Multi-Stage Stochastic Optimization for Clean Energy Transition

Mixing Dynamic Programming and Spatial Decomposition Methods

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Motivation

We consider a peer-to-peer microgrid where houses exchange energy, and we formulate it as a large-scale stochastic optimization problem.

How to manage it in an (sub)optimal manner?
Motivation

We will see that, for a large district microgrid, e.g.

- 48 buildings
- 16 batteries
- 71 edges network

methods mixing temporal decomposition (dynamic programming) and spatial decomposition (price or resource allocation) give better results than the standard SDDP algorithm (implemented using approximations)

- in terms of CPU time: $\times 3$ faster

| SDDP CPU time: | 453’ | Decompp CPU time: | 128’ |

- in terms of cost gap: 1.5% better

| SDDP policy cost: | 3550 | Decompp policy cost: | 3490 |
Lecture outline

Tools for mixing spatial and temporal decomposition methods
   Upper and lower bounds using spatial decomposition
   Temporal decomposition using dynamic programming
   The case of deterministic coordination processes

Application to the management of urban microgrids
   Nodal decomposition of a network optimization problem
   Numerical results on urban microgrids of increasing size

Another point of view: decentralized information structure
   Centralized versus decentralized information structure
   Bounds for the decentralized information structure
   Analysis of the upper bound
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An abstract optimization problem

We consider the following optimization problem

\[
V_0^\# = \min_{u^1 \in U^1_{ad}, \ldots, u^N \in U^N_{ad}} \sum_{i=1}^{N} J^i(u^i) \\
\text{s.t. } (\Theta^1(u^1), \ldots, \Theta^N(u^N)) \in S
\]

with

- \( u^i \in U^i \) be a local decision variable
- \( J^i: U^i \to \mathbb{R}, \ i \in \{1, N\} \) be a local objective
- \( U^i_{ad} \) be a subset of \( U^i \)
- \( \Theta^i: U^i \to C^i \) be a local constraint mapping
- \( S \) be a subset of \( C = C^1 \times \cdots \times C^N \)

We denote by \( S^o \) the polar cone of \( S \)

\[
S^o = \{ p \in C^* \mid \langle p, r \rangle \leq 0 \ \forall r \in S \}
\]
Price and resource value functions

For each $i \in [1, N]$, 

- for any price $p^i \in (\mathcal{C}_i)^*$, we define the local price value

$$\mathcal{V}_0^i[p^i] = \min_{u^i \in \mathcal{U}_{ad}^i} J^i(u^i) + \langle p^i, \Theta^i(u^i) \rangle$$

- for any resource $r^i \in \mathcal{C}_i$, we define the local resource value

$$\mathcal{V}_0^i[r^i] = \min_{u^i \in \mathcal{U}_{ad}^i} J^i(u^i) \text{ s.t. } \Theta^i(u^i) = r^i$$

Theorem 1 (Upper and lower bounds for optimal value)

For any

- admissible price $p = (p^1, \ldots, p^N) \in S^o$
- admissible resource $r = (r^1, \ldots, r^N) \in S$

$$\sum_{i=1}^{N} \mathcal{V}_0^i[p^i] \leq \mathcal{V}_0^\# \leq \sum_{i=1}^{N} \mathcal{V}_0^i[r^i]$$
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The case of multistage stochastic optimization

Assume that the local price value

\[ V^i_0[p^i] = \min_{u^i \in U^i_{ad}} J^i(u^i) + \langle p^i, \Theta^i(u^i) \rangle, \]

corresponds to a stochastic optimal control problem

\[
V^i_0(P^i)(x^i_0) = \min_{X^i_t, U^i_t} \mathbb{E}\left( \sum_{t=0}^{T-1} L_t^i(X^i_t, U^i_t, W_{t+1}) + \langle P^i_t, \Theta^i_t(X^i_t, U^i_t) \rangle + K^i(X^i_T) \right)
\]

s.t. \( X^i_{t+1} = g^i_t(X^i_t, U^i_t, W_{t+1}) \), \( X^i_0 = x^i_0 \)

\( \sigma(U^i_t) \subset \sigma(W_0, \cdots, W_t) \)

This local control problem can be solved by Dynamic Programming (DP) under restrictive assumptions:

- the noise process \( W \) is a white noise process
- the price process \( P \) follows a dynamics in small dimension

DP leads to a collection \( \{ V^i_t[P^i]\}_{t \in [0,T]} \) of local price value functions
Similar considerations hold true for the local resource value

\[ \overline{V}_0^i[r^i] = \min_{u^i \in U^i_{ad}} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i \]

considered as a stochastic optimal control problem

\[ \overline{V}_0^i[R^i](x^i_0) = \min_{x^i, u^i} \mathbb{E}\left( \sum_{t=0}^{T-1} L_t^i(X_t^i, U_t^i, W_{t+1}) + K^i(X_T^i) \right) \]

s.t. \( X_{t+1}^i = g_t^i(X_t^i, U_t^i, W_{t+1}) \), \( X_0^i = x_0^i \)

\( \sigma(U_t^i) \subset \sigma(W_0, \cdots, W_t) \)

\( \Theta_t^i(X_t^i, U_t^i) = R_t^i \)
Mix of spatial and temporal decompositions

For any admissible price process \( P \in S^o \) and any admissible resource process \( R \in S \), we have bounds of the optimal value \( V_0^\# \)

\[
\sum_{i=1}^{N} V_0^i[P^i](x_0^i) \leq V_0^\# \leq \sum_{i=1}^{N} V_0^i[R^i](x_0^i)
\]

1. To obtain the bounds, we perform **spatial decompositions** giving
   - a collection \( \{ V_0^i[P^i](x_0^i) \}_{i \in [1,N]} \) of price local values
   - a collection \( \{ V_0^i[R^i](x_0^i) \}_{i \in [1,N]} \) of resource local values
   *The computation of these local values can be performed in parallel*

2. To compute each local value, we perform **temporal decomposition** based on Dynamic Programming. For each \( i \), we obtain
   - a sequence \( \{ V_t^i[P^i] \}_{t \in [0,T]} \) of price local value functions
   - a sequence \( \{ V_t^i[R^i] \}_{t \in [0,T]} \) of resource local value functions
   *The computation of these local values functions is done sequentially*
Mix of spatial and temporal decompositions

Figure: The case of price decomposition
Tools for mixing spatial and temporal decomposition methods
   Upper and lower bounds using spatial decomposition
   Temporal decomposition using dynamic programming
   The case of deterministic coordination processes

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We assume that $W$ is a white noise process, and we restrict ourselves to deterministic admissible processes $p \in S^o$ and $r \in S$

- The local value functions $V_i[p^i]$ and $\overline{V}_i[r^i]$ are easy to compute because they only depend on the local state variable $x^i$.

- It is easy to obtain tighter bounds by maximizing the lower bound w.r.t. prices and minimizing the upper bound w.r.t. resources.

$$\sup_{p \in S^o} \sum_{i=1}^{N} V_i[p^i](x_0^i) \leq V^0_0 \leq \inf_{r \in S} \sum_{i=1}^{N} \overline{V}_i[r^i](x_0^i)$$
The case of deterministic price and resource processes

We assume that $\mathcal{W}$ is a white noise process, and we restrict ourselves to deterministic admissible processes $p \in S^o$ and $r \in S$

The local value functions $V_t^i[p^i]$ and $\overline{V}_t^i[r^i]$ allow the computation of global policies by solving static optimization problems

- In the case of local price value functions, the policy is obtained as

$$
\gamma_t(x^1_t, \ldots, x^N_t) \in \arg \min_{u^1_t, \ldots, u^N_t} \mathbb{E}\left( \sum_{i=1}^{N} L_t^i(x^i_t, u^i_t, \mathcal{W}_{t+1}) + \sum_{i=1}^{N} V_{t+1}^i[p^i](X_{t+1}^i) \right)
$$

s.t.

- $X_{t+1}^i = g_t(x^i_t, u^i_t, \mathcal{W}_{t+1}), \forall i \in [1, N]$

- $(\Theta_t(x^1_t, u^1_t), \ldots, \Theta_t(x^N_t, u^N_t)) \in S_t$

- A global policy based on resource value functions is also available

Estimating the expected cost of such policies by Monte Carlo simulation leads to a statistical upper bound of the optimal cost of the problem
First, we have obtained lower and upper bounds for a global optimization problem with coupling constraints thanks to two spatial decomposition schemes:
- price decomposition
- resource decomposition

Second, we have computed the lower and upper bounds by dynamic programming (temporal decomposition).

Using the price and resource Bellman value functions, we have devised two online policies for the global problem.

Now, we apply these decomposition schemes to large-scale network problems.
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Network and flows

Directed graph $G = (\mathcal{V}, \mathcal{E})$

- $Q_t^e$ flow through edge $e$,
- $F_t^i$ flow imported at node $i$

Let $A$ be the node-edge incidence matrix

Each node corresponds to a building with its own devices (battery, hot water tank, solar panel...)

At each time $t \in [0, T - 1]$, the Kirchhoff current law couples node and edge flows

$$AQ_t + F_t = 0$$
Optimization problem at a given node

At each node $i \in \mathcal{V}$, given a node flow process $F^i$, we minimize the house cost

$$J^i_V(F^i) = \min_{X^i, U^i} \mathbb{E}\left(\sum_{t=0}^{T-1} L^i_t(X^i_t, U^i_t, W^i_{t+1}) + K^i(X^i_T)\right)$$

subject to, for all $t \in [0, T - 1]$

i) **nodal dynamics constraints**

$$X^i_{t+1} = g^i_t(X^i_t, U^i_t, W^i_{t+1})$$

ii) **non-anticipativity constraints**

$$\sigma(U^i_t) \subset \sigma(W_0, \ldots, W_{t+1})$$

iii) **nodal load balance equations**

$$\Delta^i_t(X^i_t, U^i_t, W^i_{t+1}) = F^i_t$$

**Remarks**

- Local noise $W^i_t$ in the formulation of problem at node $i$
- Global noise $W_t = (W^1_{t+1}, \ldots, W^N_{t+1})$ in the non-anticipativity constraint
Transportation cost and global optimization problem

We define the network cost as the sum over time and edges of the costs of flows $Q^e_t$ through the edges of the network

$$J_{\mathcal{E}}(Q) = \mathbb{E}\left( \sum_{t=0}^{T-1} \sum_{e \in \mathcal{E}} l^e_t(Q^e_t) \right)$$

This transportation cost is additive in space, in time and in uncertainty!

The global optimization problem is obtained by gathering all elements

$$V^\#_0 = \min_{F,Q} \sum_{i \in \mathcal{V}} J^i_{\mathcal{V}}(F^i) + J_{\mathcal{E}}(Q)$$

s.t. $AQ + F = 0$
Price and resource decompositions

**Price problem:**

\[
\mathcal{V}_0[P] = \min_{F,Q} \sum_{i \in \mathcal{V}} J^i_V(F^i) + J_E(Q) + \langle P, AQ + F \rangle
\]

\[
= \sum_{i \in \mathcal{V}} \left( \min_{F_i} J^i_V(F^i) + \langle P^i, F^i \rangle \right) + \left( \min_{Q} J_E(Q) + \langle A^T P, Q \rangle \right)
\]

- **Node i’s subproblem**
- **Network subproblem**

**Resource problem:**

\[
\mathcal{V}_0[R] = \min_{F,Q} \sum_{i \in \mathcal{V}} J^i_V(F^i) + J_E(Q) \quad \text{s.t.} \quad AR + F = 0, \quad Q = R
\]

\[
= \sum_{i \in \mathcal{V}} \left( \min_{F_i} J^i_V(F^i) \quad \text{s.t.} \quad F^i = -(AR)^i \right) + \left( \min_{Q} J_E(Q) \quad \text{s.t.} \quad Q = R \right)
\]

**Objective**

Find **deterministic** processes \( \hat{p} \) and \( \hat{r} \) with a gap as small as possible

\[
\sup_p \mathcal{V}_0[p] \leq \mathcal{V}_0^\#, \quad \text{inf} \mathcal{V}_0[r] \leq \inf_r \mathcal{V}_0[r]
\]
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Different urban configurations

3-Nodes

6-Nodes

12-Nodes

24-Nodes

48-Nodes
Problem settings

Thanks to the periodicity of demands and electricity tariffs, the microgrid management problem can be solved day by day

- One day horizon with a 15mn time step: $T = 96$

- Weather corresponds to a sunny day in Paris (June 28, 2015)

- We mix three kinds of buildings
  1. battery + electrical hot water tank
  2. solar panel + electrical hot water tank
  3. electrical hot water tank

and suppose that all consumers are commoners sharing their devices
Algorithms implemented on the problem

**SDDP**
We use the SDDP algorithm to solve the problem **globally**...

▶ but noises $W^1_t, \ldots, W^N_t$ are **independent node by node**, so that the support size of the noise may be **huge** ($|\text{supp}(W^i_t)|^N$). We must **resample the noise** to be able to compute the cuts

**Price decomposition**
Spatial decomposition and maximization w.r.t. a **deterministic price** $p$

▶ Each nodal subproblem solved by a DP-like method
▶ Maximisation w.r.t. $p$ by Quasi-Newton (BFGS) method

$$p^{(k+1)} = p^{(k)} + \rho^{(k)}H^{(k)}\nabla V_0[p^{(k)}]$$

▶ **Oracle** $\nabla V_0[p]$ estimated by Monte Carlo ($N^{\text{scen}} = 1,000$)

**Resource decomposition**
Spatial decomposition and minimization w.r.t. a **deterministic resource process** $r$
Exact upper and lower bounds on the global problem

<table>
<thead>
<tr>
<th>Network</th>
<th>3-Nodes</th>
<th>6-Nodes</th>
<th>12-Nodes</th>
<th>24-Nodes</th>
<th>48-Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>State dim.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(</td>
<td>X</td>
<td>)</td>
<td>4</td>
<td>8</td>
<td>16</td>
</tr>
<tr>
<td>SDDP time</td>
<td>1'</td>
<td>3'</td>
<td>10'</td>
<td>79'</td>
<td>453'</td>
</tr>
<tr>
<td>SDDP LB</td>
<td>225.2</td>
<td>455.9</td>
<td>889.7</td>
<td>1752.8</td>
<td>3310.3</td>
</tr>
<tr>
<td>Price time</td>
<td>6'</td>
<td>14'</td>
<td>29'</td>
<td>41'</td>
<td>128'</td>
</tr>
<tr>
<td>Price LB</td>
<td>213.7</td>
<td>447.3</td>
<td>896.7</td>
<td>1787.0</td>
<td>3396.4</td>
</tr>
<tr>
<td>Resource time</td>
<td>3'</td>
<td>7'</td>
<td>22'</td>
<td>49'</td>
<td>91'</td>
</tr>
<tr>
<td>Resource UB</td>
<td>253.9</td>
<td>527.3</td>
<td>1053.7</td>
<td>2105.4</td>
<td>4016.6</td>
</tr>
</tbody>
</table>

For the **48-Nodes** microgrid,

- price decomposition gives a (slightly) better exact lower bound than SDDP

\[
\begin{align*}
\underline{3310.3} & \leq \underline{3396.4} \leq \underline{V_0^{\#}} \leq \underline{4016.6} \\
V_0^{[\text{sddp}]} & \leq V_0^{[\text{price}] \leq \overline{V_0^{[\text{resource}]}}}
\end{align*}
\]

- price decomposition is more than **3 times faster** than SDDP
Time evolution
Policy evaluation by Monte Carlo (1,000 scenarios)

<table>
<thead>
<tr>
<th></th>
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<th>48-Nodes</th>
</tr>
</thead>
<tbody>
<tr>
<td>SDDP policy</td>
<td>226 ± 0.6</td>
<td>471 ± 0.8</td>
<td>936 ± 1.1</td>
<td>1859 ± 1.6</td>
<td>3550 ± 2.3</td>
</tr>
<tr>
<td>Price policy</td>
<td>228 ± 0.6</td>
<td>464 ± 0.8</td>
<td>923 ± 1.2</td>
<td>1839 ± 1.6</td>
<td>3490 ± 2.3</td>
</tr>
<tr>
<td></td>
<td>+0.9 %</td>
<td>-1.5%</td>
<td>-1.4%</td>
<td>-1.1%</td>
<td>-1.7%</td>
</tr>
<tr>
<td>Resource policy</td>
<td>229 ± 0.6</td>
<td>471 ± 0.8</td>
<td>931 ± 1.1</td>
<td>1856 ± 1.6</td>
<td>3503 ± 2.2</td>
</tr>
<tr>
<td></td>
<td>+1.3 %</td>
<td>0.0%</td>
<td>-0.5%</td>
<td>-0.2%</td>
<td>-1.2%</td>
</tr>
</tbody>
</table>

All the cost values above are statistical upper bounds of $V_0^\#$

For the 48-Nodes microgrid,

- price policy beats SDDP policy and resource policy

\[
V_0^\# \leq \underbrace{3490}_{C[\text{price}]} \leq \underbrace{3503}_{C[\text{resource}]} \leq \underbrace{3550}_{C[\text{sddp}]}
\]

- the exact upper bound given by resource decomposition is not so tight

\[
\underbrace{3396.4}_{V_0[\text{price}]} \leq V_0^\# \leq \underbrace{3490}_{C[\text{price}]} \leq \underbrace{3503}_{C[\text{resource}]} \leq \underbrace{4016.6}_{V_0[\text{resource}]} > 18\%
\]
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Motivation for decentralized information
Centralized information structure

Up to now, we have studied the following problem

$$V_0^C = \min_{F,Q} \left( \sum_{i \in V} \min_{X_i^t, U_i^t} \mathbb{E} \left( \sum_{t=0}^{T-1} L_t^i(X_t^i, U_t^i, W_{t+1}^i) + K_i^i(X_T^i) \right) \right) + \mathbb{E} \left( \sum_{t=0}^{T-1} \sum_{e \in E} l_t^e(Q_t^e) \right)$$

subject to, for all $t \in [0, T-1]$ and for all $i \in V$

$$AQ_t + F_t = 0 \quad \text{(network constraints)}$$
$$X_{t+1}^i = g_t^i(X_t^i, U_t^i, W_{t+1}^i) \quad \text{(nodal dynamic constraints)}$$
$$\Delta_t^i(X_t^i, U_t^i, W_{t+1}^i) = F_t^i \quad \text{(nodal balance equation)}$$
$$\sigma(U_t^i) \subset \sigma(W_0, \ldots, W_{t+1}) \quad \text{(information constraints)}$$

with $W_t = (W_1^t, \ldots, W_N^t)$: global noise process
Decentralized information structure

Consider now the following problem

\[
V_0^D = \min_{F, Q} \left( \sum_{i \in V} \min_{X^i, U^i} \mathbb{E} \left( \sum_{t=0}^{T-1} L_t^i(X_t^i, U_t^i, W_{t+1}^i) + K^i(X_T^i) \right) \right)
\]

subject to, for all \( t \in [0, T - 1] \) and for all \( i \in V \)

\[
A Q_t + F_t = 0 \quad \text{(network constraints)}
\]

\[
X_{t+1}^i = g_t^i(X_t^i, U_t^i, W_{t+1}^i) \quad \text{(nodal dynamic constraints)}
\]

\[
\Delta_t^i(X_t^i, U_t^i, W_{t+1}^i) = F_t^i \quad \text{(nodal balance equation)}
\]

\[
\sigma(U_t^i) \subset \sigma(W_0^i, \ldots, W_{t+1}^i) \quad \text{(information constraints)}
\]

that is, the local control \( U_t^i \) is a feedback w.r.t. local noises \((W_0^i, \ldots, W_{t+1}^i)\)
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Bounds in the decentralized case

Consider the lower bound obtained with a deterministic price process $p$

$$V_0[p] = \sum_{i \in V} V_0^i[p^i] + V_0^E[p]$$

with

$$V_0^i[p^i] = \min_{X_t^i, U_t^i, F_t^i} \mathbb{E}\left[ \sum_{t=0}^{T-1} L_t^i(X_t^i, U_t^i, W_{t+1}) + \langle p_t^i, F_t^i \rangle + K_t^i(X_T^i) \right]$$

s.t. $X_{t+1}^i = g_t^i(X_t^i, U_t^i, W_{t+1})$, $X_0^i = x_0^i$

$$\Delta_t^i(X_t^i, U_t^i, W_{t+1}) = F_t^i$$

$$\sigma(U_t^i) \subset \sigma(W_1^i, \ldots, W_{t+1}^i)$$

Replacing the global $\sigma$-field $\sigma(W_1^i, \ldots, W_{t+1}^i)$ by the local $\sigma$-field $\sigma(W_1^i, \ldots, W_{t+1}^i)$ does not make any change in this local subproblem

**The lower bound $V_0[p]$ is the same for both information structures.**

A similar conclusion holds true for the upper bound $V_0[r]$. 
Since $\mathcal{W}_t = (\mathcal{W}_t^1, \ldots, \mathcal{W}_t^N)$, for all $i$, we have the inclusion of $\sigma$-fields

$$\sigma(\mathcal{W}_0^i, \ldots, \mathcal{W}_t^i) \subset \sigma(\mathcal{W}_0, \ldots, \mathcal{W}_t)$$

We deduce that the admissible control set in case of a decentralized information structure is smaller than the one in case of a centralized information structure, and hence

$$V_0^C \leq V_0^D$$

Finally, we obtain the following sequence of inequalities

$$\sup_p V_0[p] \leq V_0^C \leq V_0^D \leq \inf_r \overline{V}_0[r]$$
Bounds in the decentralized case

\[
\sup_p V_0[p] \leq V_0^C \leq V_0^D \leq \inf_r \overline{V}_0[r]
\]

- We have seen on the numerical experiments that the lower bound was close from the optimal value \(V_0^C\) in the centralized case

\[
\sup_p V_0[p] \leq V_0^C \approx 3\%
\]

- What can we say about the upper bound and the optimal value \(V_0^D\) in the decentralized case?

\[
V_0^C \leq \inf_r \overline{V}_0[r] \approx 18\%
\]
\[
V_0^D \leq \inf_r \overline{V}_0[r]
\]
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Analysis of the decentralized case

For the sake of brevity, we introduce the following notation

$$\mathcal{F}_t^i = \sigma(\mathcal{W}_0^i, \ldots, \mathcal{W}_t^i)$$

Consider the constraints that have to be met at node $i$ in the case of a decentralized information structure

$$X_{t+1}^i = g_t^i(X_t^i, U_t^i, W_{t+1}^i) \quad \text{(nodal dynamic constraints)}$$
$$\Delta_t^i(X_t^i, U_t^i, W_{t+1}^i) = F_t^i \quad \text{(nodal balance equation)}$$
$$\sigma(U_t^i) \subset \mathcal{F}_{t+1}^i \quad \text{(information structure)}$$

By construction, the state $X_t^i$ is a $\mathcal{F}_t^i$-measurable random variable.

Thanks to both the nodal balance equation and the information structure, we deduce that the node flow $F_t^i$ is measurable w.r.t. the $\sigma$-field $\mathcal{F}_{t+1}^i$. 
Suppose that \((W^1, \cdots, W^N)\) are independent random processes. Otherwise stated, we add an independence assumption in space.

At time \(t\), consider now the global coupling constraints \(AQ_t + F_t = 0\). Summing these constraints leads to the aggregate coupling constraint

\[
\sum_{i \in V} F^i_t = 0
\]

From the aggregate constraint and the independence assumption, we deduce that the random variables \(F_t\) (and hence \(Q_t\)) are in fact deterministic variables.
Analysis of the decentralized case

According to this conclusion, under the space independence assumption, in case of a decentralized information structure, the global minimisation problem depends on deterministic node flows $f$ and edge flows $q$ variables.

$$V_0^D = \min_{f,q} \left( \sum_{i \in \mathcal{V}} J^i_{\mathcal{V}}(f^i) + J_{\mathcal{E}}(q) \right) \quad \text{s.t.} \quad Aq + f = 0$$

$$= \inf_r \left( \sum_{i \in \mathcal{V}} \left( \min_{f^i} J^i_{\mathcal{V}}(f^i) \quad \text{s.t.} \quad f^i = -(Ar)^i \right) + \left( \min_q J_{\mathcal{E}}(q) \quad \text{s.t.} \quad q = r \right) \right)$$

$$= \inf_r \overline{V}_0[r]$$

The upper bound $\min_r \overline{V}_0[r]$ and the optimal value $V_0^D$ are the same.
Information gap

Recall the sequence of inequalities relating optimal values and bounds

\[ \sup_p V_0[p] \leq V_0^C \leq V_0^D \leq \inf_r \overline{V}_0[r] \]

Gathering all the theoretical and numerical results obtained, we have

\[ \begin{align*}
\sup_p V_0[p] & \leq V_0^C , \quad V_0^C \leq V_0^D , \quad V_0^D = \inf_r \overline{V}_0[r] \\
\approx & 3\% , \quad \approx 18\%
\end{align*} \]

that provides a way to quantify the information gap in our application.
Conclusions

- We have two algorithms that decompose spatially and temporally a large-scale optimization problem under coupling constraints.

- In our case study, price decomposition beats SDDP for large instances ($\geq 24$ nodes)
  - in computing time (more than twice faster)
  - in precision (more than 1% better)

- Price decomposition gives (in a surprising way) a tight lower bound, whereas the upper bound given by resource decomposition is weak (which is understandable on the case study)

- We have studied the case of a decentralized information structure to explain this weakness

- Can we obtain tighter bounds? especially for resource decomposition... If we select properly price $P$ and resource $R$ processes among the class of Markovian processes (instead of deterministic ones) we can obtain “better” nodal value functions (with an extended local state)
Further details in

F. Pacaud
*Decentralized Optimization Methods for Efficient Energy Management under Stochasticity*
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P. Carpentier, J.-P. Chancelier, M. De Lara and F. Pacaud
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Working paper, 2019

THANK YOU FOR YOUR ATTENTION