline{V}_0^i[p^i] \;\leq\; V_0^\sharp \;\leq\; \inf_{r \in S} \sum_{i=1}^{N} \overline{V}_0^i[r^i]$$

The case of deterministic price and resource processes II

We assume that $\boldsymbol{W}$ is a white noise process, and we restrict ourselves to **deterministic** processes $p \in S^o$ and $r \in S$

The local value functions $\underline{V}_t^i[p^i]$ and $\overline{V}_t^i[r^i]$ allow the computation of global policies by solving static optimization problems

- In the case of local price value functions, the policy is obtained as

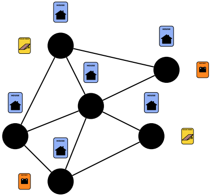
$$\underline{\pi}_t(x_t^1, \cdots, x_t^N) \in \underset{u_t^1, \cdots, u_t^N}{\arg\min} \ \mathbb{E}\left(\sum_{i=1}^N L_t^i(x_t^i, u_t^i, \boldsymbol{W}_{t+1}) + \sum_{i=1}^N \underline{V}_{t+1}^i[p^i](\boldsymbol{X}_{t+1}^i) \right)$$
$$\text{s.t.} \quad \boldsymbol{X}_{t+1}^i = g_t^i(x_t^i, u_t^i, \boldsymbol{W}_{t+1}) , \quad \forall i \in [\![1, N]\!]$$
$$\big(\Theta_t(x_t^1, u_t^1), \cdots, \Theta_t(x_t^N, u_t^N)\big) \in S_t$$

Estimating the expected cost of such a policy by Monte Carlo simulation leads to a statistical upper bound of the optimal cost of the problem

Progress status

- First, we have obtained lower and upper bounds for a global optimization problem with coupling constraints thanks to two spatial decomposition schemes
  - price decomposition
  - resource decomposition
- Second, we have computed the lower and upper bounds by dynamic programming (temporal decomposition)
- Using the price and resource Bellman value functions, we have devised two online policies for the global problem

- Now, we apply these decomposition schemes to large-scale network problems



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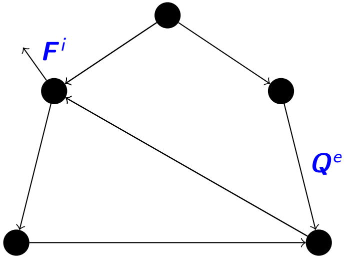
Network and flows

Directed graph $G = (\mathcal{V}, \mathcal{E})$



Each node corresponds to a building with its own devices (battery, hot water tank, solar panel. . .)

At each time $t \in [\![0, T-1]\!]$, the Kirchhoff current law couples node and edge flows

$$AQ_t + F_t = 0$$

- Q_t^e flow through edge e,
- F_t^i flow imported at node i

Let A be the *node-edge* incidence matrix

Optimization problem at a given node

At each **node** $i \in \mathcal{V}$, given a node flow process $\boldsymbol{F}^i$, we minimize the house cost

$$J^i_{\mathcal{V}}(\boldsymbol{F}^i) = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\left(\sum_{t=0}^{T-1} L^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{W}^i_{t+1}) + K^i(\boldsymbol{X}^i_T) \right)$$

subject to, for all $t \in [\![0, T-1]\!]$

i) nodal dynamics constraints (battery, hot water tank)

$$\boldsymbol{X}^i_{t+1} = g^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{W}^i_{t+1})$$

ii) non-anticipativity constraints (future remains unknown)

$$\sigma(\boldsymbol{U}^i_t) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_{t+1})$$

iii) nodal load balance equations (production + import = demand)

$$\Delta^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{F}^i_t, \boldsymbol{W}^i_{t+1}) = 0$$

Transportation cost and global optimization problem

We define the network cost as the sum over time and **edges** of the costs of flows $\boldsymbol{Q}_t^e$ through the edges of the network

$$J_{\mathcal{E}}(\boldsymbol{Q}) = \mathbb{E}\left(\sum_{t=0}^{T-1} \sum_{e \in \mathcal{E}} l_t^e(\boldsymbol{Q}_t^e)\right)$$

This transportation cost is additive in space and in time!

The global optimization problem is obtained by gathering all costs

$$V_0^\sharp = \min_{\boldsymbol{F},\boldsymbol{Q}} \sum_{i \in \mathcal{V}} J_{\mathcal{V}}^i(\boldsymbol{F}^i) + J_{\mathcal{E}}(\boldsymbol{Q})$$

$$\text{s.t. } A\boldsymbol{Q} + \boldsymbol{F} = 0$$

Price and resource decompositions

- Price problem:

$$\underline{V}_0[\boldsymbol{P}] = \min_{\boldsymbol{F},\boldsymbol{Q}} \sum_{i\in\mathcal{V}} J^i_{\mathcal{V}}(\boldsymbol{F}^i) + J_{\mathcal{E}}(\boldsymbol{Q}) + \langle \boldsymbol{P} \,, A\boldsymbol{Q} + \boldsymbol{F} \rangle$$
$$= \sum_{i\in\mathcal{V}} \underbrace{\Big(\min_{\boldsymbol{F}_i} \; J^i_{\mathcal{V}}(\boldsymbol{F}^i) + \langle \boldsymbol{P}^i \,, \boldsymbol{F}^i \rangle \Big)}_{\text{Node } i\text{'s subproblem}} + \underbrace{\Big(\min_{\boldsymbol{Q}} \; J_{\mathcal{E}}(\boldsymbol{Q}) + \langle A^\top \boldsymbol{P} \,, \boldsymbol{Q} \rangle \Big)}_{\text{Network subproblem}}$$

- Resource problem:

$$\overline{V}_0[\boldsymbol{R}] = \min_{\boldsymbol{F},\boldsymbol{Q}} \sum_{i\in\mathcal{V}} J^i_{\mathcal{V}}(\boldsymbol{F}^i) + J_{\mathcal{E}}(\boldsymbol{Q}) \quad \text{s.t.} \quad A\boldsymbol{R} + \boldsymbol{F} = 0 \,, \quad \boldsymbol{Q} = \boldsymbol{R}$$
$$= \sum_{i\in\mathcal{V}} \Big(\min_{\boldsymbol{F}_i} \; J^i_{\mathcal{V}}(\boldsymbol{F}^i) \;\text{s.t.}\; \boldsymbol{F}^i = -(A\boldsymbol{R})^i \Big) + \Big(\min_{\boldsymbol{Q}} \; J_{\mathcal{E}}(\boldsymbol{Q}) \;\text{s.t.}\; \boldsymbol{Q} = \boldsymbol{R} \Big)$$

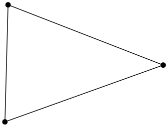
Objective

Find **deterministic** processes $\widehat{p}$ and $\widehat{r}$ with a gap as small as possible

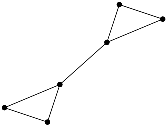
$$\max_{p} \; \underline{V}_0[p] \;\leq\; V_0^\sharp \;\leq\; \min_{r} \; \overline{V}_0[r]$$

Different urban configurations

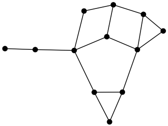
3-Nodes

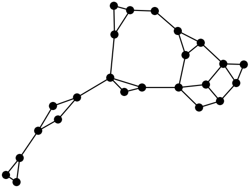


6-Nodes

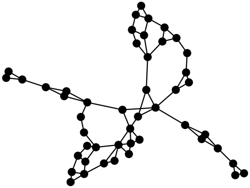


12-Nodes





24-Nodes



48-Nodes

Problem settings

Thanks to the periodicity of demands and electricity tariffs, the microgrid management problem can be solved day by day

- One day horizon with a 15mn time step: $T = 96$
- Weather corresponds to a sunny day in Paris *(June 28, 2015)*
- We mix three kinds of buildings
  1. battery + electrical hot water tank
  2. solar panel + electrical hot water tank
  3. electrical hot water tank

  and suppose that all consumers are commoners sharing their devices

Algorithms implemented on the problem

SDDP

We use the SDDP algorithm to solve the problem globally...

- but noises $W_t^1, \cdots, W_t^N$ are independent node by node, so that the support size of the noise may be huge ($|\mathrm{supp}(W_t^i)|^N$). We must resample the noise to be able to compute the cuts

Price decomposition

Spatial decomposition and maximization w.r.t. a deterministic price p

- Each nodal subproblem solved by a DP-like method
- Maximisation w.r.t. p by Quasi-Newton (BFGS) method

$$p^{(k+1)} = p^{(k)} + \rho^{(k)} H^{(k)} \nabla \underline{V}_0[p^{(k)}]$$

- Oracle $\nabla \underline{V}_0[p]$ estimated by Monte Carlo ($N^{scen} = 1,000$)

Resource decomposition

Spatial decomposition and minimization w.r.t. a deterministic resource process r

Exact upper and lower bounds on the global problem

	Network	3-Nodes	6-Nodes	12-Nodes	24-Nodes	48-Nodes
State dim.	$\lvert\mathbb{X}\rvert$	4	8	16	32	64
SDDP	time	1'	3'	10'	79'	453'
SDDP	LB	225.2	455.9	889.7	1752.8	3310.3
Price	time	6'	14'	29'	41'	128'
Price	LB	213.7	447.3	896.7	1787.0	3396.4
Resource	time	3'	7'	22'	49'	91'
Resource	UB	253.9	527.3	1053.7	2105.4	4016.6

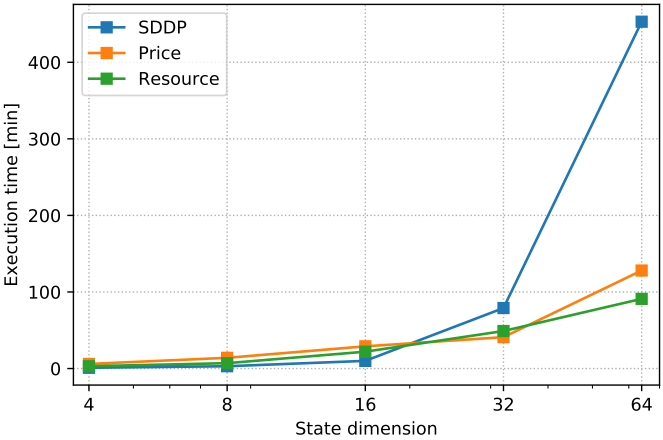
For the **48-Nodes** microgrid,

- price decomposition gives a better exact lower bound than SDDP

$$\underbrace{3310.3}_{\underline{V}_0[sddp]} \leq \underbrace{3396.4}_{\underline{V}_0[price]} \leq V_0^\sharp \leq \underbrace{4016.6}_{\overline{V}_0[resource]}$$

- **price decomposition is more than 3 times faster than SDDP**

Time evolution



Policy evaluation by Monte Carlo (1,000 scenarios)

	3-Nodes	6-Nodes	12-Nodes	24-Nodes	48-Nodes
SDDP policy	226 ± 0.6	471 ± 0.8	936 ± 1.1	1859 ± 1.6	3550 ± 2.3
Price policy	228 ± 0.6	464 ± 0.8	923 ± 1.2	1839 ± 1.6	3490 ± 2.3
Gap	+0.9 %	-1.5%	-1.4%	-1.1%	-1.7%
Resource policy	229 ± 0.6	471 ± 0.8	931 ± 1.1	1856 ± 1.6	3503 ± 2.2
Gap	+1.3 %	0.0%	-0.5%	-0.2%	-1.2%

All the cost values above are statistical upper bounds of $V_0^\sharp$

For the **48-Nodes** microgrid,

- price policy beats SDDP policy and resource policy

$$V_0^\sharp \leq \underbrace{3490}_{C[price]} \leq \underbrace{3503}_{C[resource]} \leq \underbrace{3550}_{C[sddp]}$$

- the exact upper bound given by resource decomposition is not so tight

$$\underbrace{3396.4}_{\underline{V}_0[price]} \leq V_0^\sharp \leq \underbrace{3490}_{C[price]} \leq \underbrace{3503}_{C[resource]} \leq \underbrace{4016.6}_{\overline{V}_0[resource]}$$

gap <3% ≈ 3% >18%

Conclusion

- We have two algorithms that decompose spatially and temporally a large-scale optimization problem under coupling constraints
- On this case study, price decomposition beats SDDP for large instances ($\geq$ 24 nodes)
  - in computing time (more than twice faster)
  - in precision (more than 1% better)
- Can we obtain tighter bounds? *(especially for resource decomposition. . .)* If we select properly price $\boldsymbol{P}$ and resource $\boldsymbol{R}$ processes among the class of Markovian processes (instead of deterministic ones) we can obtain "better" nodal value functions (with an extended local state)

Further details in

F. Pacaud. *Decentralized Optimization Methods for Efficient Energy Management under Stochasticity*. PhD Thesis, Université Paris Est, 2018.

P. Carpentier, J.-P. Chancelier, M. De Lara and F. Pacaud. *Upper and lower bounds for Bellman functions by spatial decomposition*. Working paper.