Decomposition/Coordination Methods for Multistage Stochastic Optimization Problems

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Lecture outline

Decomposition and coordination
The three dimensions of stochastic optimization problems
A bird’s eye view of decomposition methods: the cube

A brief insight into scenario decomposition methods
Scenario decomposition methods “à la Progressive Hedging”
Handling risk with scenario decomposition methods

A brief insight into spatial decomposition methods
Spatial decomposition methods in the deterministic case
The stochastic case raises specific obstacles

Summary and research agenda
Outline of the presentation

Decomposition and coordination

A brief insight into scenario decomposition methods

A brief insight into spatial decomposition methods

Summary and research agenda
A long-term effort in our group (I)


A long-term effort in our group (II)


A long-term effort in our group (III)


2018 H. Gérard, “Décomposition de problèmes d’optimisation stochastique de grande dimension, avec mesure de risque”, *Thèse de l’Université Paris-Est*, octobre 2018
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Summary and research agenda
Decomposition-coordination: divide and conquer

- **Spatial decomposition**
  - Multiple players with their local information
  - Network with decision-makers located at nodes where they control local storage and flows through edges

- **Temporal decomposition**
  - A state is an information summary
  - Time coordination realized through dynamic programming, by value functions
  - Hard nonanticipativity constraints

- **Scenario decomposition**
  - Along each scenario, sub-problems are deterministic (powerful algorithms)
  - Scenario coordination realized through Progressive Hedging, by updating nonanticipativity multipliers
  - Soft nonanticipativity constraints
Let us fix problem and notations

\[
\begin{align*}
\min_{U,X} & \quad \mathbb{E} \left[ \sum_{i=1}^{N} \left( \sum_{t=0}^{T-1} L_t^i(X_t^i, U_t^i, W_{t+1}) + K_t^i(X_T^i) \right) \right] \\
\text{subject to dynamics constraints} & \\
X_{t+1}^i & = f_t^i(X_t^i, U_t^i, W_{t+1}), \quad X_0^i = f_{-1}(W_0) \\
\text{to measurability constraints on the control } U_t^i & \\
\sigma(U_t^i) & \subset \sigma(W_0, \ldots, W_t) \iff U_t^i = \mathbb{E} \left( U_t^i \bigg| W_0, \ldots, W_t \right) \\
\text{and to instantaneous coupling constraints} & \\
\sum_{i=1}^{N} Y_t^i(X_t^i, U_t^i) & = 0
\end{align*}
\]

(The letter \( U \) stands for the Russian word for control: \textit{upravlenie})
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Summary and research agenda
Couplings for stochastic problems

\[
\begin{align*}
\min & \sum_{\omega} \sum_{i} \sum_{t} \pi_{\omega} L_{t}^{i}(X_{t}^{i}(\omega), U_{t}^{i}(\omega), W_{t+1}(\omega)) \\
\text{s.t.} & \quad X_{t+1}^{i}(\omega) = f_{t}^{i}(X_{t}^{i}(\omega), U_{t}^{i}(\omega), W_{t}^{i}(\omega)) \\
& \quad U_{t}^{i} = E \left( U_{t}^{i} | W_{0}, \ldots, W_{t} \right) \\
& \quad \sum_{i} Y_{t}^{i}(X_{t}^{i}, U_{t}^{i}) = 0
\end{align*}
\]
Couplings for stochastic problems: in time

\[
\min \sum_{\omega} \sum_{i} \sum_{t} \pi_{\omega} L^{i}_{t}(X^{i}_{t}(\omega), U^{i}_{t}(\omega), W_{t+1}(\omega)) \\
\text{s.t. } X^{i}_{t+1} = f^{i}_{t}(X^{i}_{t}, U^{i}_{t}, W_{t+1})
\]
Couplings for stochastic problems: in uncertainty

\[ \min \sum_{\omega} \sum_{i} \sum_{t} \pi_{\omega} L_{t}^{i}(X_{t}(\omega), U_{t}^{i}(\omega), W_{t+1}(\omega)) \]

s.t. \[ X_{t+1}^{i} = f_{t}^{i}(X_{t}^{i}, U_{t}^{i}, W_{t+1}) \]

\[ U_{t}^{i} = \mathbb{E} \left( U_{t}^{i} \mid W_{0}, \ldots, W_{t} \right) \]
Couplings for stochastic problems: in space

\[ \min \sum_{\omega} \sum_{i} \sum_{t} \pi(\omega) L_{t}^{i}(X_{t}^{i}(\omega), U_{t}^{i}(\omega), W_{t+1}^{i}(\omega)) \]

s.t. \( X_{t+1}^{i} = f_{t}^{i}(X_{t}^{i}, U_{t}^{i}, W_{t+1}) \)

\[ U_{t}^{i} = \mathbb{E}\left( U_{t}^{i} \bigg| W_{0}, \ldots, W_{t} \right) \]

\[ \sum_{i} Y_{t}^{i}(X_{t}^{i}, U_{t}^{i}) = 0 \]
Can we decouple stochastic problems?

\[
\min \sum_{\omega} \sum_{i} \sum_{t} \pi_{\omega} L_t^i (X_t^i(\omega), U_t^i(\omega), W_{t+1}(\omega))
\]

s.t. \( X_{t+1}^i = f_t^i (X_t^i, U_t^i, W_{t+1}) \)

\[
U_t^i = \mathbb{E} \left( U_t^i \bigg| W_0, \ldots, W_t \right)
\]

\[
\sum_i Y_t^i (X_t^i, U_t^i) = 0
\]
Sequential decomposition in time

\[
\min \sum_{\omega} \sum_{i} \sum_{t} \pi_{\omega} L_{t}^{i}(X_{t}(\omega), U_{t}(\omega), W_{t+1}(\omega))
\]

s.t. \[
X_{t+1}^{i} = f_{t}^{i}(X_{t}^{i}, U_{t}^{i}, W_{t+1})
\]

\[
U_{t}^{i} = \mathbb{E}\left(U_{t}^{i} \mid W_{0}, ..., W_{t}\right)
\]

\[
\sum_{i} Y_{t}^{i}(X_{t}^{i}, U_{t}^{i}) = 0
\]

Dynamic Programming

Bellman (56)
Parallel decomposition in uncertainty/scenarios

\[
\min \sum_{\omega} \sum_{i} \sum_{t} \pi_{\omega} L_{t}^{i}(X_{t}^{i}(\omega), U_{t}^{i}(\omega), W_{t+1}(\omega))
\]

s.t. \( X_{t+1}^{i} = f_{t}^{i}(X_{t}^{i}, U_{t}^{i}, W_{t+1}) \)

\[
U_{t}^{i} = \mathbb{E}\left( U_{t}^{i} \mid W_{0}, \ldots, W_{t} \right)
\]

\[
\sum_{i} Y_{t}^{i}(X_{t}^{i}, U_{t}^{i}) = 0
\]

Progressive Hedging
Rockafellar - Wets (91)
Parallel decomposition in space/units

\[
\min \sum_{\omega} \sum_{i} \sum_{t} \pi_{\omega} L_{t}^{i}(X_{t}^{i}(\omega), U_{t}^{i}(\omega), W_{t+1}(\omega))
\]

s.t. \( X_{t+1}^{i} = f_{t}^{i}(X_{t}^{i}, U_{t}^{i}, W_{t+1}) \)

\[
U_{t}^{i} = \mathbb{E}\left( U_{t}^{i} \mid W_{0}, \ldots, W_{t} \right)
\]

\[
\sum_{i} Y_{t}^{i}(X_{t}^{i}, U_{t}^{i}) = 0
\]

Price and Quantity Decompositions with DP
Outline of the presentation

Decomposition and coordination

A brief insight into scenario decomposition methods

A brief insight into spatial decomposition methods

Summary and research agenda
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   A bird’s eye view of decomposition methods: the cube

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   The stochastic case raises specific obstacles

Summary and research agenda
Non-anticipativity constraints are linear

- From tree to scenarios (fan)
- Equivalent formulations of the non-anticipativity constraints
  - pairwise equalities
  - all equal to their mathematical expectation
- Linear structure

\[ U_t = \mathbb{E} \left( U_t \mid W_0, \ldots, W_t \right) \]
Progressive Hedging stands as a scenario decomposition method

We dualize the non-anticipativity constraints

- When the criterion is strongly convex, we use a Lagrangian relaxation (algorithm “à la Uzawa”) to obtain a scenario decomposition
- When the criterion is linear, Rockafellar - Wets (91) propose to use an augmented Lagrangian, and obtain the Progressive Hedging algorithm
Data: step $\rho > 0$, initial multipliers $\{\lambda_s^{(0)}\}_{s \in S}$ and mean first decision $\overline{x}^{(0)}$;
Result: optimal first decision $x$;
repeat
  forall scenarios $s \in S$ do
    Solve the deterministic minimization problem for scenario $s$, with a penalization $+\lambda_s^{(k)} \left( x_s^{(k+1)} - \overline{x}^{(k)} \right)$, and obtain optimal first decision $x_s^{(k+1)}$;
    Update the mean first decisions
    $$\overline{x}^{(k+1)} = \sum_{s \in S} \pi_s x_s^{(k+1)};$$
    Update the multiplier by
    $$\lambda_s^{(k+1)} = \lambda_s^{(k)} + \rho \left( x_s^{(k+1)} - \overline{x}^{(k+1)} \right), \ \forall s \in S;$$
  until $x_s^{(k+1)} - \sum_{s' \in S} \pi_{s'} x_{s'}^{(k+1)} = 0$, $\forall s \in S;$
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Summary and research agenda
Suppose you had to manage a day-ahead energy market
You would have to fix reserves by night
and adjust in the morning with recourse energies
From linear to stochastic programming

- The linear program

\[
\begin{align*}
\min_{x \in \mathbb{R}^n} & \quad \langle c, x \rangle \\
A x + b & \geq 0 \quad (\in \mathbb{R}^m)
\end{align*}
\]

- becomes a stochastic program

\[
\begin{align*}
\min_{x \in \mathbb{R}^n} & \quad \sum_{s \in S} \pi_s \langle c_s, x \rangle \\
A_s x + b_s & \geq 0 , \quad \forall s \in S
\end{align*}
\]

- We observe that there are as many (vector) inequalities as there are possible scenarios \( s \in S \)

\[
A_s x + b_s \geq 0 , \quad \forall s \in S
\]

and these inequality constraints can delineate an empty domain for optimization
Recourse variables need be introduced for feasibility issues

- We denote by \( s \in S \) any possible value of the random variable \( \xi \),
  with corresponding probability \( \pi_s \)
- and we introduce a recourse variable \( y = (y_s)_{s \in S} \) and the program

\[
\min_{x, (y_s)_{s \in S}} \sum_{s \in S} \pi_s \left( \langle c_s, x \rangle + \langle p_s, y_s \rangle \right)
\]

\[
y_s \geq 0, \quad \forall s \in S
\]

\[
A_s x + b_s + y_s \geq 0, \quad \forall s \in S
\]

- so that the inequality \( A_s x + b_s + y_s \geq 0 \) is now possible,
  at (unitary recourse) price vector \( p = (p_s, s \in S) \)
- Observe that such stochastic programs are huge problems,
  with solution \( (x, (y_s)_{s \in S}) \), but remain linear
Minimizing the Tail Value at Risk of costs: linear programming formulation

- The risk-averse stochastic linear program with recourse

\[
\min_{x, (y_s)_{s \in S}} \min_{r \in \mathbb{R}} \left\{ r + \frac{1}{1 - \lambda} \sum_{s \in S} \pi_s \left( \langle c_s, x \rangle + \langle p_s, y_s \rangle \right) + \right\}
\]

- can be written as the linear program

\[
\min_{x, (y_s)_{s \in S}} \min_{r} \min_{(v_s)_{s \in S}} r + \frac{1}{1 - \lambda} \sum_{s \in S} \pi_s v_s
\]

\[
v_s - \langle c_s, x \rangle - \langle p_s, y_s \rangle \geq 0, \ \forall s \in S
\]

\[
v_s \geq 0, \ \forall s \in S
\]

\[
y_s \geq 0, \ \forall s \in S
\]

\[
A_s x + b_s + y_s \geq 0, \ \forall s \in S
\]
Minimizing a mixture: linear programming formulation

The risk-averse stochastic linear program with recourse

\[
\min_{x,(y_s)_{s \in S}} \min_{r \in \mathbb{R}} \left\{ \theta \sum_{s \in S} \pi_s \left( \langle c_s, x \rangle + \langle p_s, y_s \rangle \right) + (1 - \theta) r + \frac{1 - \theta}{1 - \lambda} \sum_{s \in S} \pi_s \left( \langle c_s, x \rangle + \langle p_s, y_s \rangle \right) \right\}
\]

can be written as the linear program

\[
\min_{x,(y_s)_{s \in S}} \min_{r \in \mathbb{R}} \min_{(u_s,v_s)_{s \in S}} \sum_{s \in S} \pi_s \left\{ \theta u_s + (1 - \theta) r + \frac{1 - \theta}{1 - \lambda} v_s \right\}
\]

\[
\begin{align*}
u_s - \langle c_s, x \rangle - \langle p_s, y_s \rangle & \geq 0, \forall s \in S \\
v_s - u_s + r & \geq 0, \forall s \in S \\
v_s & \geq 0, \forall s \in S \\
y_s & \geq 0, \forall s \in S \\
A_s x + b_s + y_s & \geq 0, \forall s \in S
\end{align*}
\]
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Summary and research agenda
Decomposition and coordination

- The system to be optimized consists of interconnected subsystems
- We want to use this structure to formulate optimization subproblems of reasonable complexity
- But the presence of interactions requires a level of coordination
- Coordination iteratively provides a local model of the interactions for each subproblem
- We expect to obtain the solution of the overall problem by concatenation of the solutions of the subproblems
Example: the “flower model”

\[
\begin{align*}
\min & \sum_{i=1}^{N} J_i(u_i) \\
\text{s.t.} & \sum_{i=1}^{N} \theta_i(u_i) = 0
\end{align*}
\]

Unit Commitment Problem
Intuition of spatial decomposition

- Purpose: satisfy a demand with \( N \) production units, at minimal cost
- Price decomposition

Diagram:
- Coordinator
- Unit 1
- Unit 2
- Unit 3
Intuition of spatial decomposition

- Purpose: satisfy a demand with $N$ production units, at minimal cost
- Price decomposition
  - the coordinator sets a price $\lambda$

Diagram:

- Coordinator
- Unit 1
- Unit 2
- Unit 3
Intuition of spatial decomposition

- Purpose: satisfy a demand with $N$ production units, at minimal cost
- Price decomposition
  - the coordinator sets a price $\lambda$
  - the units send their optimal decision $u_i$
Intuition of spatial decomposition

Purpose: satisfy a demand with $N$ production units, at minimal cost

Price decomposition

- the coordinator sets a price $\lambda$
- the units send their optimal decision $u_i$
- the coordinator compares total production $\sum_{i=1}^{N} \theta_i(u_i)$ and demand, and then updates the price accordingly
Intuition of spatial decomposition

Purpose: satisfy a demand with $N$ production units, at minimal cost

Price decomposition

- the coordinator sets a price $\lambda$
- the units send their optimal decision $u_i$
- the coordinator compares total production $\sum_{i=1}^{N} \theta_i(u_i)$ and demand, and then updates the price accordingly
- and so on...
Intuition of spatial decomposition

Purpose: satisfy a demand with $N$ production units, at minimal cost

Price decomposition
- the coordinator sets a price $\lambda$
- the units send their optimal decision $u_i$
- the coordinator compares total production $\sum_{i=1}^{N} \theta_i(u_i)$ and demand, and then updates the price accordingly
- and so on...
Intuition of spatial decomposition

Purpose: satisfy a demand with $N$ production units, at minimal cost

Price decomposition

- the coordinator sets a price $\lambda$
- the units send their optimal decision $u_i$
- the coordinator compares total production $\sum_{i=1}^{N} \theta_i(u_i)$ and demand, and then updates the price accordingly
- and so on...
Price decomposition relies on dualization

\[
\min_{u_i \in U_i, i=1 \ldots N} \sum_{i=1}^{N} J_i(u_i) \quad \text{subject to} \quad \sum_{i=1}^{N} \theta_i(u_i) = 0
\]

1. Form the Lagrangian and assume that a saddle point exists

\[
\max_{\lambda \in \mathcal{V}} \min_{u_i \in U_i, i=1 \ldots N} \sum_{i=1}^{N} \left( J_i(u_i) + \langle \lambda, \theta_i(u_i) \rangle \right)
\]

2. Solve this problem by the dual gradient algorithm "à la Uzawa"

\[
u_i^{(k+1)} \in \arg \min_{u_i \in U_i} J_i(u_i) + \langle \lambda^{(k)}, \theta_i(u_i) \rangle, \quad i = 1 \ldots, N
\]

\[
\lambda^{(k+1)} = \lambda^{(k)} + \rho \sum_{i=1}^{N} \theta_i(u_i^{(k+1)})
\]
Remarks on decomposition methods

- The theory is available for infinite dimensional Hilbert spaces, and thus applies in the **stochastic framework**, that is, when the $U_i$ are spaces of **random variables**.

- The **minimization algorithm** used for solving the subproblems is not specified in the decomposition process.

- **New variables** $\lambda^{(k)}$ appear in the subproblems arising at iteration $k$ of the optimization process:

  $$\min_{u_i \in U_i} J_i(u_i) + \langle \lambda^{(k)} , \theta_i(u_i) \rangle$$

- These variables are **fixed** when solving the subproblems, and do not cause any difficulty, at least in the **deterministic** case.
Price decomposition applies to various couplings
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Summary and research agenda
Stochastic optimal control (SOC) problem formulation

Consider the following SOC problem

$$\min_{U, X} \mathbb{E} \left( \sum_{i=1}^{N} \left( \sum_{t=0}^{T-1} L_t^i(X_t^i, U_t^i, W_{t+1}) + K_t^i(X_T^i) \right) \right)$$

subject to the constraints

$$X_0^i = f_{-1}^i(W_0), \quad i = 1 \ldots N$$

$$X_{t+1}^i = f_t^i(X_t^i, U_t^i, W_{t+1}), \quad t = 0 \ldots T-1, \quad i = 1 \ldots N$$

$$\sigma(U_t^i) \subset \sigma(W_0, \ldots, W_t), \quad t = 0 \ldots T-1, \quad i = 1 \ldots N$$

$$\sum_{i=1}^{N} \theta_t^i(X_t^i, U_t^i) = 0, \quad t = 0 \ldots T-1$$
Consider the following SOC problem

\[
\min_{U,X} \sum_{i=1}^{N} \left( \mathbb{E}\left( \sum_{t=0}^{T-1} L_t^i(X_t^i, U_t^i, W_{t+1}) + K_t^i(X_{T}^i) \right) \right)
\]

subject to the constraints

\[
\begin{align*}
X_0^i &= f_{-1}^i(W_0), & i &= 1 \ldots N \\
X_{t+1}^i &= f_t^i(X_t^i, U_t^i, W_{t+1}), & t &= 0 \ldots T-1, \ i &= 1 \ldots N \\
\sigma(U_t^i) &\subset \sigma(W_0, \ldots, W_t), & t &= 0 \ldots T-1, \ i &= 1 \ldots N \\
\sum_{i=1}^{N} \theta_t^i(X_t^i, U_t^i) &= 0, & t &= 0 \ldots T-1
\end{align*}
\]
Dynamic programming yields centralized controls

- As we want to solve this SOC problem using dynamic programming (DP), we suppose to be in the Markovian setting, that is, $W_0, \ldots, W_T$ are a white noise

- The system is made of $N$ interconnected subsystems, with the control $U^i_t$ and the state $X^i_t$ of subsystem $i$ at time $t$

- The optimal control $U^i_t$ of subsystem $i$ is a function of the whole system state $(X^1_t, \ldots, X^N_t)$

  $$U^i_t = \lambda^i_t(X^1_t, \ldots, X^N_t)$$

*Naive decomposition should lead to decentralized feedbacks*

  $$U^i_t = \hat{\lambda}^i_t(X^i_t)$$

*which are, in most cases, far from being optimal...*
The crucial point is that the optimal feedback of a subsystem a priori depends on the state of all other subsystems, so that using a decomposition scheme by subsystems is not obvious...

As far as we have to deal with dynamic programming, the central concern for decomposition/coordination purpose boils down to

- how to decompose a feedback $\lambda_t$ w.r.t. its domain $X_t$ rather than its range $U_t$?

And the answer is

- impossible in the general case!
Price decomposition and dynamic programming

When applying price decomposition to the problem by dualizing the (almost sure) coupling constraint \( \sum_i \theta_{it}(X^i_t, U^i_t) = 0 \), multipliers \( \Lambda_t^{(k)} \) appear in the subproblems arising at iteration \( k \)

\[
\min_{U^i_t, X^i_t} \mathbb{E} \left[ \sum_t L^i_t(X^i_t, U^i_t, W_{t+1}) + \Lambda_t^{(k)} \cdot \theta_t^i(X^i_t, U^i_t) \right]
\]

- The variables \( \Lambda_t^{(k)} \) are fixed random variables, so that the random process \( \Lambda^{(k)} \) acts as an additional input noise in the subproblems
- But this process may be correlated in time, so that the white noise assumption has no reason to be fulfilled
- DP cannot be applied in a straightforward manner!

**Question:** how to handle the coordination instruments \( \Lambda_t^{(k)} \) to obtain (an approximation of) the overall optimum?
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Summary and research agenda
Let us move to broader stochastic optimization challenges

- Stochastic optimization requires to make risk attitudes explicit
  - robust, worst case, risk measures, in probability, almost surely
- Stochastic dynamic optimization requires to make online information explicit
  - State-based functional approach
  - Scenario-based measurability approach

Numerical walls

- in dynamic programming, the bottleneck is the dimension of the state
- in stochastic programming, the bottleneck is the number of stages
Here is our research agenda for stochastic decomposition

- Designing risk criteria compatible with decomposition
  - thèse d'Adrien Le Franc (2018—)
- Combining different decomposition methods
  - time: dynamic programming
  - scenario: Progressive Hedging
  - space: decomposition by prices or by quantities
- into blends
  - time + space: Pierre Carpentier talk
    nodal decomposition by prices or by quantities
    + dynamic programming within node
  - time + scenario: Jean-Philippe Chancelier talk
    dynamic programming across time blocks
    + Progressive Hedging within time blocks