Multi-Stage Stochastic Optimization for Clean Energy Transition

An Overview of Decomposition/Coordination Methods for Multistage Stochastic Optimization Problems

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Motivation
Lecture outline

Decomposition and coordination
   The three dimensions of stochastic optimization problems
   A bird’s eye view of decomposition methods: the cube

A brief insight into three decomposition methods
   Scenario decomposition methods
   Spatial (price/resource) decomposition methods
   Time decomposition methods

Summary and research agenda
Outline of the presentation

Decomposition and coordination

A brief insight into three decomposition methods

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Summary and research agenda
Temporal, scenario and spatial structures in multistage stochastic optimization problems

In multistage stochastic optimization problems, we consider that the control variable

\[ U_t^i(\omega) \]

is indexed by

- Time/stages \( t \in T \) (\( = \{0, \ldots, T - 1\} \))
- Scenarios \( \omega \in \Omega \)
- Space/units/agents \( i \in I \) (\( = \{1, \ldots, N\} \))

The letter \( U \) comes from the Russian word *upravlenie* for control
Let us fix problem and notations

\[
\min_{U, X} \mathbb{E} \left( \sum_{i \in I} \sum_{t \in T} L_t^i (X_t^i, U_t^i, W_{t+1}) \right)
\]

subject to

dynamics constraints

\[
X_{t+1}^i = g_t^i (X_t^i, U_t^i, W_{t+1}) , \quad X_0^i = g_{-1}^i (W_0)
\]

measurability constraints (nonanticipativity of the control \(U_t^i\))

\[
\sigma(U_t^i) \subset \sigma(W_0, \ldots, W_t) \iff U_t^i = \mathbb{E}(U_t^i \mid W_0, \ldots, W_t)
\]

spatially coupling constraints

\[
\sum_{i \in I} \Theta_t^i (X_t^i, U_t^i) = 0
\]
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Summary and research agenda
Couplings for stochastic problems

\[
\min \quad \mathbb{E}\left( \sum_i \sum_t L_t^i (X_t^i, U_t^i, W_{t+1}) \right)
\]
Couplings for stochastic problems: in time

\[
\begin{align*}
\min \mathbb{E} & \left( \sum_i \sum_t L^i_t(X^i_t, U^i_t, W_{t+1}) \right) \\
\text{s.t.} & \quad X^i_{t+1} = g^i_t(X^i_t, U^i_t, W_{t+1})
\end{align*}
\]
Couplings for stochastic problems: in uncertainty

\[
\begin{align*}
\min \mathbb{E} \left( \sum_i \sum_t L_t^i(X_t^i, U_t^i, W_{t+1}) \right) \\
\text{s.t.} \quad X_{t+1}^i &= g_t^i(X_t^i, U_t^i, W_{t+1}) \\
U_t^i &= \mathbb{E}(U_t^i \mid W_0, \ldots, W_t)
\end{align*}
\]
Couplings for stochastic problems: in space

\[
\min \mathbb{E} \left( \sum_i \sum_t L_t^i (X_t^i, U_t^i, W_{t+1}) \right)
\]

s.t. \( X_{t+1}^i = g_t^i (X_t^i, U_t^i, W_{t+1}) \)

\( U_t^i = \mathbb{E} (U_t^i \mid W_0, \ldots, W_t) \)

\[
\sum_i \Theta_t^i (X_t^i, U_t^i) = 0
\]
Can we decouple stochastic optimization problems?

\[
\begin{align*}
\min & \quad \mathbb{E} \left( \sum_i \sum_t L_t^i (X_t^i, U_t^i, W_{t+1}) \right) \\
\text{s.t.} & \quad X_{t+1}^i = g_t^i (X_t^i, U_t^i, W_{t+1}) \\
& \quad U_t^i = \mathbb{E} (U_t^i \mid W_0, \ldots, W_t) \\
& \quad \sum_i \Theta_t^i (X_t^i, U_t^i) = 0
\end{align*}
\]
Sequential decomposition in time

\[
\min \mathbb{E} \left( \sum_i \sum_t L_t^i(X_t^i, U_t^i, W_{t+1}) \right)
\]

s.t. \( X_{t+1}^i = g_t^i(X_t^i, U_t^i, W_{t+1}) \)

\( U_t^i = \mathbb{E}(U_t^i \mid W_0, \ldots, W_t) \)

\[
\sum_i \Theta_t^i(X_t^i, U_t^i) = 0
\]

Dynamic Programming (DP)

Bellman (56)
Parallel decomposition in uncertainty/scenarios

\[
\min \mathbb{E} \left( \sum_i \sum_t L_t^i (X_t^i, U_t^i, W_{t+1}) \right)
\]

s.t. \[ X_{t+1}^i = g_t^i (X_t^i, U_t^i, W_{t+1}) \]

\[ U_t^i = \mathbb{E} (U_t^i \mid W_0, \ldots, W_t) \]

\[ \sum_i \Theta_t^i (X_t^i, U_t^i) = 0 \]

Progressive Hedging

Rockafellar-Wets (91)
Parallel decomposition in space/units

\[ \min \mathbb{E} \left( \sum_i \sum_t L_t^i (X_t^i, U_t^i, W_{t+1}) \right) \]

s.t. \[ X_{t+1}^i = g_t^i (X_t^i, U_t^i, W_{t+1}) \]

\[ U_t^i = \mathbb{E} (U_t^i \mid W_0, \ldots, W_t) \]

\[ \sum_i \Theta_t^i (X_t^i, U_t^i) = 0 \]

Price and Resource Decompositions
Decomposition-coordination: divide and conquer

- **Temporal decomposition**
  - A state is an information summary
  - Time coordination realized through Dynamic Programming, by value functions (of the state)
  - Hard nonanticipativity constraints

- **Scenario decomposition**
  - Along each scenario, subproblems are deterministic (powerful algorithms)
  - Scenario coordination realized through Progressive Hedging, by updating nonanticipativity multipliers
  - Soft nonanticipativity constraints

- **Spatial decomposition**
  - By prices (multipliers of the spatial coupling constraint)
  - By resources (splitting the spatial coupling constraint)
Outline of the presentation

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A brief insight into three decomposition methods

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Summary and research agenda
Moving from tree to fan (and scenarios)
Equivalent formulations of the nonanticipativity constraints

On a (scenario) tree, the nonanticipativity constraints
\[ \sigma(U_t) \subset \sigma(W_0, \ldots, W_t) \]
are “hardwired”

On a fan, the nonanticipativity constraints write as linear equality constraints
\[ U_t = \mathbb{E}(U_t \mid W_0, \ldots, W_t) \]
Progressive Hedging stands as a scenario decomposition method

Rockafellar-Wets (91) dualize the nonanticipativity constraints

$$U_t = \mathbb{E}(U_t \mid W_0, \ldots, W_t)$$

- When the criterion is strongly convex, one uses a Lagrangian relaxation (algorithm “à la Uzawa”) to obtain a scenario decomposition
- When the criterion is linear, Rockafellar-Wets (91) propose to use an augmented Lagrangian, and obtain the Progressive Hedging algorithm
Data: step $\rho > 0$, initial multipliers $\{\lambda_s^{(0)}\}_{s \in S}$ and mean first decision $\overline{u}^{(0)}$;

Result: optimal first decision $u$;

repeat

forall scenarios $s \in S$ do

Solve the deterministic minimization problem for scenario $s$,

with a penalization $+\lambda_s^{(k)} \left( u_s^{(k+1)} - \overline{u}^{(k)} \right)$,

and obtain optimal first decision $u_s^{(k+1)}$;

Update the mean first decisions

$$\overline{u}^{(k+1)} = \sum_{s \in S} \pi_s u_s^{(k+1)};$$

Update the multiplier by

$$\lambda_s^{(k+1)} = \lambda_s^{(k)} + \rho \left( u_s^{(k+1)} - \overline{u}^{(k+1)} \right), \ \forall s \in S;$$

until $u_s^{(k+1)} - \sum_{s' \in S} \pi_{s'} u_{s'}^{(k+1)} = 0, \ \forall s \in S;$. 
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Summary and research agenda
We consider an additive model

Consider the following minimization problem

\[
\min_{u \in U_{ad} \subseteq U} J(u) \quad \text{subject to} \quad \Theta(u) - \theta = 0 \in \mathcal{V}
\]

for which exists a decomposition of the space \( U = U_1 \times \ldots \times U_N \), so that \( u \in U \) writes \( u = (u^1, \ldots, u^N) \) with \( u^i \in U^i \), and also

- \( U_{ad} = U^1_{ad} \times \ldots \times U^N_{ad} \), \( U^i_{ad} \subseteq U^i \)
- \( J(u) = J^1(u^1) + \ldots + J^N(u^N) \)
- \( \Theta(u) = \Theta^1(u^1) + \ldots + \Theta^N(u^N) \)

Then the problem displays the following additive structure

\[
\min_{u^1 \in U^1_{ad}} \ldots \min_{u^N \in U^N_{ad}} \sum_{i=1}^{N} J^i(u^i) \quad \text{subject to} \quad \sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0
\]
Additive model — Price decomposition

\[
\min_{u \in \mathcal{U}_{ad}} \sum_{i=1}^{N} J^i(u^i) \quad \text{subject to} \quad \sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0
\]

1. Form the Lagrangian of the problem
   We assume that a saddle point exists,
   so that solving the initial problem is equivalent to
   \[
   \max_{\lambda \in \mathcal{V}} \min_{u \in \mathcal{U}_{ad}} \sum_{i=1}^{N} \left( J^i(u^i) + \langle \lambda, \Theta^i(u^i) \rangle \right) - \langle \lambda, \theta \rangle
   \]

2. Solve this problem by the Uzawa algorithm
   \[
   u^{i,(k+1)} \in \arg \min_{u^i \in \mathcal{U}_{ad}^i} J^i(u^i) + \langle \lambda^{(k)}, \Theta^i(u^i) \rangle, \quad i = 1 \ldots, N
   \]
   \[
   \lambda^{(k+1)} = \lambda^{(k)} + \rho \left( \sum_{i=1}^{N} \Theta^i(u^{i,(k+1)}) - \theta \right)
   \]
Additive model — Price decomposition II

Coordination

\[ \lambda^{(k+1)} = \lambda^{(k)} + \rho \left( \sum \Theta^i(u^i,(k+1)) - \theta \right) \]

Subproblem 1
\[
\min J^1(u^1) + \langle \lambda^{(k)}, \Theta^1(u^1) \rangle
\]

Subproblem \( i \)
\[
\min J^i(u^i) + \langle \lambda^{(k)}, \Theta^i(u^i) \rangle
\]

Subproblem \( N \)
\[
\min J^N(u^N) + \langle \lambda^{(k)}, \Theta^N(u^N) \rangle
\]
Additive model — Resource allocation

\[
\min_{u \in \mathcal{U}_{ad}} \sum_{i=1}^{N} J^i(u^i) \quad \text{subject to} \quad \sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0
\]

1. Write the constraint in an equivalent manner by introducing new variables \( v = (v^1, \ldots, v^N) \) (the so-called “allocation”)

\[
\sum_{i=1}^{N} \Theta^i(u^i) - \theta = 0 \iff \Theta^i(u^i) - v^i = 0 \quad \text{and} \quad \sum_{i=1}^{N} v^i = \theta
\]

and minimize the criterion w.r.t. \( u \) and \( v \)

\[
\min_{v \in \mathcal{V}^N} \sum_{i=1}^{N} \left( \min_{u^i \in \mathcal{U}^i_{ad}} J^i(u^i) \right) \quad \text{s.t.} \quad \Theta^i(u^i) - v^i = 0 \quad \text{s.t.} \quad \sum_{i=1}^{N} v^i = \theta
\]
Additive model — Resource allocation II

\[
\min_{v \in \mathcal{V}^N} \sum_{i=1}^{N} \left( \min_{u_i \in \mathcal{U}_{ad}^i} J_i(u_i) \right. \quad \text{s.t.} \quad \Theta_i(u_i) - v_i^i = 0 \left. \right) \quad \text{s.t.} \quad \sum_{i=1}^{N} v_i^i = \theta
\]

\[
G_i(v_i^i) \quad \updownarrow
\]

\[
\min_{v \in \mathcal{V}^N} \sum_{i=1}^{N} G_i(v_i^i) \quad \text{s.t.} \quad \sum_{i=1}^{N} v_i^i = \theta
\]

2. Solve the last problem using a projected gradient method

\[
G_i(v_i^{i,(k)}) = \min_{u_i \in \mathcal{U}_{ad}^i} J_i(u_i) \quad \text{s.t.} \quad \Theta_i(u_i) - v_i^{i,(k)} = 0 \quad \leadsto \quad \lambda_i^{i,(k+1)}
\]

\[
v_i^{i,(k+1)} = v_i^{i,(k)} + \rho \left( \lambda_i^{i,(k+1)} - \frac{1}{N} \sum_{j=1}^{N} \lambda_j^{j,(k+1)} \right)
\]
Additive model — Resource allocation

Subproblem

Coordination

Subproblem $i$

$\nu^{i,(k+1)} = \nu^{i,(k)} + \rho \left( \lambda^{i,(k+1)} - \frac{1}{N} \sum \lambda^{j,(k+1)} \right)$

Subproblem 1

$\min J^1(u^1)$

$s.t. \Theta^1(u^1) - \nu^{1,(k)} = 0$

Subproblem $i$

$\min J^i(u^i)$

$s.t. \Theta^i(u^i) - \nu^{i,(k)} = 0$

Subproblem $N$

$\min J^N(u^N)$

$s.t. \Theta^N(u^N) - \nu^{N,(k)} = 0$
Preparing Pierre Carpentier’s talk
We can also use price/resource decomposition to bound a minimization problem

\[ V_0^* = \inf_{u^1 \in U^1_{ad}, \ldots, u^N \in U^N_{ad}} \sum_{i=1}^{N} J^i(u^i) \]

s.t. \( (\Theta^1(u^1), \ldots, \Theta^N(u^N)) \in S \)

\[
\text{coupling constraint}
\]

- \( u^i \in U^i \) be a local decision variable
- \( J^i: U^i \rightarrow \mathbb{R}, \ i \in [1, N] \) be a local objective function
- \( U^i_{ad} \) be a subset of the local decision set \( U^i \)
- \( \Theta^i: U^i \rightarrow C^i \) be a local constraint mapping
- \( S \) be a subset of \( C = C^1 \times \cdots \times C^N \)

We denote by \( S^o \) the polar cone of \( S \)

\[ S^o = \{ p \in C^* \mid \langle p, r \rangle \leq 0, \ \forall r \in S \} \]
Price and resource local value functions

For each \( i \in [1, N] \),

- for any price \( p^i \in (C^i)^* \), we define the local price value

\[
V_0^i[p^i] = \inf_{u^i \in \mathbb{U}_i^{ad}} J^i(u^i) + \langle p^i, \Theta^i(u^i) \rangle
\]

- for any resource \( r^i \in C^i \), we define the local resource value

\[
\overline{V}_0^i[r^i] = \inf_{u^i \in \mathbb{U}_i^{ad}} J^i(u^i) \quad \text{s.t.} \quad \Theta^i(u^i) = r^i
\]

Proposition (upper and lower bounds for optimal value)

- For any admissible price \( p = (p^1, \cdots, p^N) \in S^o \)
- For any admissible resource \( r = (r^1, \cdots, r^N) \in S \)

\[
\sum_{i=1}^{N} V_0^i[p^i] \leq V^*_0 \leq \sum_{i=1}^{N} \overline{V}_0^i[r^i]
\]
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Summary and research agenda
## Brief literature review on dynamic programming

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<td>$X$</td>
<td>$X$</td>
<td>$X$</td>
<td>$-$</td>
<td>$(\omega, U_{1:t-1})$</td>
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<tr>
<td><strong>Dynamics</strong></td>
<td>$f(X, U, W)$</td>
<td>$P^u_{X,x'}$</td>
<td>$f(X, U, W)$</td>
<td>$-$</td>
<td>$X_t = (X_{t-1}, U_t)$</td>
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<td><strong>Uncertainties</strong></td>
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<td><strong>Cost</strong></td>
<td>$\sum_t$</td>
<td>$\sum_t$</td>
<td>$\sum_t$</td>
<td>$j(\omega, U)$</td>
<td>$j(\omega, U)$</td>
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<tr>
<td><strong>Controls</strong></td>
<td>$\gamma(X)$</td>
<td>$\gamma(X) \gamma(H)$</td>
<td>$\gamma(X) \gamma(H)$</td>
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<td>$\gamma(x_t) \mathcal{I}_t$-meas.</td>
</tr>
<tr>
<td><strong>History</strong></td>
<td>$-$</td>
<td>$(X, U, \ldots)_t$</td>
<td>$(W, U, \ldots)_t$</td>
<td>$-$</td>
<td>$X_t$</td>
</tr>
</tbody>
</table>
We introduce the history

- The timeline is

\[ w_0 \rightarrow u_0 \rightarrow w_1 \rightarrow u_1 \rightarrow \ldots \rightarrow w_{T-1} \rightarrow u_{T-1} \rightarrow w_T \]

- and the history is

\[
\begin{align*}
\text{history} \\
\hat{h}_t &= (w_0, u_0, w_1, u_1, \ldots, u_{t-1}, w_t) \\
\in \mathbb{H}_t &= \mathbb{W}_0 \times \prod_{s=0}^{t-1} (\mathbb{U}_s \times \mathbb{W}_{s+1})
\end{align*}
\]
History is the largest state

The history follows the dynamics

\[
h_{t+1} = \left( w_0, u_0, w_1, u_1, \ldots, u_{t-1}, w_t, u_t, w_{t+1} \right) \\
= \left( h_t, u_t, w_{t+1} \right)
\]

history \( h_t \)

control uncertainty

control

uncertainty
We formulate a sequence of minimization problems over increasing history spaces

- Once given
  - a criterion $j : \mathbb{H}_T \to \mathbb{R}$
  - a sequence of stochastic kernels $\rho_{t:t+1} : \mathbb{H}_t \to \Delta(\mathbb{W}_{t+1})$

- we define, for any history $h_t$, a minimization problem

$$V_t(h_t) = \inf_{\gamma_{t:T-1} \in \Gamma_{t:T-1}} \int_{\mathbb{H}_T} \left[ j(h'_{T}) \right] \rho_{t:T}^{\gamma}(h_t, dh'_{T})$$

controlled stochastic kernel

history feedbacks
There is a Bellman equation involving value functions over increasing history spaces without white noise assumption

\[ V_T = j \]
\[ V_t = B_{t+1:t} V_{t+1} \]

with

\[ (B_{t+1:t} \varphi)(h_t) = \inf_{u_t \in \mathbb{U}_t} \int_{W_{t+1}} \varphi(h_t, u_t, w_{t+1}) \rho_{t:t+1}(h_t, dw_{t+1}) \]
Preparing Jean-Philippe Chancelier’s talk
Towards state reduction by time blocks

- History $h_t$ is itself a canonical state variable, which lives in the history space
  \[ \mathbb{H}_t = \mathbb{W}_0 \times \prod_{s=0}^{t-1} (\mathbb{U}_s \times \mathbb{W}_{s+1}) \]

- However the size of this canonical state increases with $t$, which is a nasty feature for dynamic programming

- We will now
  - introduce "state" spaces $\mathbb{X}_t$
  - and then reduce the history with a mapping $\theta_r : \mathbb{H}_r \rightarrow \mathbb{X}_r$
  - to obtain a compressed "state" variable $\theta_t(h_t) = x_t \in \mathbb{X}_t$
  - but only at some specified times $0 = t_0 < t_1 < \cdots < t_N = T$

- As an application, we will handle
  stochastic independence between time blocks
  but possible dependence \textit{within} time blocks
The triplet \((\theta_r, \theta_t, f_{r:t})\) is a state reduction across \((r : t)\) if

- the following diagram, for the dynamics, commutes

\[
\begin{array}{c}
\mathbb{H}_r \times \mathbb{H}_{r+1:t} \\
\downarrow \theta_r \\
\mathbb{X}_r \times \mathbb{H}_{r+1:t}
\end{array} \xrightarrow{I_d} \begin{array}{c}
\mathbb{H}_t \\
\downarrow \theta_t \\
\mathbb{X}_t
\end{array}
\]

- the following diagrams, for the stochastic kernels, commute

\[
\begin{array}{c}
\mathbb{H}_r \times \mathbb{H}_{r+1:s-1} \\
\downarrow \theta_r \\
\mathbb{X}_r \times \mathbb{H}_{r+1:s-1}
\end{array} \xrightarrow{\rho_{s-1:s}} \begin{array}{c}
\Delta(\mathcal{W}_s) \\
\tilde{\rho}_{s-1:s}
\end{array}
\]

\[
\begin{array}{c}
\mathbb{H}_r \times \mathbb{H}_{r+1:s-1} \\
\downarrow \theta_r \\
\mathbb{X}_r \times \mathbb{H}_{r+1:s-1}
\end{array} \xrightarrow{I_d} \begin{array}{c}
\Delta(\mathcal{W}_s)
\end{array}
\]
Bellman operator across \((r : t)\)

\[
\mathcal{B}_{r:t} : \mathbb{L}^0_+ (\mathbb{H}_r, \mathcal{H}_r) \rightarrow \mathbb{L}^0_+ (\mathbb{H}_t, \mathcal{H}_t)
\]
is defined by

\[
\mathcal{B}_{r:t} = \mathcal{B}_{t+1:t} \circ \cdots \circ \mathcal{B}_{r:r-1}
\]

where the one time step operators \(\mathcal{B}_{s:s-1}\) are

\[
(\mathcal{B}_{s:s-1} \varphi)(h_{s-1}) = \inf_{u_{s-1} \in \mathcal{U}_{s-1}} \int_{\mathcal{W}_s} \varphi(h_{s-1}, u_{s-1}, w_s) \rho_{s-1:s}(h_{s-1}, dw_s)
\]
State reduction and Dynamic Programming

Denoting by $\theta_r^*: L_0^+(X_r, X_r) \to L_0^+(H_r, H_r)$ the operator defined by

$$\theta_r^*(\tilde{\varphi}_r) = \tilde{\varphi}_r \circ \theta_r^r, \quad \forall \tilde{\varphi}_r \in L_0^+(X_r, X_r),$$

there exists a reduced Bellman operator across $(r: t)$ such that

$$\theta_t^* \circ \tilde{B}_{r:t} = B_{r:t} \circ \theta_r^*,$$

that is, the following diagram is commutative:

$$\begin{array}{ccc}
L_0^+(H_t, H_t) & \xrightarrow{B_{t:r}} & L_0^+(H_r, H_r) \\
\uparrow \theta_t^* & & \uparrow \theta_r^* \\
L_0^+(X_t, X_t) & \xrightarrow{\tilde{B}_{t:r}} & L_0^+(X_r, X_r)
\end{array}$$
Outline of the presentation

Decomposition and coordination

A brief insight into three decomposition methods

Summary and research agenda
We have sketched three main decomposition methods in multistage stochastic optimization

- **time:** Dynamic Programming
- **scenario:** Progressive Hedging
- **space:** decomposition by prices or by resources

**Numerical walls are well-known**

- in dynamic programming, the bottleneck is the dimension of the state
- in stochastic programming, the bottleneck is the number of stages
Here is our research agenda for stochastic decomposition

- Designing risk criteria compatible with decomposition
- Combining different decomposition methods
  - time: Dynamic Programming
  - scenario: Progressive Hedging
  - space: decomposition by prices or by resources
- to produce blends and tackle large scale energy applications
  - time blocks + prices/resources
    (talk of Jean-Philippe Chancelier)
    - dynamic programming across time blocks
    + prices/resources decomposition by time block
    - application to two time scales battery management
- time + space
  (talk of Pierre Carpentier)
  - nodal decomposition by prices or by resources
    + dynamic programming within node
  - application to large scale microgrid management