Multi-Stage Stochastic Optimization for Clean Energy Transition

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Mixing Dynamic Programming and Spatial Decomposition Methods

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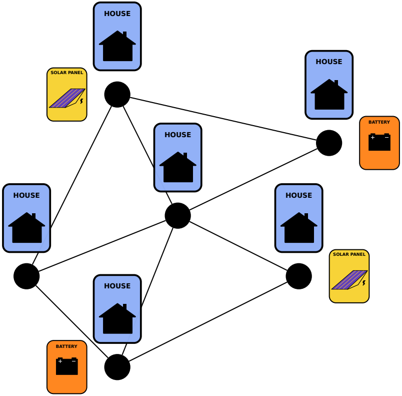
Motivation

We consider a *peer-to-peer* microgrid where houses exchange energy, and we formulate it as a large-scale stochastic optimization problem



How to manage it in an (sub)optimal manner?

Motivation

We will see that, for a large district microgid, e.g.

- 48 buildings
- 16 batteries
- 71 edges network

methods mixing temporal decomposition (dynamic programming) and spatial decomposition (price or resource allocation) give better results than the standard SDDP algorithm (implemented using approximations)

- in terms of CPU time: $\times 3$ faster

SDDP CPU time: 453'	Decomp CPU time: 128'

- in terms of cost gap: 1.5% better

SDDP policy cost: 3550	Decomp policy cost: 3490

Lecture outline

Tools for mixing spatial and temporal decomposition methods

- Upper and lower bounds using spatial decomposition
- Temporal decomposition using dynamic programming
- The case of deterministic coordination processes

Application to the management of urban microgrids

- Nodal decomposition of a network optimization problem
- Numerical results on urban microgrids of increasing size

Another point of view: decentralized information structure

- Centralized versus decentralized information structure
- Bounds for the decentralized information structure
- Analysis of the upper bound

Tools for mixing spatial and temporal decomposition methods

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An abstract optimization problem

We consider the following optimization problem

$$V_0^\sharp = \min_{u^1 \in \mathcal{U}^1_{\mathrm{ad}}, \cdots, u^N \in \mathcal{U}^N_{\mathrm{ad}}} \sum_{i=1}^{N} J^i(u^i)$$

$$\text{s.t.} \quad \underbrace{\left(\Theta^1(u^1), \cdots, \Theta^N(u^N)\right) \in S}_{\text{coupling constraint}}$$

with

- $u^i \in \mathcal{U}^i$ be a local decision variable
- $J^i : \mathcal{U}^i \to \mathbb{R}, \; i \in [\![1, N]\!]$ be a local objective
- $\mathcal{U}^i_{\mathrm{ad}}$ be a subset of $\mathcal{U}^i$
- $\Theta^i : \mathcal{U}^i \to \mathcal{C}^i$ be a local constraint mapping
- S be a subset of $\mathcal{C} = \mathcal{C}^1 \times \cdots \times \mathcal{C}^N$

We denote by S^o the polar cone of S

$$S^o = \left\{ p \in \mathcal{C}^\star \mid \langle p \, , r \rangle \leq 0 \quad \forall r \in S \right\}$$

Price and resource value functions

For each $i \in [\![1, N]\!]$,

- for any price $p^i \in (\mathcal{C}^i)^\star$, we define the local price value

$$\underline{V}_0^i[p^i] = \min_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} J^i(u^i) + \left\langle p^i \,, \Theta^i(u^i) \right\rangle$$

- for any resource $r^i \in \mathcal{C}^i$, we define the local resource value

$$\overline{V}_0^i[r^i] = \min_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i$$

Theorem 1 (Upper and lower bounds for optimal value)

For any

- *admissible price* $p = (p^1, \cdots, p^N) \in S^o$
- *admissible resource* $r = (r^1, \cdots, r^N) \in S$

$$\sum_{i=1}^N \underline{V}_0^i[p^i] \;\leq\; V_0^\sharp \;\leq\; \sum_{i=1}^N \overline{V}_0^i[r^i]$$

The case of multistage stochastic optimization I

Assume that the local price value

$$\underline{V}_0^i[p^i] = \min_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} J^i(u^i) + \left\langle p^i , \Theta^i(u^i) \right\rangle ,$$

corresponds to a stochastic optimal control problem

$$\underline{V}_0^i[\boldsymbol{P}^i](x_0^i) = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\bigg(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + \left\langle \boldsymbol{P}_t^i , \Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) \right\rangle + K^i(\boldsymbol{X}_T^i) \bigg)$$

$$\text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) , \quad \boldsymbol{X}_0^i = x_0^i$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots , \boldsymbol{W}_t)$$

This local control problem can be solved by Dynamic Programming (DP) under restrictive assumptions:

- the noise process $\boldsymbol{W}$ is a white noise process
- the price process $\boldsymbol{P}$ follows a dynamics in small dimension

DP leads to a collection $\left\{ \underline{V}_t^i[\boldsymbol{P}^i] \right\}_{t \in [\![0, T]\!]}$ of local price value functions

The case of multistage stochastic optimization II

Similar considerations hold true for the local resource value

$$\overline{V}_0^i[r^i] = \min_{u^i \in \mathcal{U}_{\mathrm{ad}}^i} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i$$

considered as a stochastic optimal control problem

$$\begin{aligned}
\overline{V}_0^i[\boldsymbol{R}^i](x_0^i) = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\Big(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) + K^i(\boldsymbol{X}_T^i) \Big) \\
\text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}) \,, \quad \boldsymbol{X}_0^i = x_0^i \\
\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_t) \\
\Theta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i) = \boldsymbol{R}_t^i
\end{aligned}$$

Mix of spatial and temporal decompositions I

For any admissible price process $\boldsymbol{P} \in S^o$ and any admissible resource process $\boldsymbol{R} \in S$, we have bounds of the optimal value $V_0^\sharp$

$$\sum_{i=1}^{N} \underline{V}_0^i[\boldsymbol{P}^i](x_0^i) \;\leq\; V_0^\sharp \;\leq\; \sum_{i=1}^{N} \overline{V}_0^i[\boldsymbol{R}^i](x_0^i)$$

1. To obtain the bounds, we perform spatial decompositions giving
   - a collection $\left\{\underline{V}_0^i[\boldsymbol{P}^i](x_0^i)\right\}_{i \in [\![1,N]\!]}$ of price local values
   - a collection $\left\{\overline{V}_0^i[\boldsymbol{R}^i](x_0^i)\right\}_{i \in [\![1,N]\!]}$ of resource local values

   The computation of these local values can be performed in parallel

2. To compute each local value, we perform temporal decomposition based on Dynamic Programming. For each i, we obtain
   - a sequence $\left\{\underline{V}_t^i[\boldsymbol{P}^i]\right\}_{t \in [\![0,T]\!]}$ of price local value functions
   - a sequence $\left\{\overline{V}_t^i[\boldsymbol{R}^i]\right\}_{t \in [\![0,T]\!]}$ of resource local value functions

   The computation of these local values functions is done sequentially

Mix of spatial and temporal decompositions II

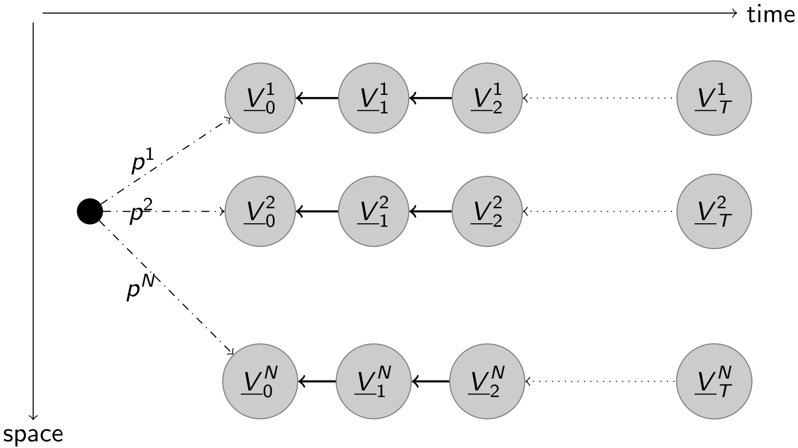

Figure: The case of price decomposition

Tools for mixing spatial and temporal decomposition methods
- Upper and lower bounds using spatial decomposition
- Temporal decomposition using dynamic programming
- The case of deterministic coordination processes

Application to the management of urban microgrids
- Nodal decomposition of a network optimization problem
- Numerical results on urban microgrids of increasing size

Another point of view: decentralized information structure
- Centralized versus decentralized information structure
- Bounds for the decentralized information structure
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The case of deterministic price and resource processes I

We assume that $\boldsymbol{W}$ is a white noise process, and we restrict ourselves to **deterministic** admissible processes $p \in S^o$ and $r \in S$

- The local value functions $\underline{V}_t^i[p^i]$ and $\overline{V}_t^i[r^i]$ are easy to compute because they only depend on the local state variable x^i

- It is easy to obtain tighter bounds by maximizing the lower bound w.r.t. prices and minimizing the upper bound w.r.t. resources

$$\sup_{p \in S^o} \sum_{i=1}^{N} \underline{V}_0^i[p^i](x_0^i) \;\leq\; V_0^\sharp \;\leq\; \inf_{r \in S} \sum_{i=1}^{N} \overline{V}_0^i[r^i](x_0^i)$$

The case of deterministic price and resource processes II

We assume that $\boldsymbol{W}$ is a white noise process, and we restrict ourselves to **deterministic** admissible processes $p \in S^o$ and $r \in S$

The local value functions $\underline{V}_t^i[p^i]$ and $\overline{V}_t^i[r^i]$ allow the computation of global policies by solving static optimization problems

- In the case of local price value functions, the policy is obtained as

$$\underline{\gamma}_t(x_t^1, \cdots, x_t^N) \in \operatorname*{arg\,min}_{u_t^1, \cdots, u_t^N} \mathbb{E}\bigg(\sum_{i=1}^{N} L_t^i(x_t^i, u_t^i, \boldsymbol{W}_{t+1}) + \sum_{i=1}^{N} \underline{V}_{t+1}^i[p^i](\boldsymbol{X}_{t+1}^i) \bigg)$$

$$\text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(x_t^i, u_t^i, \boldsymbol{W}_{t+1}) , \ \forall i \in [\![1, N]\!]$$

$$\big(\Theta_t(x_t^1, u_t^1), \cdots, \Theta_t(x_t^N, u_t^N)\big) \in S_t$$

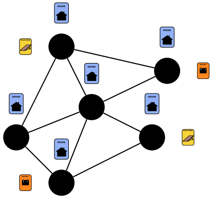
- A global policy based on resource value functions is also available

Estimating the expected cost of such policies by Monte Carlo simulation leads to a statistical upper bound of the optimal cost of the problem

Progress status

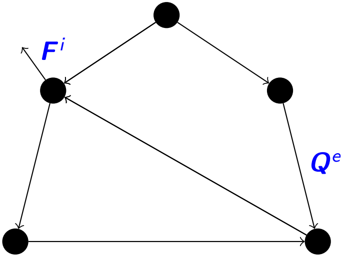
- First, we have obtained lower and upper bounds
  for a global optimization problem with coupling constraints
  thanks to two spatial decomposition schemes
  - price decomposition
  - resource decomposition
- Second, we have computed the lower and upper bounds
  by dynamic programming (temporal decomposition)
- Using the price and resource Bellman value functions,
  we have devised two online policies for the global problem

- Now, we apply these decomposition schemes
  to large-scale network problems



Network and flows

Directed graph $G = (\mathcal{V}, \mathcal{E})$



- $\boldsymbol{Q}_t^e$ flow through edge e,
- $\boldsymbol{F}_t^i$ flow imported at node i

Let A be the *node-edge* incidence matrix

Each node corresponds to a building with its own devices (battery, hot water tank, solar panel. . .)

At each time $t \in [\![0, T-1]\!]$, the Kirchhoff current law couples node and edge flows

$$A\boldsymbol{Q}_t + \boldsymbol{F}_t = 0$$

Optimization problem at a given node

At each **node** $i \in \mathcal{V}$, given a node flow process $\boldsymbol{F}^i$, we minimize the house cost

$$J^i_{\mathcal{V}}(\boldsymbol{F}^i) = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i} \mathbb{E}\left(\sum_{t=0}^{T-1} L^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{W}^i_{t+1}) + K^i(\boldsymbol{X}^i_T) \right)$$

subject to, for all $t \in [\![0, T-1]\!]$

i) nodal dynamics constraints (battery, hot water tank)

$$\boldsymbol{X}^i_{t+1} = g^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{W}^i_{t+1})$$

ii) non-anticipativity constraints (future remains unknown)

$$\sigma(\boldsymbol{U}^i_t) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_{t+1})$$

iii) nodal load balance equations (demand - production = import)

$$\Delta^i_t(\boldsymbol{X}^i_t, \boldsymbol{U}^i_t, \boldsymbol{W}^i_{t+1}) = \boldsymbol{F}^i_t$$

Remarks

- Local noise $\boldsymbol{W}^i_t$ in the formulation of problem at node i
- Global noise $\boldsymbol{W}_t = (\boldsymbol{W}^1_{t+1}, \ldots, \boldsymbol{W}^N_{t+1})$ in the non -anticipativity constraint

Transportation cost and global optimization problem

We define the network cost as the sum over time and **edges** of the costs of flows $\boldsymbol{Q}_t^e$ through the edges of the network

$$J_{\mathcal{E}}(\boldsymbol{Q}) = \mathbb{E}\left(\sum_{t=0}^{T-1} \sum_{e \in \mathcal{E}} l_t^e(\boldsymbol{Q}_t^e) \right)$$

This transportation cost is additive in space, in time and in uncertainty!

The global optimization problem is obtained by gathering all elements

$$V_0^\sharp \; = \; \min_{\boldsymbol{F},\boldsymbol{Q}} \; \sum_{i \in \mathcal{V}} J_{\mathcal{V}}^i(\boldsymbol{F}^i) + J_{\mathcal{E}}(\boldsymbol{Q})$$

$$\text{s.t. } A\boldsymbol{Q} + \boldsymbol{F} = 0$$

Price and resource decompositions

- Price problem:

$$\underline{V}_0[\boldsymbol{P}] = \min_{\boldsymbol{F},\boldsymbol{Q}} \sum_{i \in \mathcal{V}} J^i_{\mathcal{V}}(\boldsymbol{F}^i) + J_{\mathcal{E}}(\boldsymbol{Q}) + \left\langle \boldsymbol{P} \,, A\boldsymbol{Q} + \boldsymbol{F} \right\rangle$$
$$= \sum_{i \in \mathcal{V}} \underbrace{\left(\min_{\boldsymbol{F}_i} \; J^i_{\mathcal{V}}(\boldsymbol{F}^i) + \left\langle \boldsymbol{P}^i \,, \boldsymbol{F}^i \right\rangle \right)}_{\text{Node } i\text{'s subproblem}} + \underbrace{\left(\min_{\boldsymbol{Q}} \; J_{\mathcal{E}}(\boldsymbol{Q}) + \left\langle A^\top \boldsymbol{P} \,, \boldsymbol{Q} \right\rangle \right)}_{\text{Network subproblem}}$$

- Resource problem:

$$\overline{V}_0[\boldsymbol{R}] = \min_{\boldsymbol{F},\boldsymbol{Q}} \sum_{i \in \mathcal{V}} J^i_{\mathcal{V}}(\boldsymbol{F}^i) + J_{\mathcal{E}}(\boldsymbol{Q}) \quad \text{s.t.} \quad A\boldsymbol{R} + \boldsymbol{F} = 0 \,, \quad \boldsymbol{Q} = \boldsymbol{R}$$
$$= \sum_{i \in \mathcal{V}} \left(\min_{\boldsymbol{F}_i} \; J^i_{\mathcal{V}}(\boldsymbol{F}^i) \text{ s.t. } \boldsymbol{F}^i = -(A\boldsymbol{R})^i \right) + \left(\min_{\boldsymbol{Q}} \; J_{\mathcal{E}}(\boldsymbol{Q}) \text{ s.t. } \boldsymbol{Q} = \boldsymbol{R} \right)$$

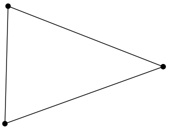
Objective

Find **deterministic** processes $\widehat{p}$ and $\widehat{r}$ with a gap as small as possible

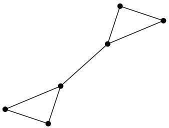
$$\sup_p \underline{V}_0[p] \;\leq\; V_0^\sharp \;\leq\; \inf_r \overline{V}_0[r]$$

Different urban configurations

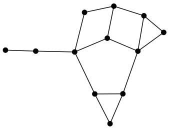
3-Nodes

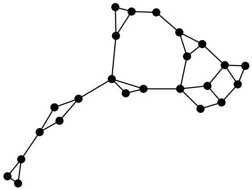


6-Nodes

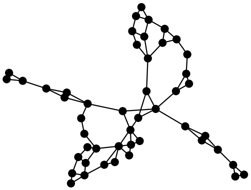


12-Nodes





24-Nodes



48-Nodes

Problem settings

Thanks to the periodicity of demands and electricity tariffs, the microgrid management problem can be solved day by day

- One day horizon with a 15mn time step: $T = 96$
- Weather corresponds to a sunny day in Paris *(June 28, 2015)*
- We mix three kinds of buildings
  1. battery + electrical hot water tank
  2. solar panel + electrical hot water tank
  3. electrical hot water tank

  and suppose that all consumers are commoners sharing their devices

Algorithms implemented on the problem

SDDP

We use the SDDP algorithm to solve the problem globally...

- but noises $\boldsymbol{W}_t^1, \cdots, \boldsymbol{W}_t^N$ are independent node by node, so that the support size of the noise may be huge ($|\mathrm{supp}(\boldsymbol{W}_t^i)|^N$). We must resample the noise to be able to compute the cuts

Price decomposition

Spatial decomposition and maximization w.r.t. a deterministic price p

- Each nodal subproblem solved by a DP-like method
- Maximisation w.r.t. p by Quasi-Newton (BFGS) method

$$p^{(k+1)} = p^{(k)} + \rho^{(k)} H^{(k)} \nabla \underline{V}_0[p^{(k)}]$$

- Oracle $\nabla \underline{V}_0[p]$ estimated by Monte Carlo ($N^{scen} = 1,000$)

Resource decomposition

Spatial decomposition and minimization w.r.t. a deterministic resource process r

Exact upper and lower bounds on the global problem

	Network	**3-Nodes**	**6-Nodes**	**12-Nodes**	**24-Nodes**	**48-Nodes**
State dim.	$\lvert\mathbb{X}\rvert$	4	8	16	32	64
SDDP	time	1'	3'	10'	79'	453'
SDDP	LB	225.2	455.9	889.7	1752.8	3310.3
Price	time	6'	14'	29'	41'	128'
Price	LB	213.7	447.3	896.7	1787.0	3396.4
Resource	time	3'	7'	22'	49'	91'
Resource	UB	253.9	527.3	1053.7	2105.4	4016.6

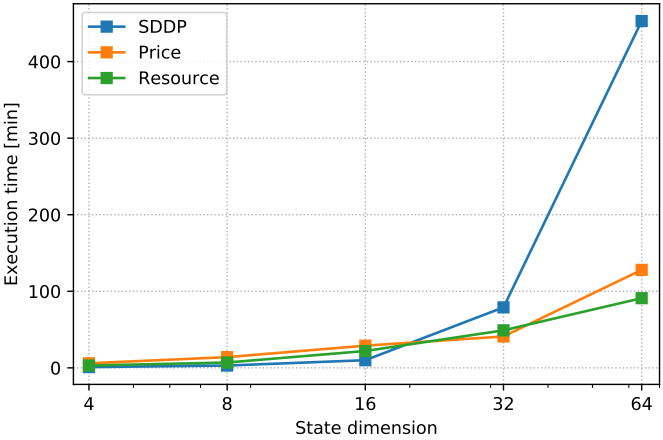
For the **48-Nodes** microgrid,

- price decomposition gives a (slightly) better exact lower bound than SDDP

$$\underbrace{3310.3}_{\underline{V}_0[sddp]} \leq \underbrace{3396.4}_{\underline{V}_0[price]} \leq V_0^\sharp \leq \underbrace{4016.6}_{\overline{V}_0[resource]}$$

- **price decomposition is more than 3 times faster than SDDP**

Time evolution



Policy evaluation by Monte Carlo (1,000 scenarios)

	3-Nodes	6-Nodes	12-Nodes	24-Nodes	48-Nodes
SDDP policy	226 ± 0.6	471 ± 0.8	936 ± 1.1	1859 ± 1.6	3550 ± 2.3
Price policy	228 ± 0.6	464 ± 0.8	923 ± 1.2	1839 ± 1.6	3490 ± 2.3
Gap	+0.9 %	-1.5%	-1.4%	-1.1%	-1.7%
Resource policy	229 ± 0.6	471 ± 0.8	931 ± 1.1	1856 ± 1.6	3503 ± 2.2
Gap	+1.3 %	0.0%	-0.5%	-0.2%	-1.2%

All the cost values above are statistical upper bounds of $V_0^\sharp$

For the **48-Nodes** microgrid,

- price policy beats SDDP policy and resource policy

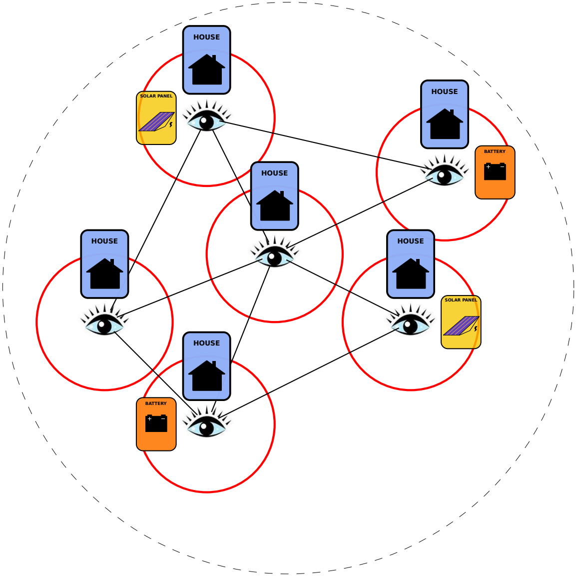
$$V_0^\sharp \leq \underbrace{3490}_{C[price]} \leq \underbrace{3503}_{C[resource]} \leq \underbrace{3550}_{C[sddp]}$$

- the exact upper bound given by resource decomposition is not so tight

$$\underbrace{3396.4}_{\underline{V}_0[price]} \leq V_0^\sharp \leq \underbrace{3490}_{C[price]} \leq \underbrace{3503}_{C[resource]} \leq \underbrace{4016.6}_{\overline{V}_0[resource]}$$

gap: $<3\%$ (C[price]), $\approx 3\%$ (C[resource]), $>18\%$ ($\overline{V}_0[resource]$)

Motivation for decentralized information



Centralized information structure

Up to now, we have studied the following problem

$$V_0^{\mathrm{C}} = \min_{\boldsymbol{F},\boldsymbol{Q}} \Bigg(\sum_{i\in\mathcal{V}} \underbrace{\min_{\boldsymbol{X}^i,\boldsymbol{U}^i} \mathbb{E}\bigg(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) + K^i(\boldsymbol{X}_T^i) \bigg)}_{J_{\mathcal{V}}^i(\boldsymbol{F}^i)} + \underbrace{\mathbb{E}\bigg(\sum_{t=0}^{T-1} \sum_{e\in\mathcal{E}} l_t^e(\boldsymbol{Q}_t^e) \bigg)}_{J_{\mathcal{E}}(\boldsymbol{Q})} \Bigg)$$

subject to, for all $t \in [\![0, T-1]\!]$ and for all $i \in \mathcal{V}$

$$A\boldsymbol{Q}_t + \boldsymbol{F}_t = 0 \qquad \text{(network constraints)}$$

$$\boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) \qquad \text{(nodal dynamic constraints)}$$

$$\Delta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) = \boldsymbol{F}_t^i \qquad \text{(nodal balance equation)}$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0, \cdots, \boldsymbol{W}_{t+1}) \qquad \text{(information constraints)}$$

with $\boldsymbol{W}_t = (\boldsymbol{W}_t^1, \ldots, \boldsymbol{W}_t^N)$: global noise process

Decentralized information structure

Consider now the following problem

$$V_0^{\mathrm{D}} = \min_{\boldsymbol{F},\boldsymbol{Q}} \Bigg(\sum_{i\in\mathcal{V}} \underbrace{\min_{\boldsymbol{X}^i,\boldsymbol{U}^i} \mathbb{E}\bigg(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) + K^i(\boldsymbol{X}_T^i) \bigg)}_{J_{\mathcal{V}}^i(\boldsymbol{F}^i)} + \underbrace{\mathbb{E}\bigg(\sum_{t=0}^{T-1} \sum_{e\in\mathcal{E}} l_t^e(\boldsymbol{Q}_t^e) \bigg)}_{J_{\mathcal{E}}(\boldsymbol{Q})} \Bigg)$$

subject to, for all $t \in [\![0, T-1]\!]$ and for all $i \in \mathcal{V}$

$$A\boldsymbol{Q}_t + \boldsymbol{F}_t = 0 \qquad \text{(network constraints)}$$

$$\boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) \qquad \text{(nodal dynamic constraints)}$$

$$\Delta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) = \boldsymbol{F}_t^i \qquad \text{(nodal balance equation)}$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_0^i, \cdots, \boldsymbol{W}_{t+1}^i) \qquad \text{(information constraints)}$$

that is, the local control $\boldsymbol{U}_t^i$ is a feedback w.r.t. local noises $(\boldsymbol{W}_0^i, \ldots, \boldsymbol{W}_{t+1}^i)$

Bounds in the decentralized case I

Consider the lower bound obtained with a deterministic price process p

$$\underline{V}_0[p] = \sum_{i \in \mathcal{V}} V_0^i[p^i] + V_0^{\mathcal{E}}[p] \ , \qquad \text{with}$$

$$V_0^i[p^i] = \min_{\boldsymbol{X}^i, \boldsymbol{U}^i, \boldsymbol{F}^i} \mathbb{E}\Big[\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) + \langle p_t^i \ , \boldsymbol{F}_t^i \rangle + K^i(\boldsymbol{X}_T^i) \Big]$$

$$\text{s.t. } \boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) \ , \ \ \boldsymbol{X}_0^i = x_0^i$$

$$\Delta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) = \boldsymbol{F}_t^i$$

$$\sigma(\boldsymbol{U}_t^i) \subset \sigma(\boldsymbol{W}_1, \ldots, \boldsymbol{W}_{t+1})$$

Replacing the global σ-field $\sigma(\boldsymbol{W}_1, \ldots, \boldsymbol{W}_{t+1})$ by the local σ-field $\sigma(\boldsymbol{W}_1^i, \ldots, \boldsymbol{W}_{t+1}^i)$ does not make any change in this local subproblem

The lower bound $\underline{V}_0[p]$ is the same for both information structures

A similar conclusion holds true for the upper bound $\overline{V}_0[r]$

Bounds in the decentralized case II

Since $\boldsymbol{W}_t = (\boldsymbol{W}_t^1, \ldots, \boldsymbol{W}_t^N)$, for all i, we have the inclusion of σ-fields

$$\sigma(\boldsymbol{W}_0^i, \ldots, \boldsymbol{W}_t^i) \subset \sigma(\boldsymbol{W}_0, \ldots, \boldsymbol{W}_t)$$

We deduce that the admissible control set in case of a decentralized information structure is smaller that the one in case of a centralized information structure, and hence

$$V_0^{\mathrm{C}} \leq V_0^{\mathrm{D}}$$

Finally, we obtain the following sequence of inequalities

$$\sup_p \underline{V}_0[p] \leq V_0^{\mathrm{C}} \leq V_0^{\mathrm{D}} \leq \inf_r \overline{V}_0[r]$$

Bounds in the decentralized case III

$$\sup_p \underline{V}_0[p] \leq V_0^{\mathrm{C}} \leq V_0^{\mathrm{D}} \leq \inf_r \overline{V}_0[r]$$

- We have seen on the numerical experiments that the lower bound was close from the optimal value V_0^{C} in the centralized case

$$\underbrace{\sup_p \underline{V}_0[p] \leq V_0^{\mathrm{C}}}_{\approx 3\%}$$

- What can we say about the upper bound and the optimal value V_0^{D} in the decentralized case?

$$\underbrace{V_0^{\mathrm{C}} \leq \inf_r \overline{V}_0[r]}_{\approx 18\%} \quad , \quad \underbrace{V_0^{\mathrm{D}} \leq \inf_r \overline{V}_0[r]}_{\textbf{Value of the gap?}}$$

Tools for mixing spatial and temporal decomposition methods
- Upper and lower bounds using spatial decomposition
- Temporal decomposition using dynamic programming
- The case of deterministic coordination processes

Application to the management of urban microgrids
- Nodal decomposition of a network optimization problem
- Numerical results on urban microgrids of increasing size

Another point of view: decentralized information structure
- Centralized versus decentralized information structure
- Bounds for the decentralized information structure
- Analysis of the upper bound

Analysis of the decentralized case

For the shake of brevity, we introduce the following notation

$$\mathcal{F}_t^i = \sigma(\boldsymbol{W}_0^i, \ldots, \boldsymbol{W}_t^i)$$

Consider the constraints that have to be met at node i in the case of a decentralized information structure

$$\boldsymbol{X}_{t+1}^i = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) \qquad \text{(nodal dynamic constraints)}$$

$$\Delta_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_{t+1}^i) = \boldsymbol{F}_t^i \qquad \text{(nodal balance equation)}$$

$$\sigma(\boldsymbol{U}_t^i) \subset \mathcal{F}_{t+1}^i \qquad \text{(information structure)}$$

By construction, the state $\boldsymbol{X}_t^i$ is a $\mathcal{F}_t^i$-measurable random variable

Thanks to both the nodal balance equation and the information structure, we deduce that the node flow $\boldsymbol{F}_t^i$ is measurable w.r.t. the σ-field $\mathcal{F}_{t+1}^i$

Analysis of the decentralized case II

Suppose that $(\boldsymbol{W}^1, \cdots, \boldsymbol{W}^N)$ are independent random processes
Otherwise stated, we add an **independence assumption in space**

At time t, consider now the global coupling constraints $A\boldsymbol{Q}_t + \boldsymbol{F}_t = 0$.
Summing these constraints leads to the aggregate coupling constraint

$$\sum_{i \in \mathcal{V}} \boldsymbol{F}_t^i = 0$$

From the aggregate constraint and the independence assumption,
we deduce that the random variables $\boldsymbol{F}_t$ (and hence $\boldsymbol{Q}_t$) are in fact
deterministic variables

Analysis of the decentralized case III

According to this conclusion, under the space independence assumption, in case of a decentralized information structure, the global minimisation problem depends on deterministic node flows f and edge flows q variables

$$
\begin{aligned}
V_0^{\mathrm{D}} &= \min_{f,q} \left(\sum_{i\in\mathcal{V}} J_{\mathcal{V}}^i(f^i) + J_{\mathcal{E}}(q) \right) \quad \text{s.t.} \quad Aq + f = 0 \\
&= \inf_r \left(\sum_{i\in\mathcal{V}} \left(\min_{f_i} \ J_{\mathcal{V}}^i(f^i) \ \text{s.t.} \ f^i = -(Ar)^i \right) \ + \ \left(\min_q \ J_{\mathcal{E}}(q) \ \text{s.t.} \ q = r \right) \right) \\
&= \inf_r \overline{V}_0[r]
\end{aligned}
$$

The upper bound $\min_r \overline{V}_0[r]$ and the optimal value V_0^{D} are **the same**

Information gap

Recall the sequence of inequalities relating optimal values and bounds

$$\sup_p \underline{V}_0[p] \;\leq\; V_0^{\mathrm{C}} \;\leq\; V_0^{\mathrm{D}} \;\leq\; \inf_r \overline{V}_0[r]$$

Gathering all the theoretical and numerical results obtained, we have

$$\underbrace{\sup \underline{V}_0[p] \;\leq\; V_0^{\mathrm{C}}}_{\approx 3\%} \;,\; \underbrace{V_0^{\mathrm{C}} \;\leq\; V_0^{\mathrm{D}}}_{\approx 18\%} \;,\; V_0^{\mathrm{D}} \;=\; \inf \overline{V}_0[r]$$

that provides a way to **quantify the information gap** in our application.

Conclusions

- We have two algorithms that decompose spatially and temporally a large-scale optimization problem under coupling constraints.
- In our case study, price decomposition beats SDDP for large instances (≥ 24 nodes)
  - in computing time (more than twice faster)
  - in precision (more than 1% better)
- Price decomposition gives (in a surprising way) a tight lower bound, whereas the upper bound given by resource decomposition is weak (which is understandable on the case study)
- We have studied the case of a decentralized information structure to explain this weakness
- Can we obtain tighter bounds? *especially for resource decomposition...*
  If we select properly price $\boldsymbol{P}$ and resource $\boldsymbol{R}$ processes among the class of Markovian processes (instead of deterministic ones) we can obtain "better" nodal value functions (with an extended local state)

Further details in

F. Pacaud
Decentralized Optimization Methods for Efficient Energy Management under Stochasticity
PhD Thesis, Université Paris Est, 2018

P. Carpentier, J.-P. Chancelier, M. De Lara and F. Pacaud
Computation by Decomposition of Upper and Lower Bounds for Large Scale Multistage Stochastic Optimization Problems
Working paper, 2019

THANK YOU FOR YOUR ATTENTION