Signal Processing

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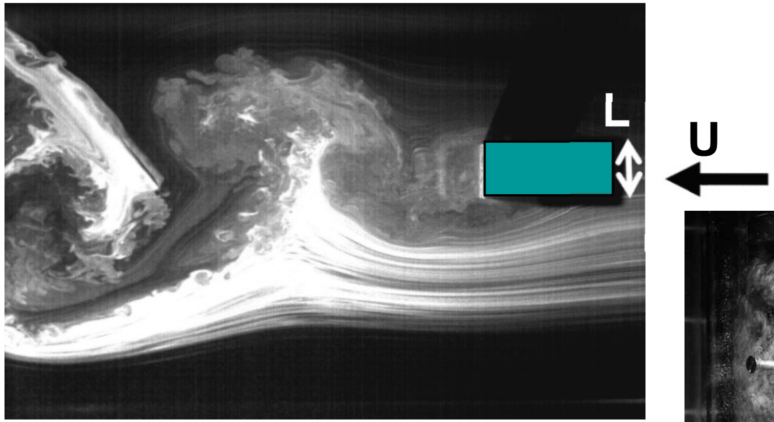
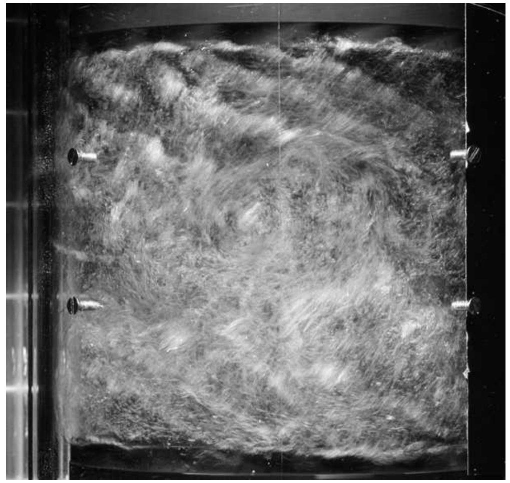
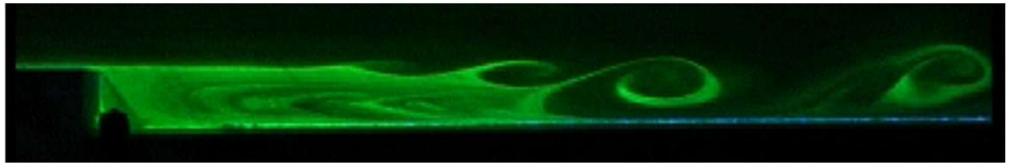
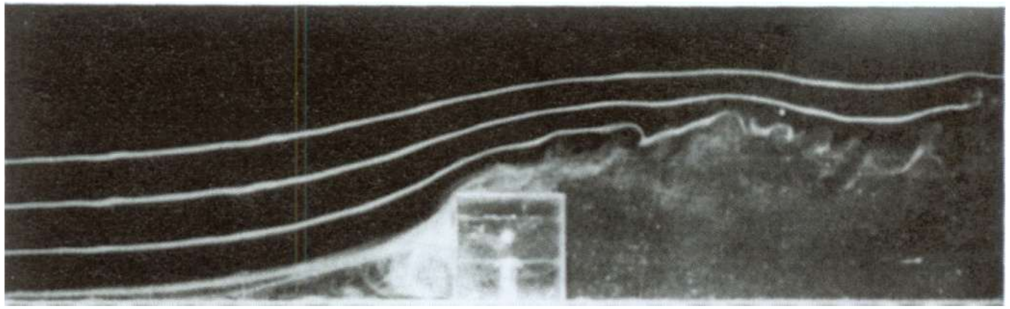
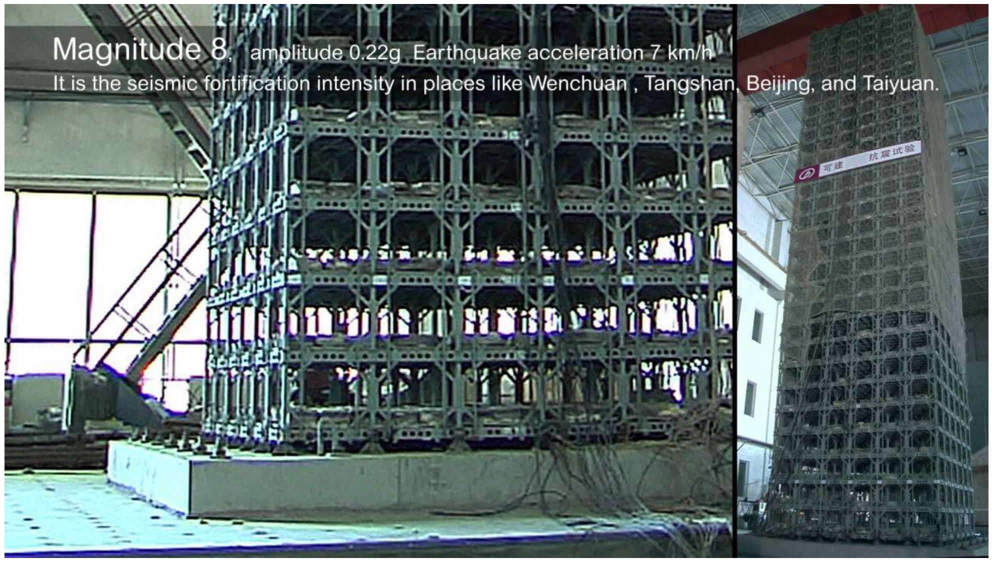
Motivations





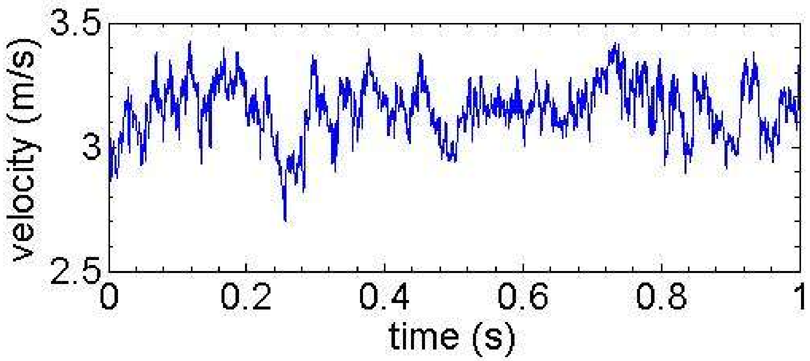




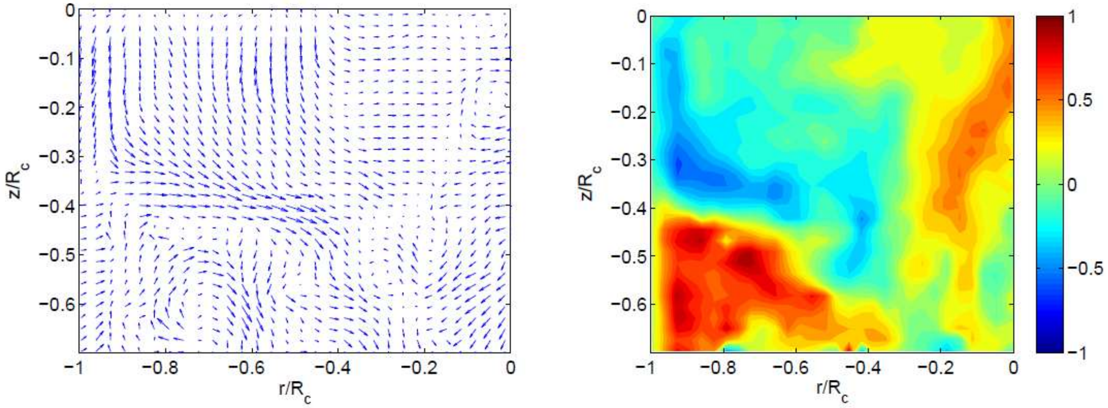


⟹ **Visualise, quantify, caracterise**

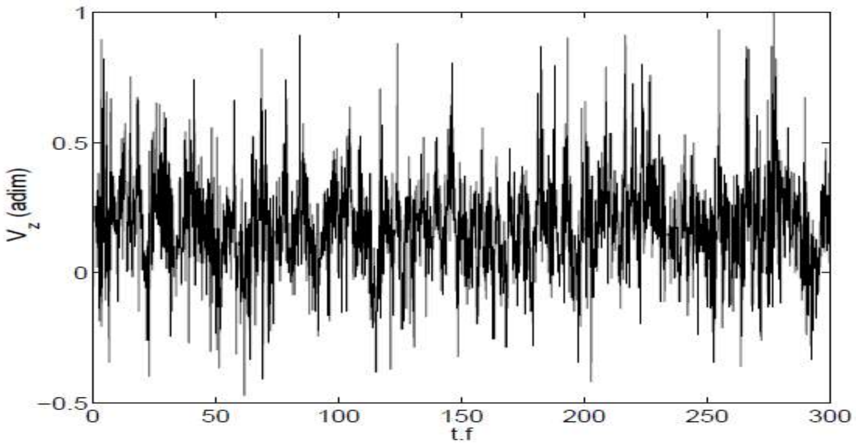
Turbulent jet: hot-wire acquisition



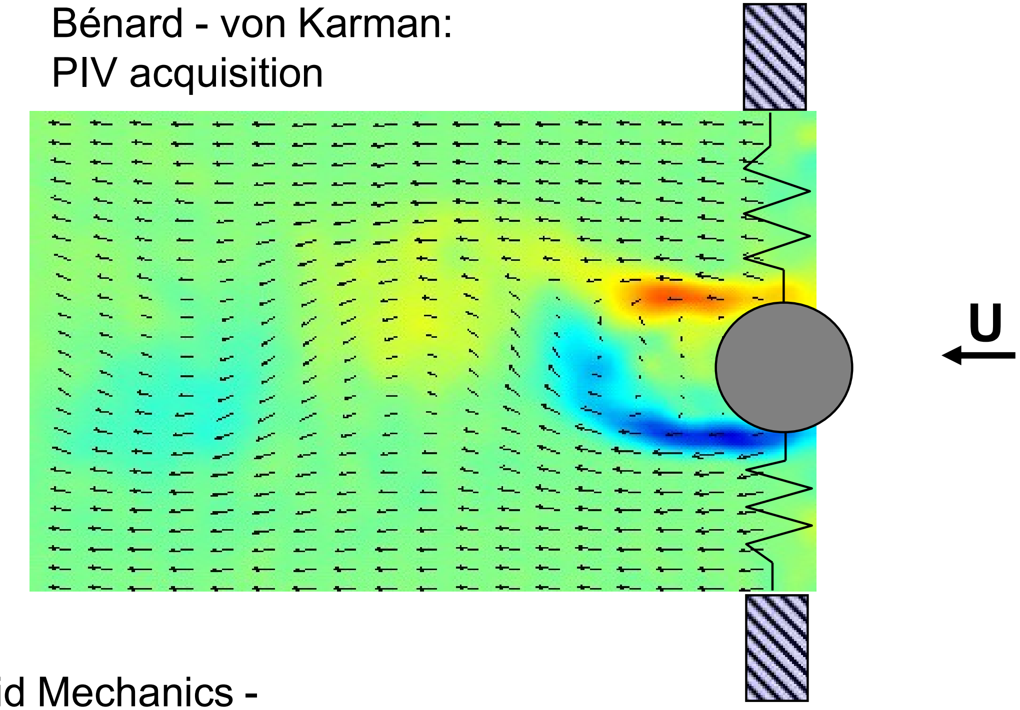
von Karman flow: SPIV acquisition



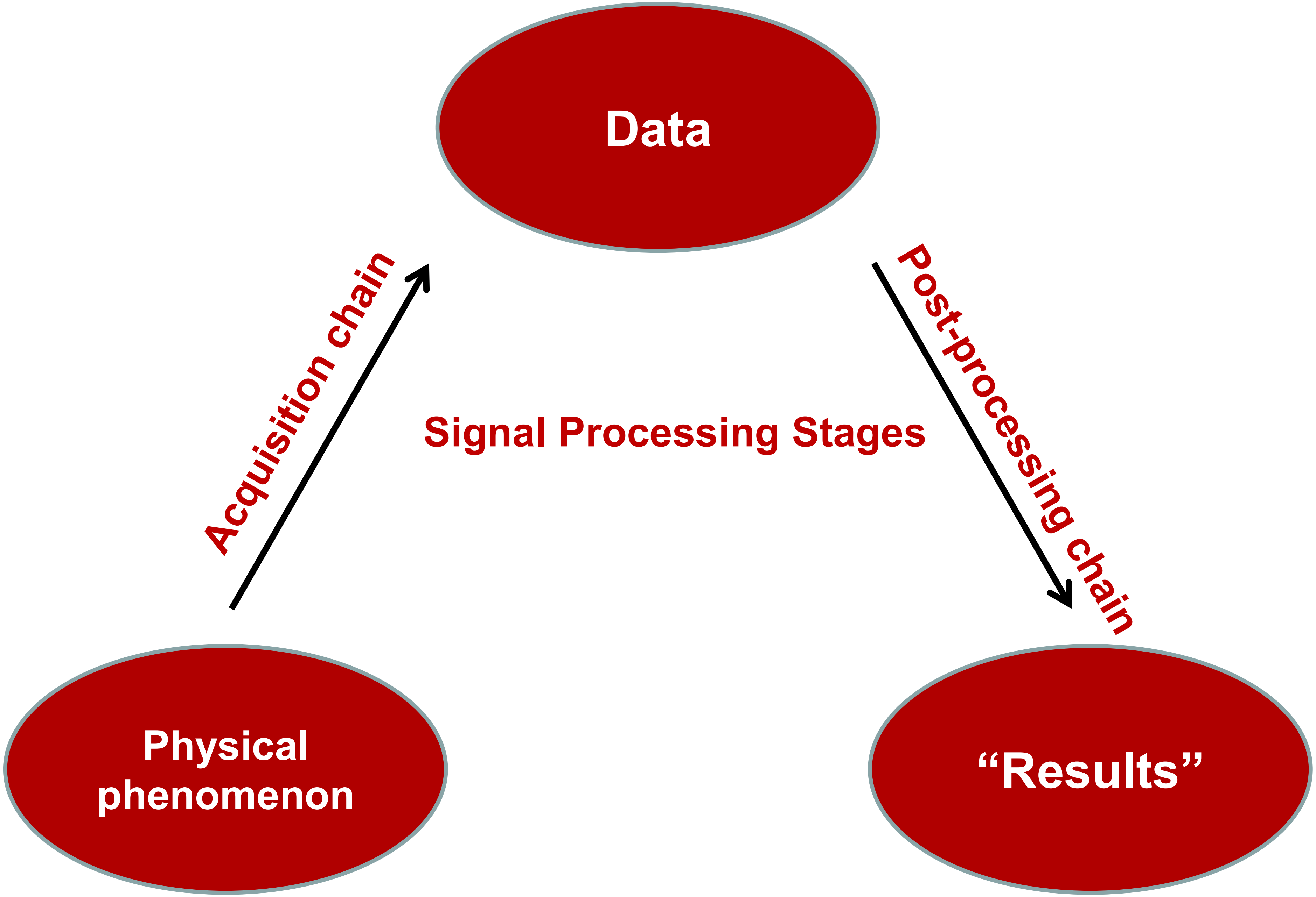
von Karman flow: LDA acquisition



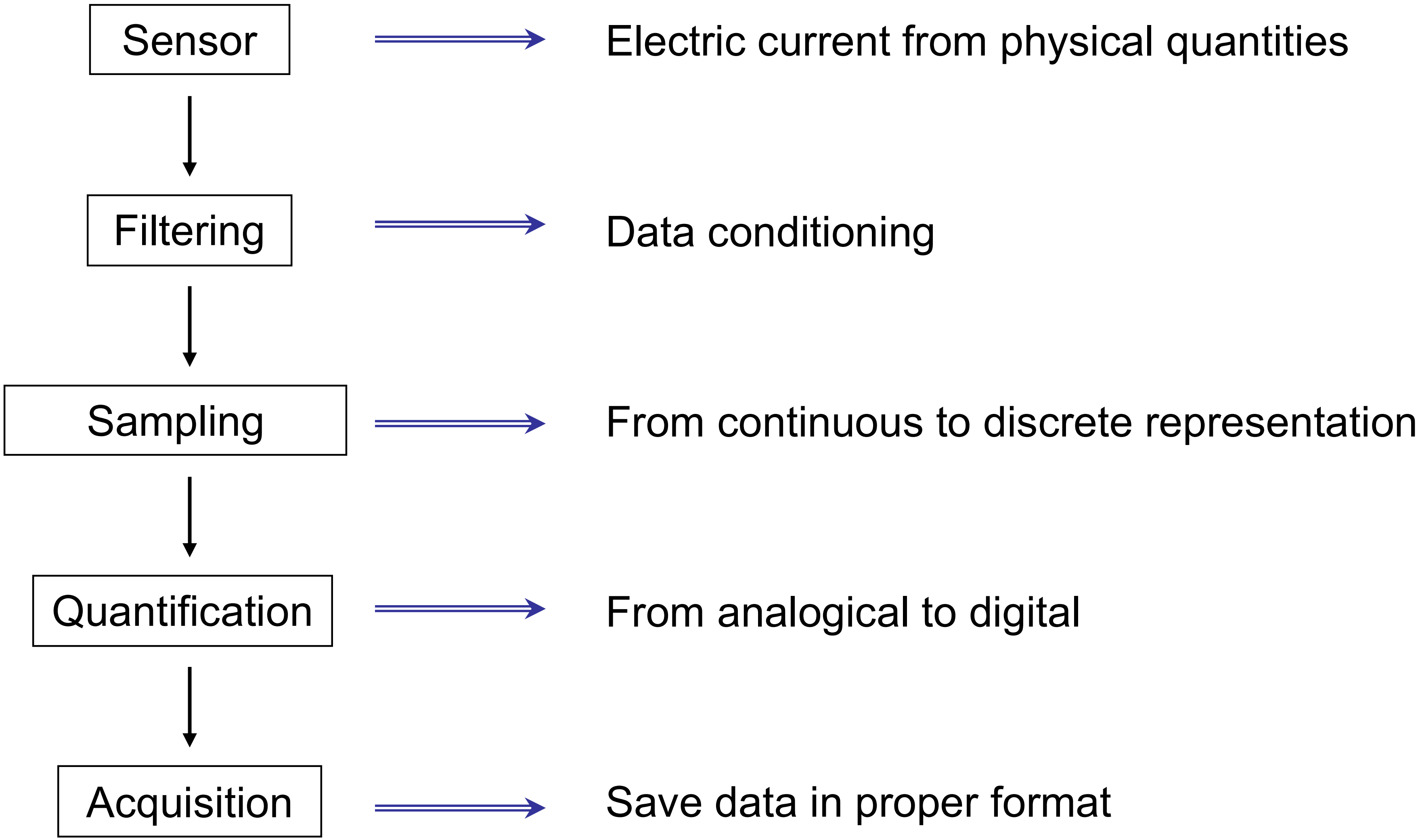
Bénard - von Karman:
PIV acquisition



Motivations - Outlines



Signal processing: acquisition chain



Signal processing: acquisition chain

Sensors

Choose physical quantities
Pression
Position
Displacement
Velocity
Acceleration
Vorticity
Deformation
Force
…

Choose the sensor:
Sampling frequency
Sensitivity, dynamical range
Spatial extend / Integration
Intrusivity or not ?
Life time
…

⟹ **Direct Measurement**

⟹ **Indirect Measurement**

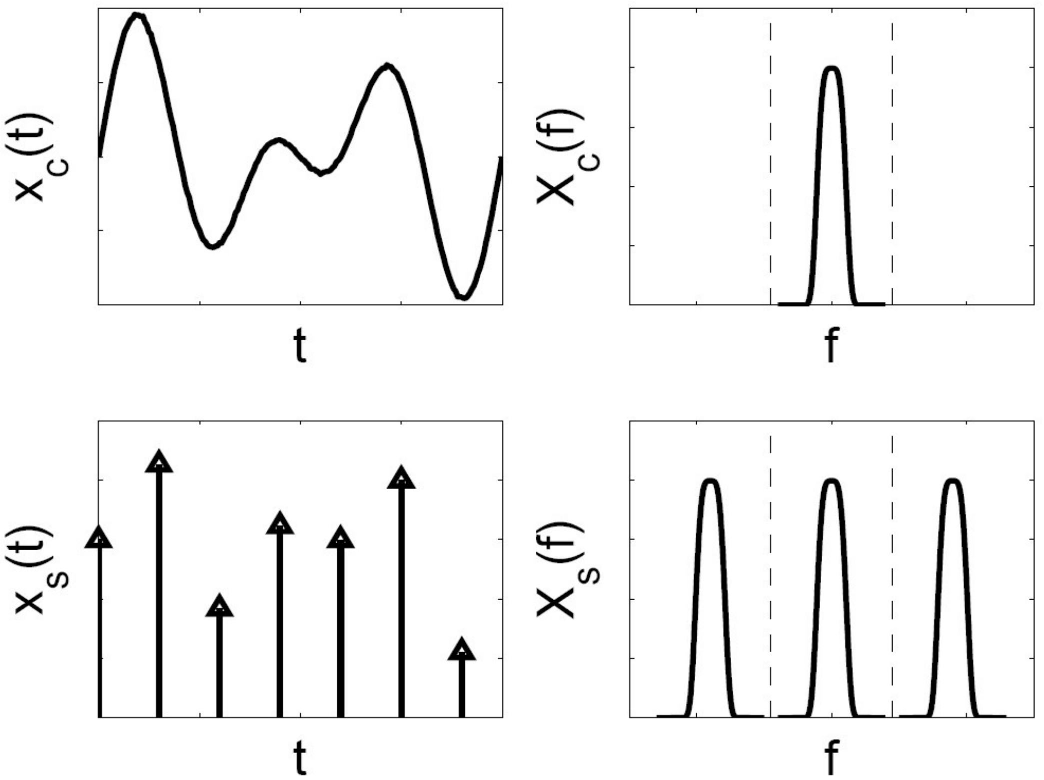
Sensor | Filtering | Sampling | Quantification | Acquisition

Sampling (continuous – discrete)

Signal | Fourier Transform

Original signal:

Sampled signal:

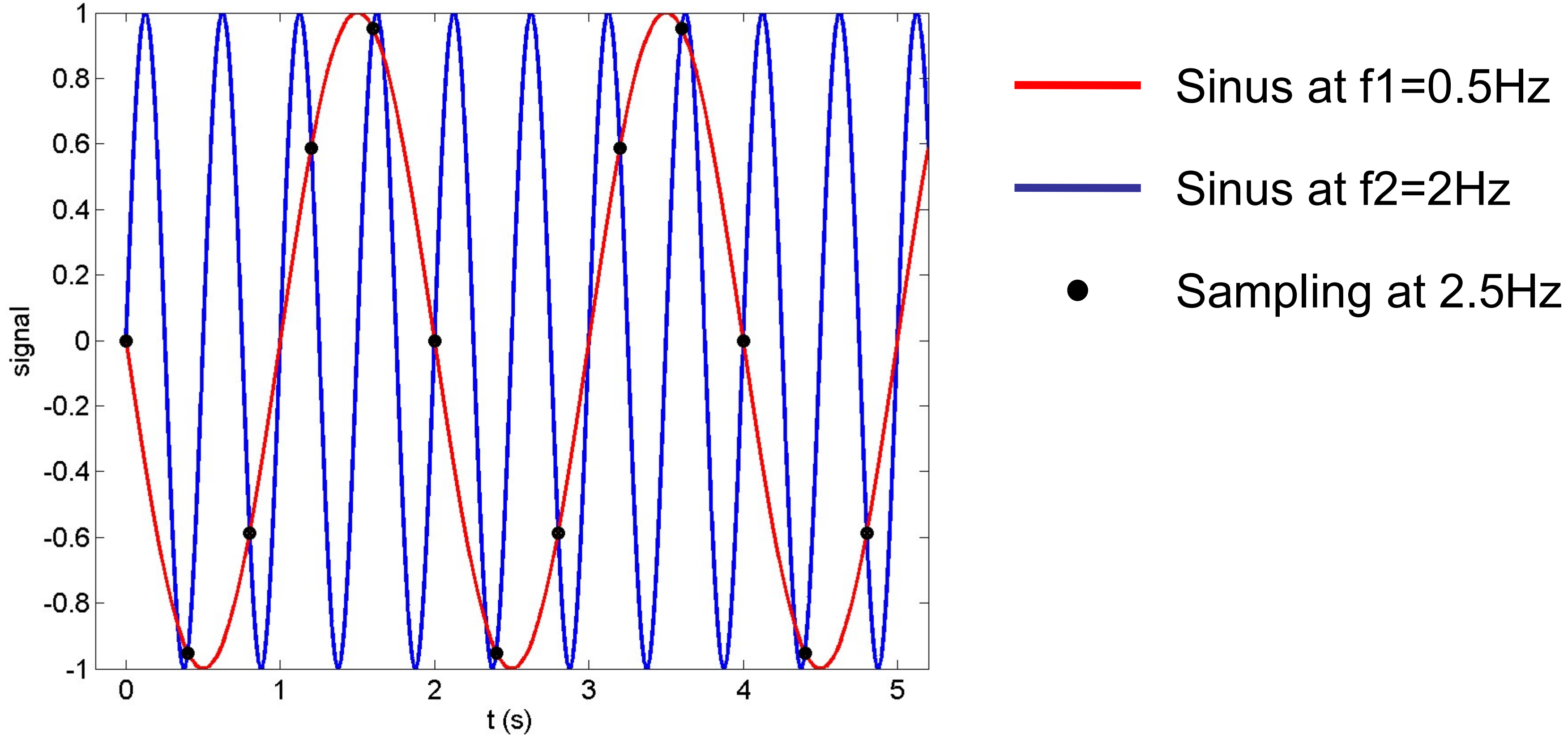


$$X_s(f) = \int_{-\infty}^{+\infty} x_s(t) e^{-2\pi j f t} \, dt$$

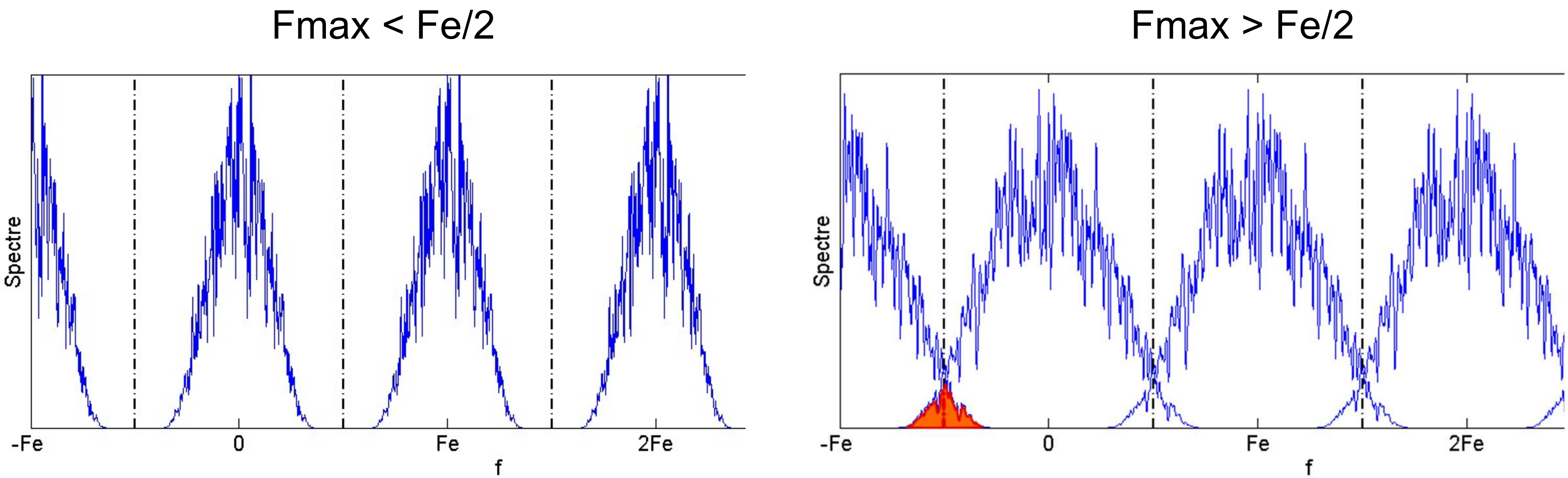
$$x_s(t) = x_c(t) \sum_{k=-\infty}^{+\infty} T_s \delta(t - kT_s)$$

$$X_s(f) = \frac{1}{T_s} \sum_{k=-\infty}^{+\infty} X_c(f - \frac{k}{T_s})$$

Sampling (aliasing)

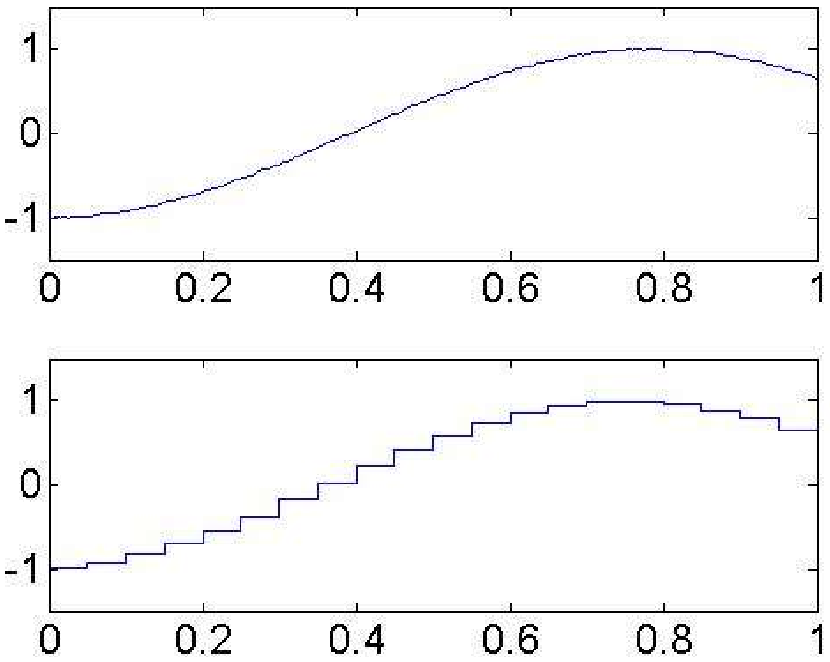


Sampling (aliasing)



⟹ **Shannon criterion**

Quantification (analog- digital)



Quantification level: n bits

Signal dynamics: $\pm V_{max}$

No quantification: $\delta V = \frac{2V_{max}}{2^n}$

Error spanned uniformly on: $-\delta V/2 < e(n) < \delta V/2$

Noise power:

$$\sigma_e^2 = \int_{-\delta V/2}^{\delta V/2} e^2 \frac{1}{\delta V} = \frac{\delta V^2}{12} = \frac{V_{max}^2 2^{-2n}}{3}$$

Quantification noise:

Signal to noise ratio due to quantification

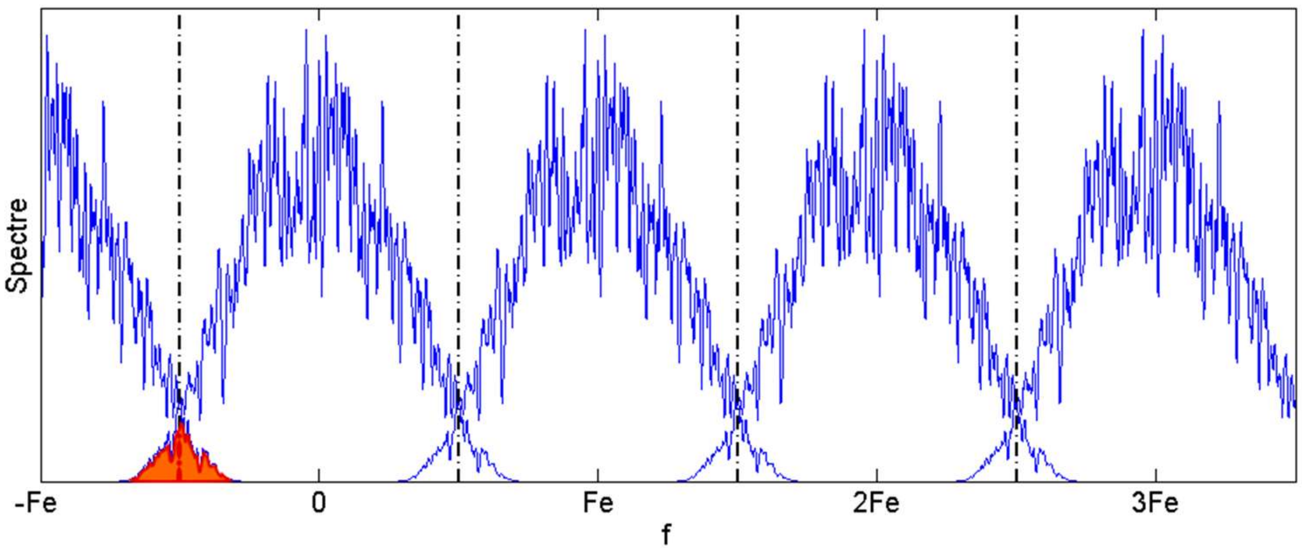
Nb bits	8	12	16
S/B	53 dB	77 dB	110 dB

Acquisition

Irreversible action (cf. filtering)

Data flux:
- data quantity *vs* band width
- writting speed
- memory (RAM)

FIltering



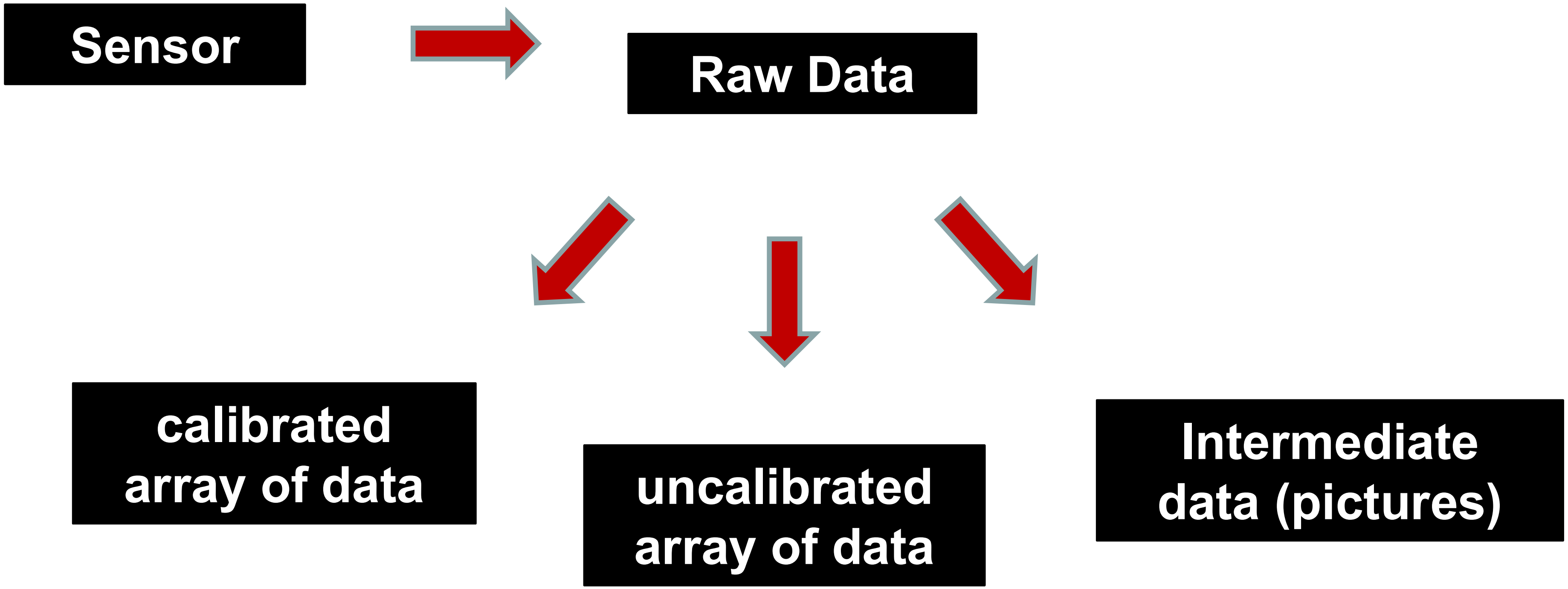
Low pass
High pass
Band pass

To respect Shannon criterion

To limit data volume

To reduce noise level

Data for science: saving data



Wish list
- accessible
- portable
- understandable
- long-life
- space saving
- ...

Data representation
- Text files (.csv, .txt, ...)
- Binary files
- Proprietary formats
- .hdf5 or equivalent
- ...

Data location
- on computer
- on hard drive
- on servers
- on cloud
- ...

Signal processing: acquisition chain

Sensor → Electric current from physical quantities

↓

Filtering → Data conditioning

↓

Sampling → From continuous to discrete representation

↓

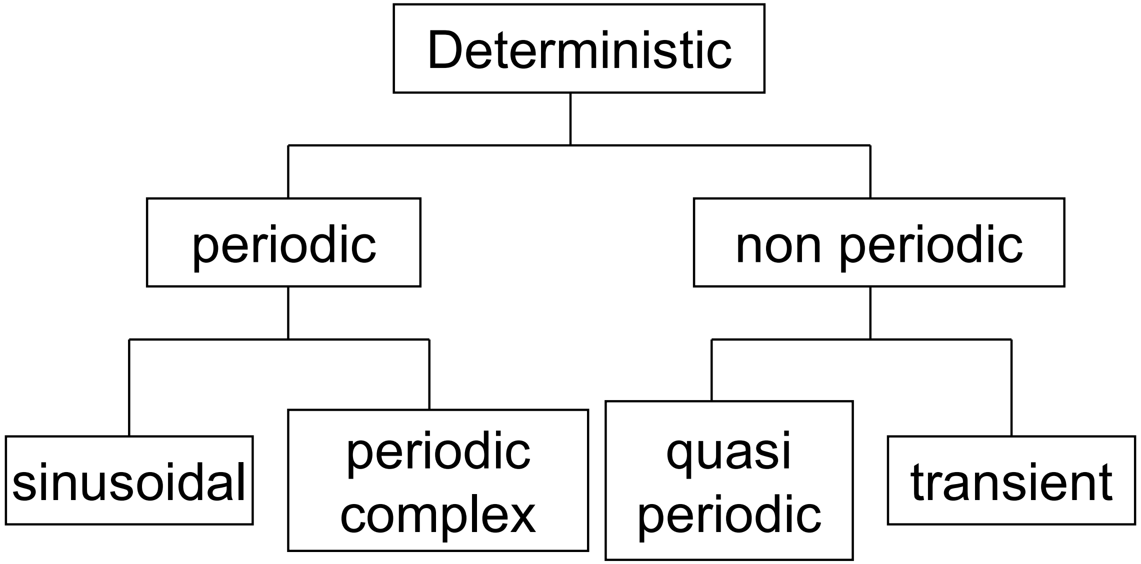
Quantification → From analogical to digital

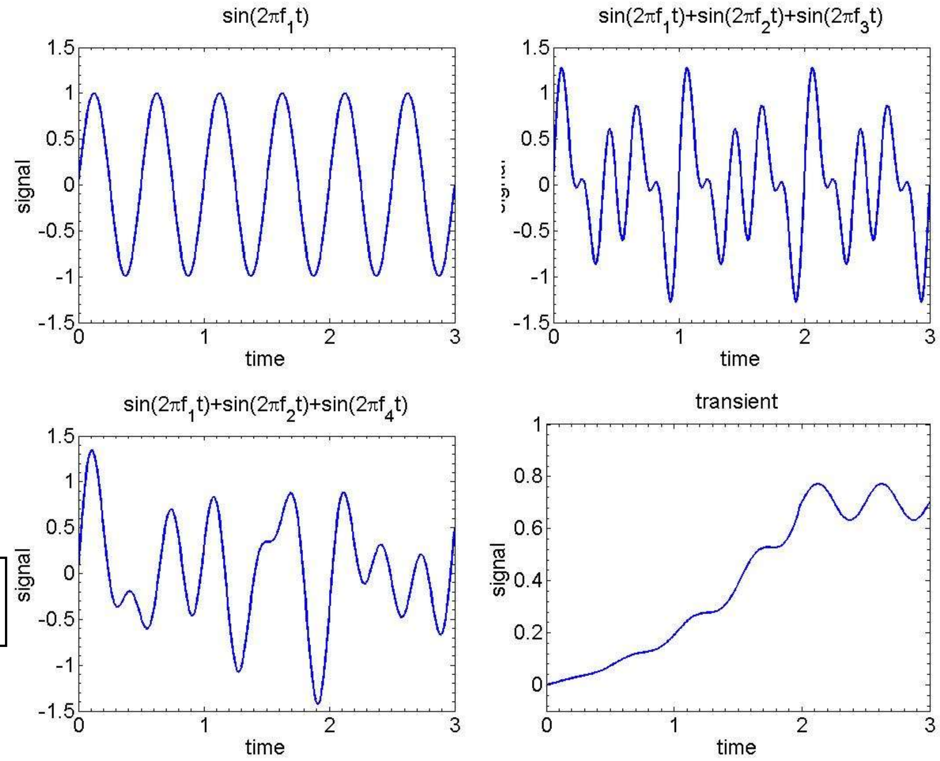
↓

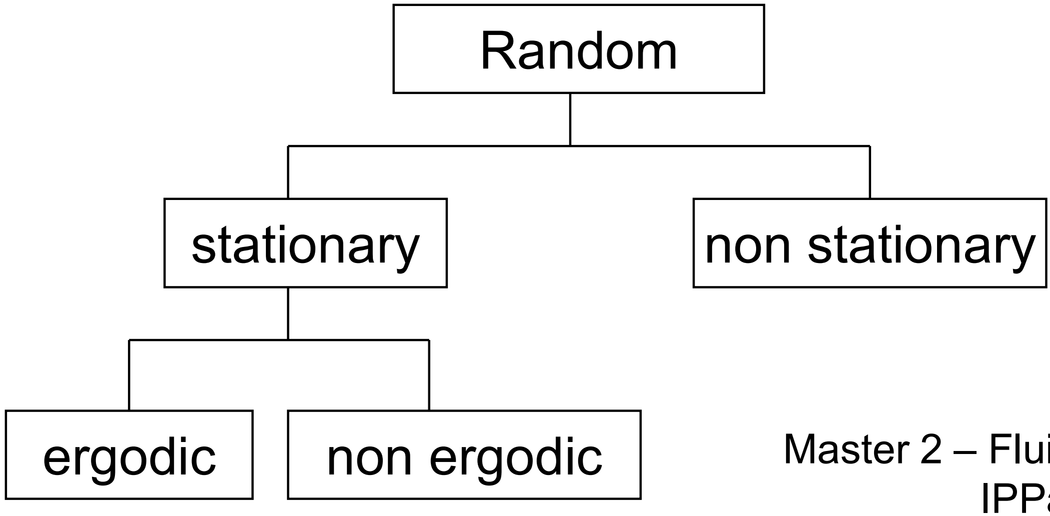
Acquisition → Save data in proper format

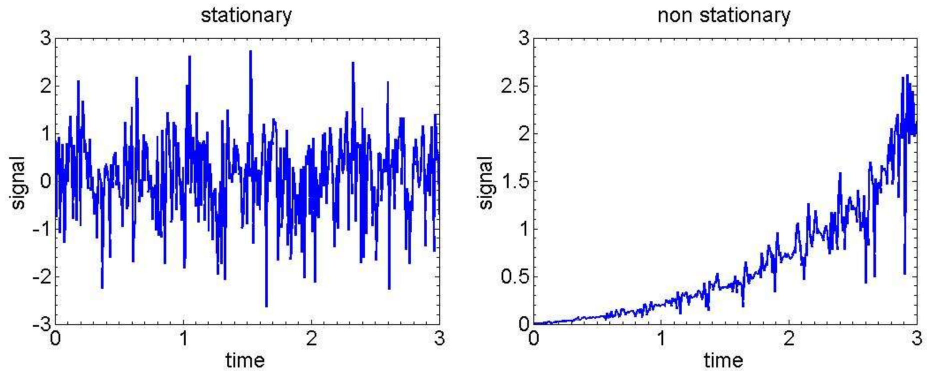
→ **Time to analyse these signals**

Deterministic vs random signals

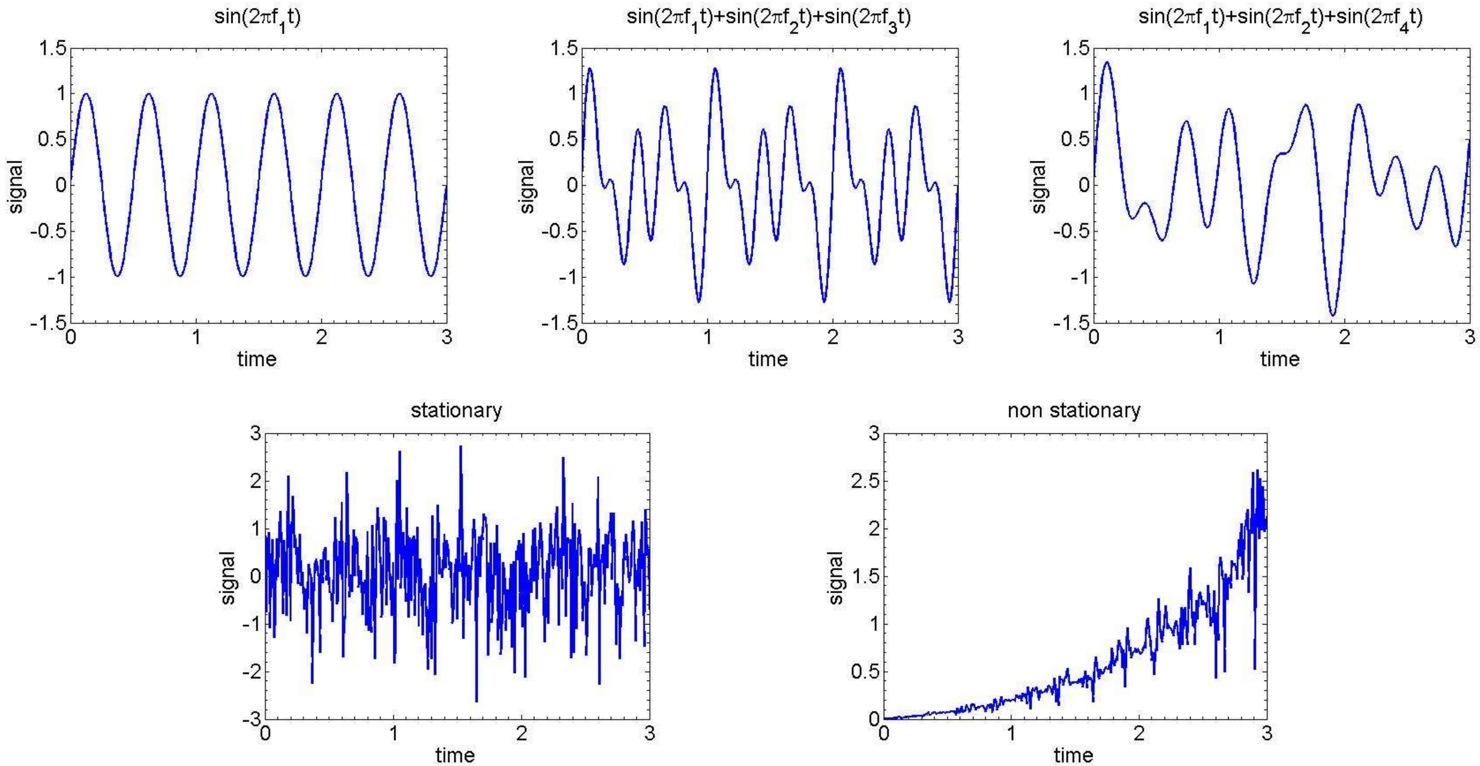








Post-processing challenges



Statistical quantities:
- mean, satndard deviation
- correlations

Probability densities:
- fluctuation asymetry
- complex effect identification

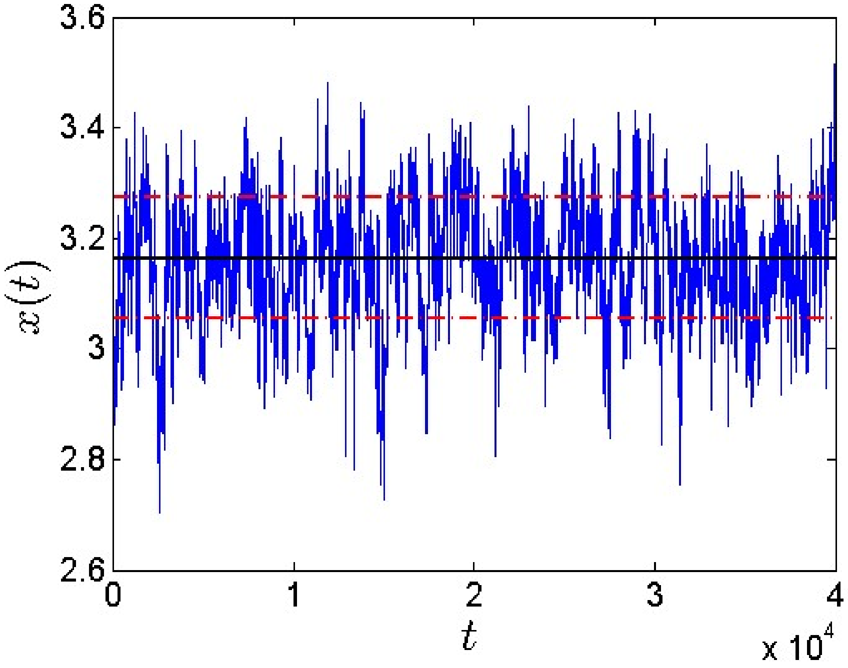
Spectral content:
- rich informations (see practical session)

Drifts:
- in space or time
- data quality, non trivial long time effects

Mean and standard deviation

$$\overline{p} = \frac{1}{M} \sum_{n=0}^{M-1} p[n]$$

$$p_{rms} = \left(\frac{1}{M-1} \sum_{n=0}^{M-1} (p[n] - \overline{p})^2 \right)^{1/2}$$



Standard deviation:

- carefull to estimation bias!
- fluctuation measurement
- distribution width
- data dispersion
- empirical error bars

Steady signal:

- obvious meaning

Unsteady signal:

- meaningless quantities
- moving moment estimation

Averages:

Ensemble average :

$$< T(\vec{x},t) >= \lim_{N\to\infty} \frac{1}{N} \sum_{i=1}^{N} T_i(\vec{x},t)$$

N times the same experiment

Average on realisations

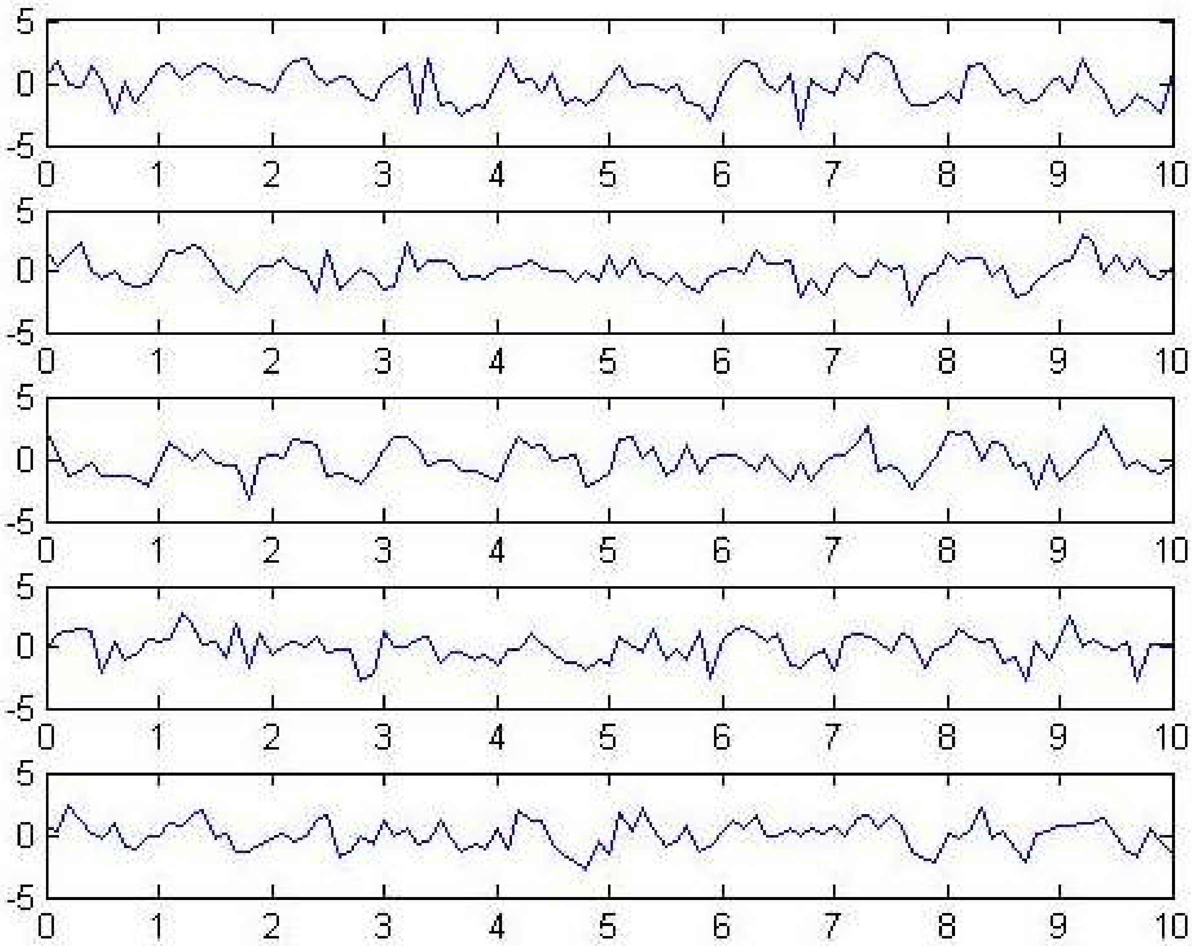
Time average :

$$\overline{T}_a(\vec{x},t) = \frac{1}{2a} \int_{-a}^{a} T(\vec{x},t+s)ds$$

$$\overline{T}(\vec{x}) = \lim_{a\to\infty} \overline{T}_a(\vec{x},t)$$

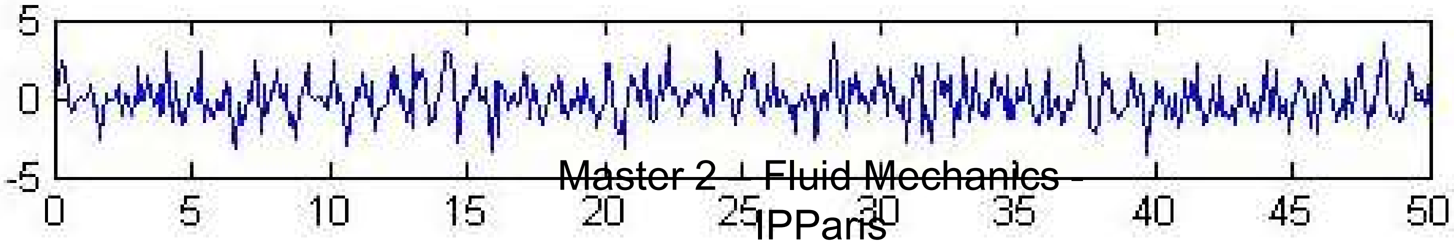
Ergodicity if both are identical

Averages:



5 realisations
over 10 time units

1 realisation over 50 time units

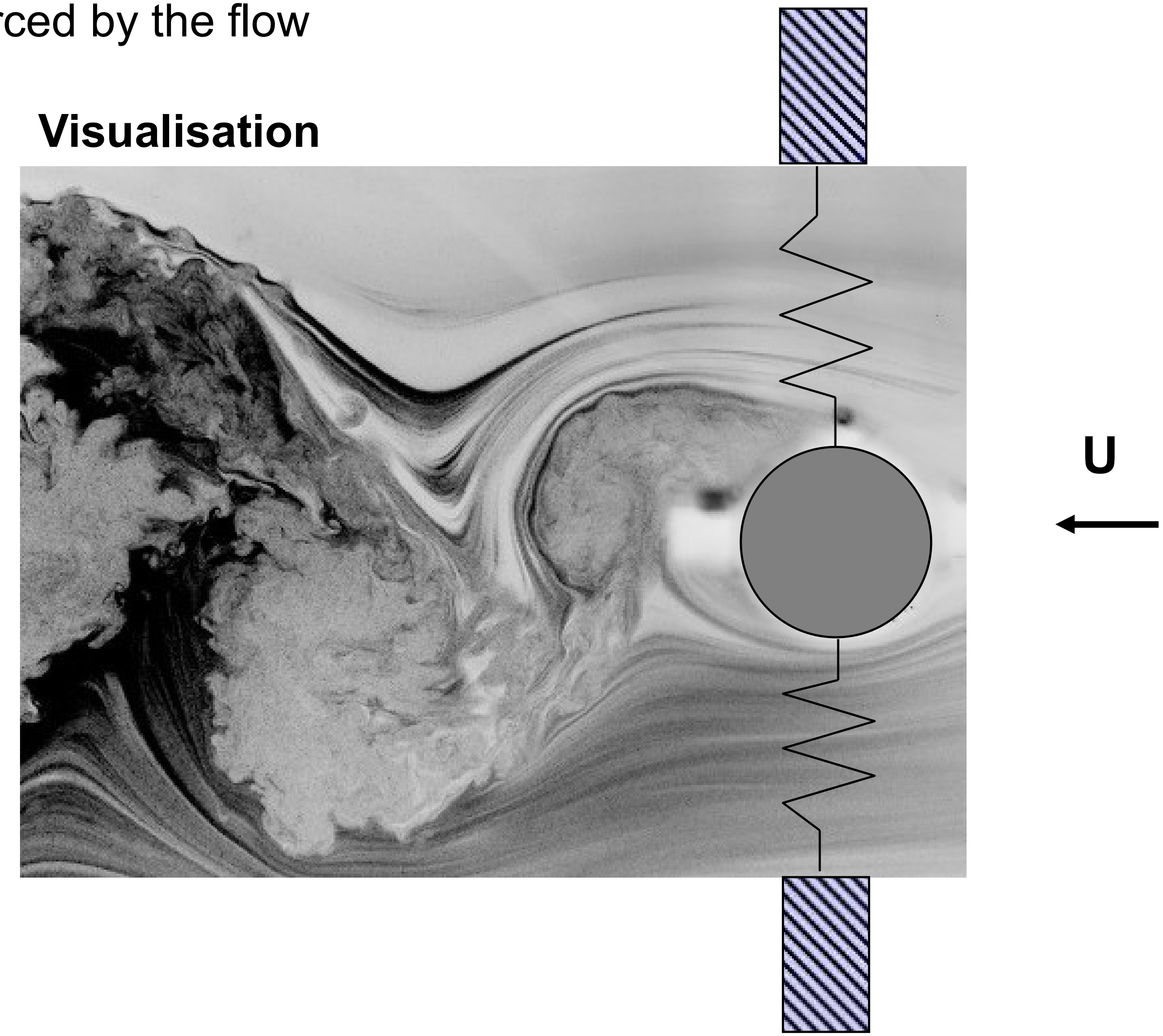


Signal processing: post-processing chain

Time and ensemble averages :

Cylinder wake with oscillations
forced by the flow

Visualisation

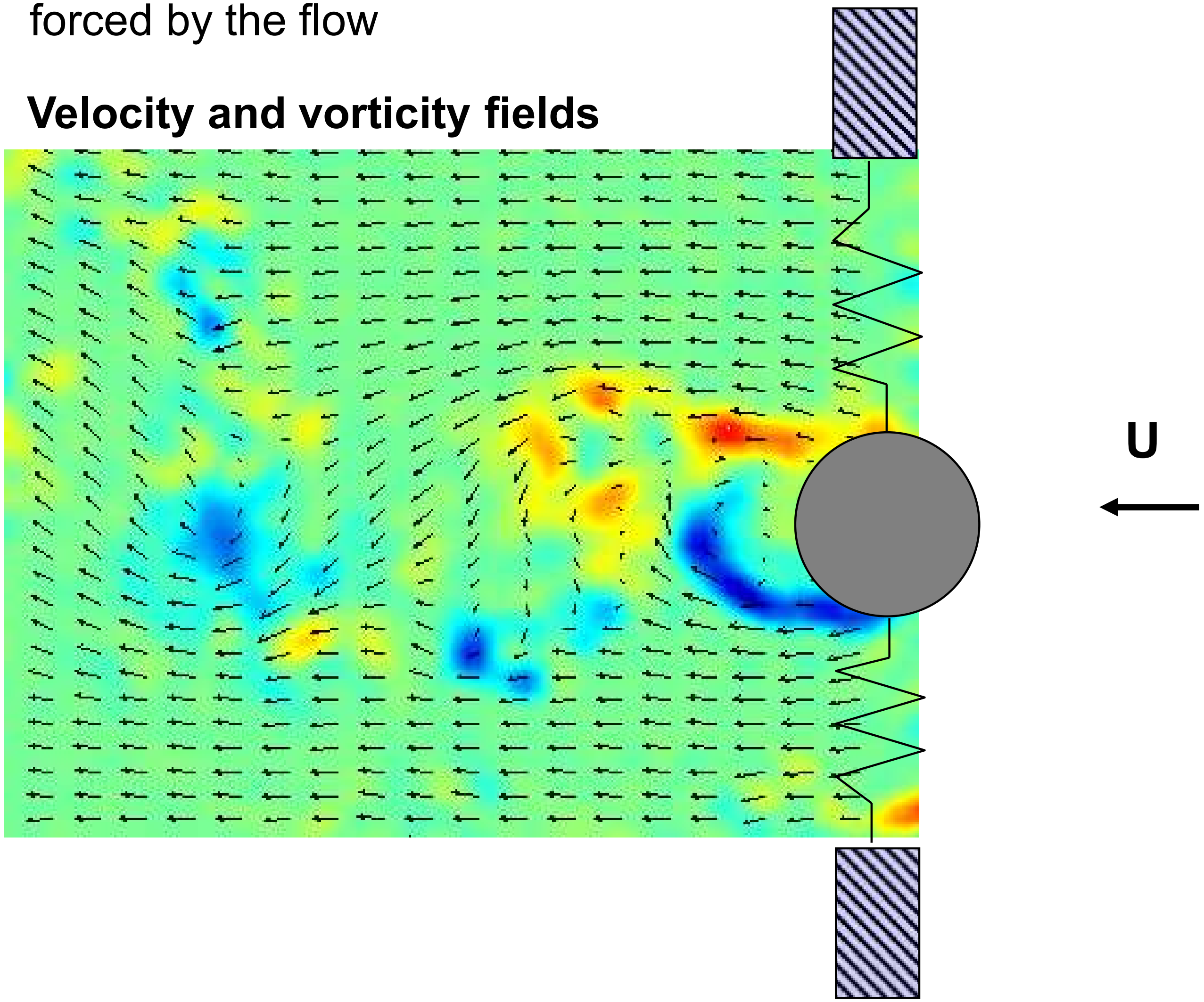


Signal processing: post-processing chain

Time and ensemble averages :

Cylinder wake with oscillations forced by the flow

Velocity and vorticity fields



Time and ensemble averages :

Cylinder wake with oscillations
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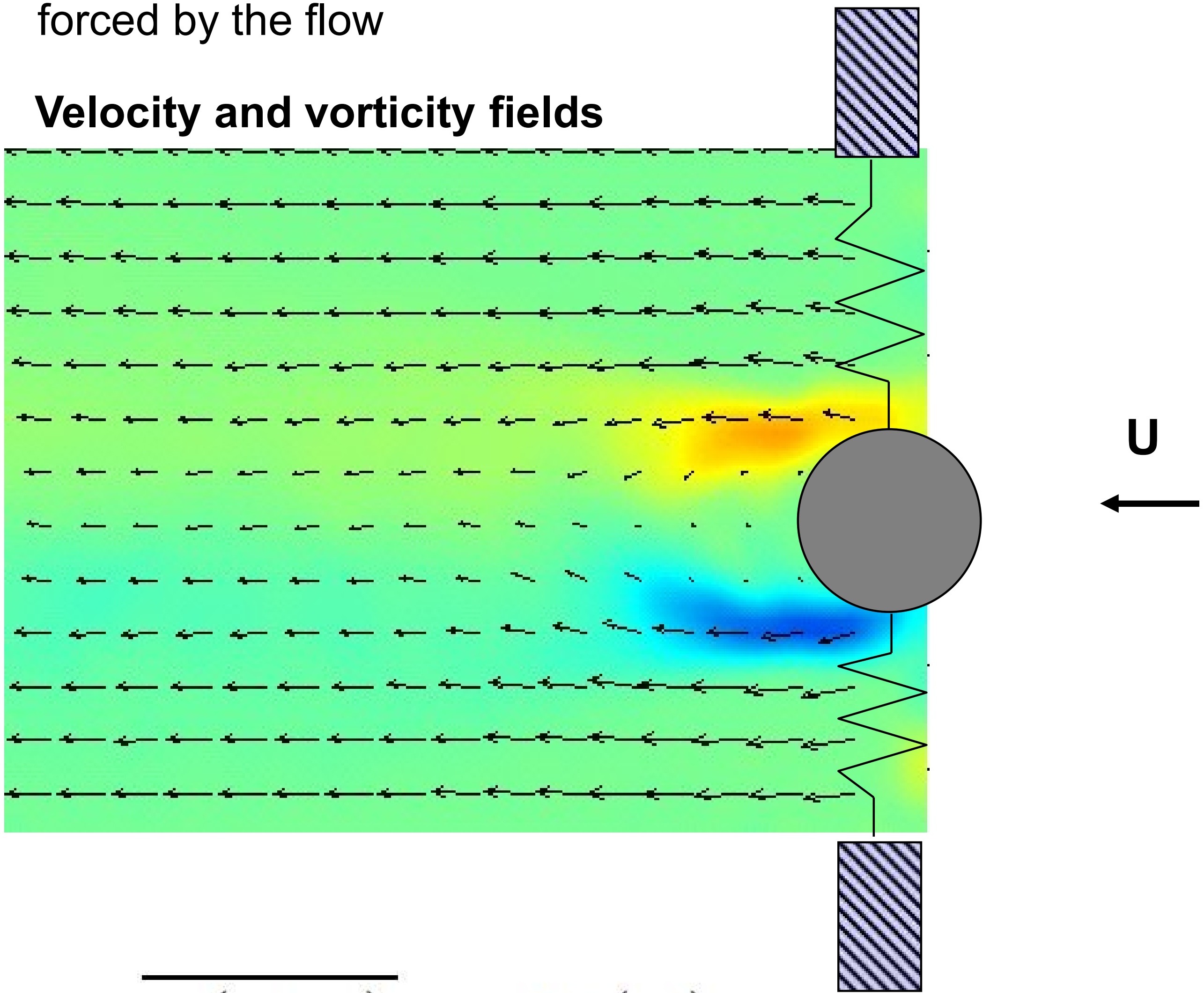
Velocity and vorticity fields

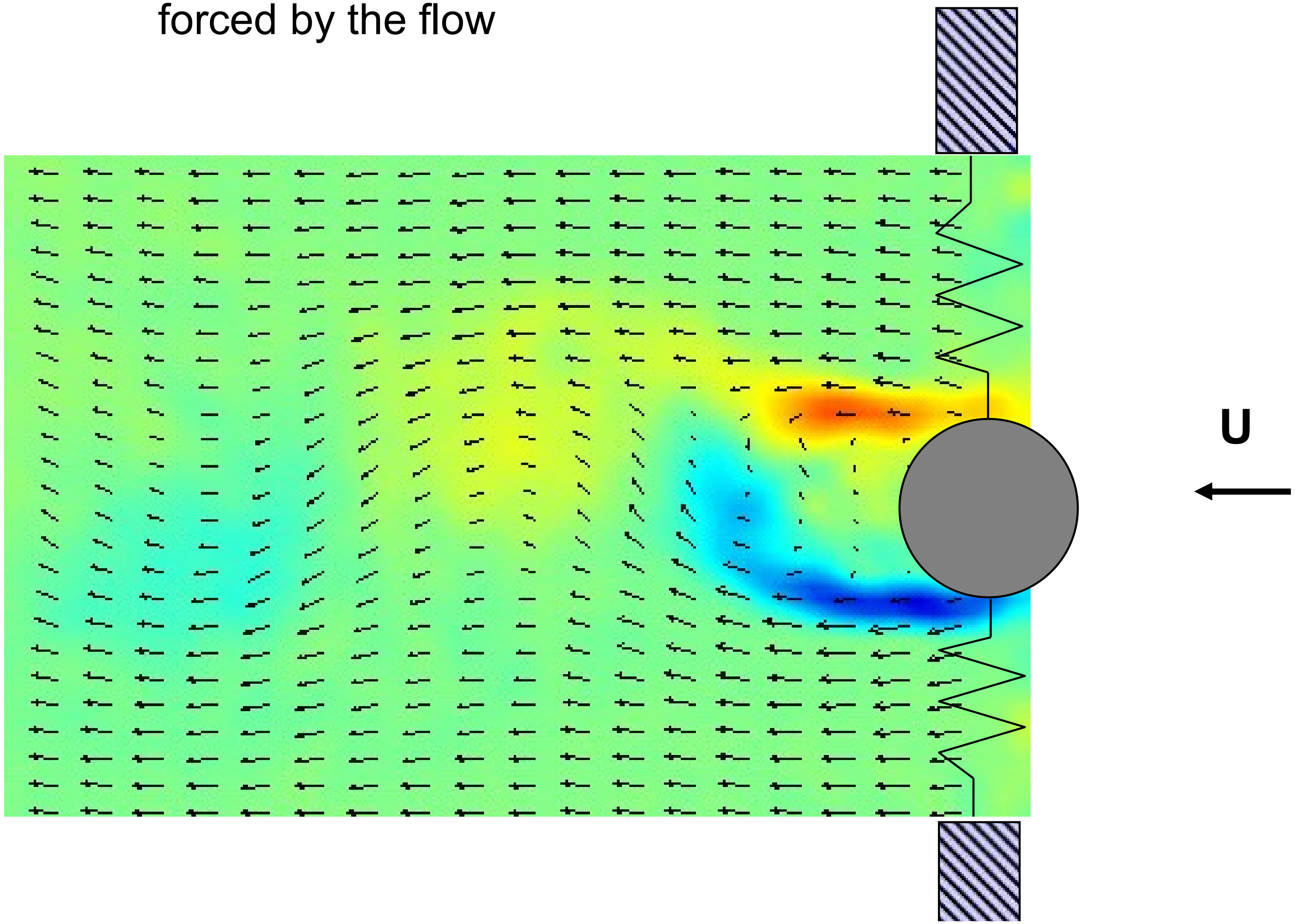


Time average : $\overline{u(\vec{x},t)} = U(\vec{x})$

Signal processing: post-processing chain

Time and ensemble averages :

Cylinder wake with oscillations forced by the flow



Ensemble average : $$\overline{u(\vec{x}, t_1)} = U(\vec{x}, t_1)$$

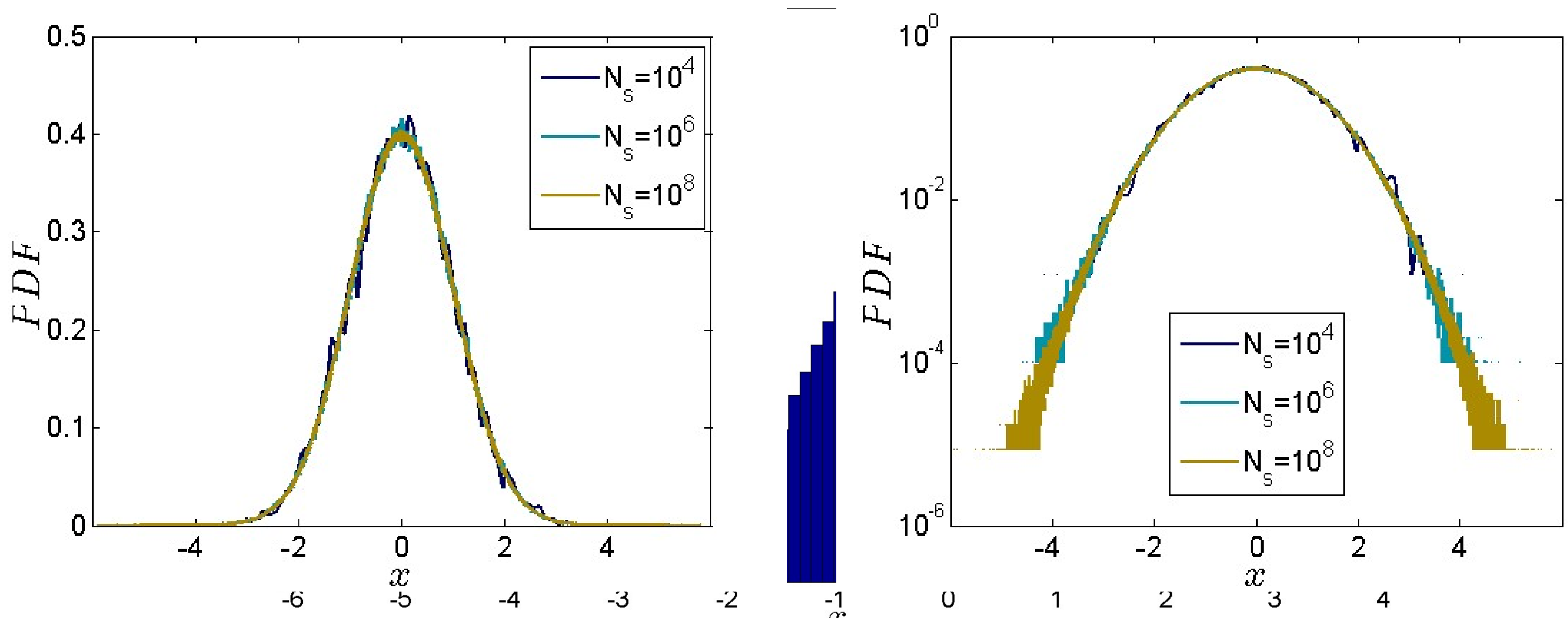
Probability density function

Cumulated probability: $$\mathcal{F}(x) = P(X < x)$$

Probability density function: $$f(x) = \frac{d\mathcal{F}}{dx}$$

Practically:

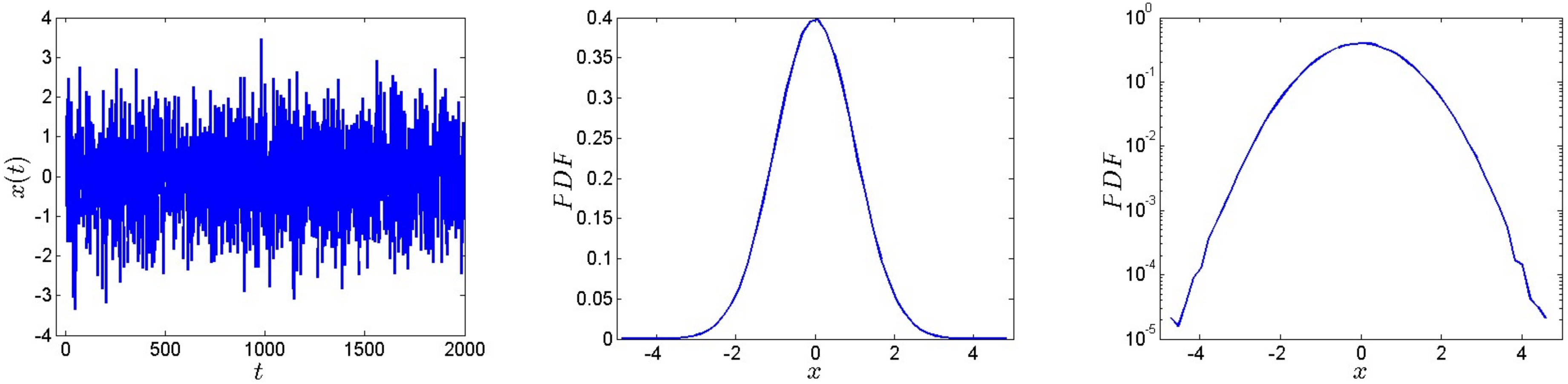
- histogram estimation
- histogram normalisation



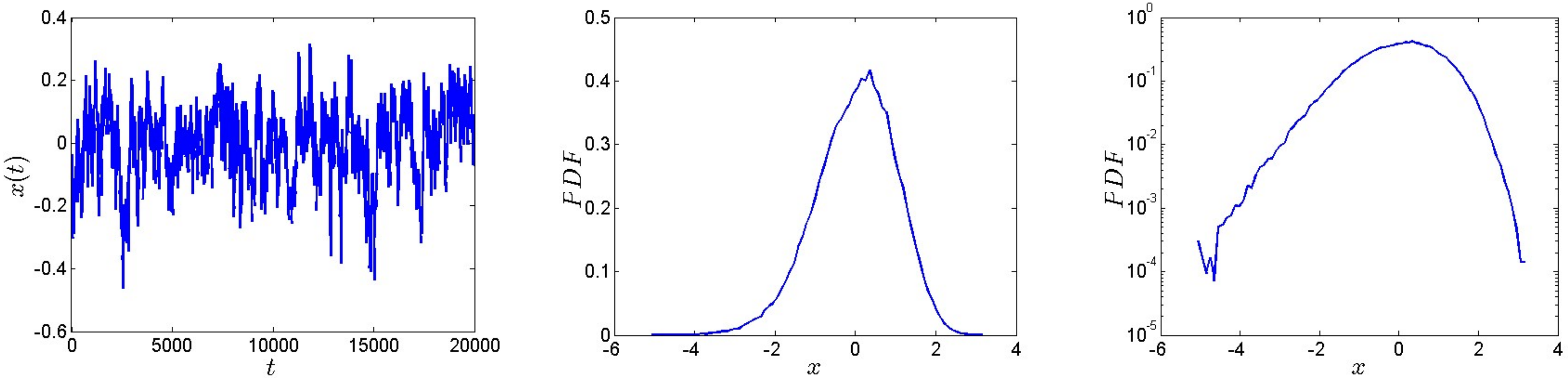
Signal processing: post-processing chain

Probability density

Gaussian white noise



Turbulent jet



Power spectrum (definition)

Fourier transform (continuous signals)

$$X(f) = < x, e^{2\pi j f t} > = \int_{-\infty}^{+\infty} x(\tau) e^{-2\pi j f \tau} \, d\tau$$

$$x(t) = \int_{-\infty}^{+\infty} X(f) e^{2\pi j f \tau} \, df.$$

Fourier transform (discrete signals)

$$X(\lambda) = \sum_{n=-\infty}^{+\infty} x[n] e^{-2j\pi\lambda n}, \quad \lambda \in [-0.5, 0.5]$$

$$x[n] = \int_{-0.5}^{0.5} X(\lambda) e^{2j\pi\lambda n} \, d\lambda$$

Power spectrum (definition)

Energy *vs.* power

$$P(x) = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{+T/2} |x(t)|^2 \, dt.$$ (infinite signals)

$$E(x) = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt.$$ (finite signals)

Parseval theorem

$$E(x) = \int_{-\infty}^{+\infty} |x(t)|^2 \, dt = \int_{-\infty}^{+\infty} |X(f)|^2 \, df$$

⟹ **Energy can be measured in spectral or physical space**

Power spectrum (practical calculation)

Mean periodogram:

$$\mathrm{PSD}_x(f) = \frac{1}{M}\sum_{i=0}^{M} |X_i(f)|^2$$

with $X_i(f) = FFT[f(n)x(n+iP)]$

⟹

Required:
- segment number
- segment length
- windowing

Correlogram:

$$\mathrm{PSD}_x(f) = \int_{-\infty}^{\infty} R_{xx}(t)\, dt$$

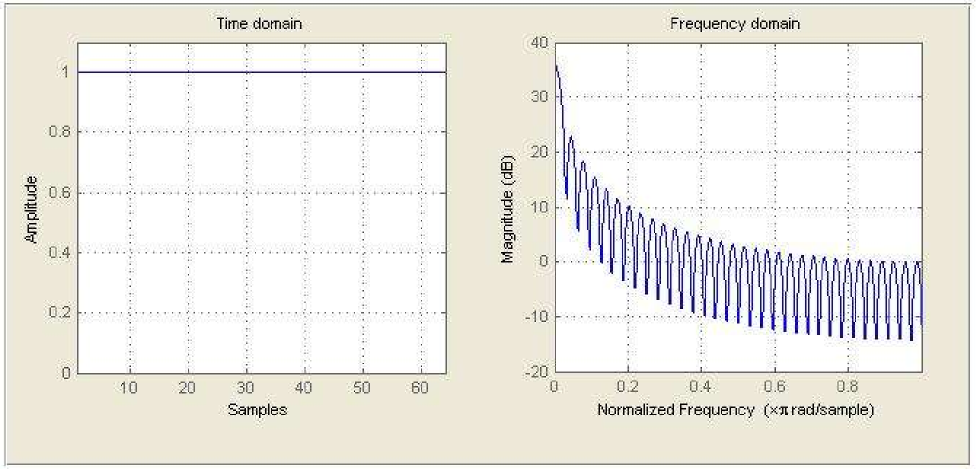
Wiener-Khintchine formula

⟹

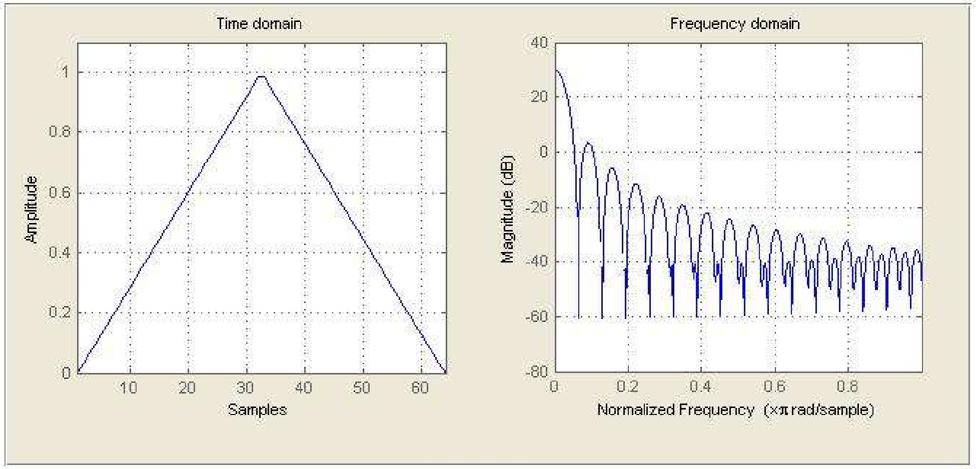
Required:
- autocorrélation estimation
- Fourier transform calculation

Power spectrum (windows)

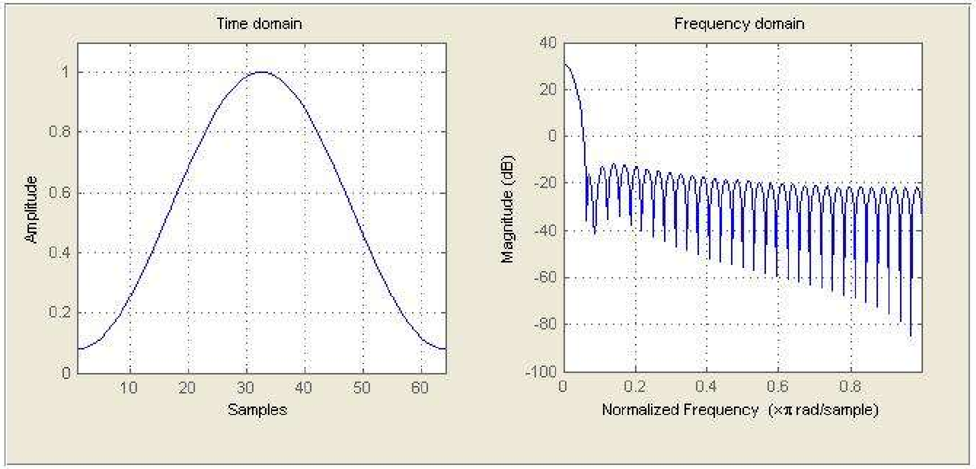
Rectangle



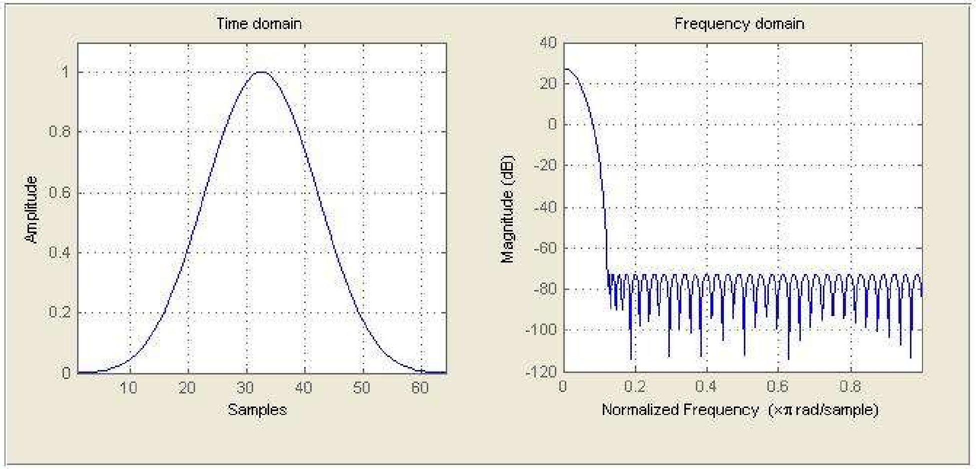
Bartlet



Hamming



Chebyshev



⇒ **No ideal choice**
Case to case decision

Correlations

$$R_x(\tau) = \overline{x(t)x(t+\tau)}$$

$$R_f(\tau) = \int_{-\infty}^{+\infty} f(t)f(t-\tau)dt$$

Auto-correlation

$$R_{xy}(\tau) = \overline{x(t)y(t+\tau)}$$

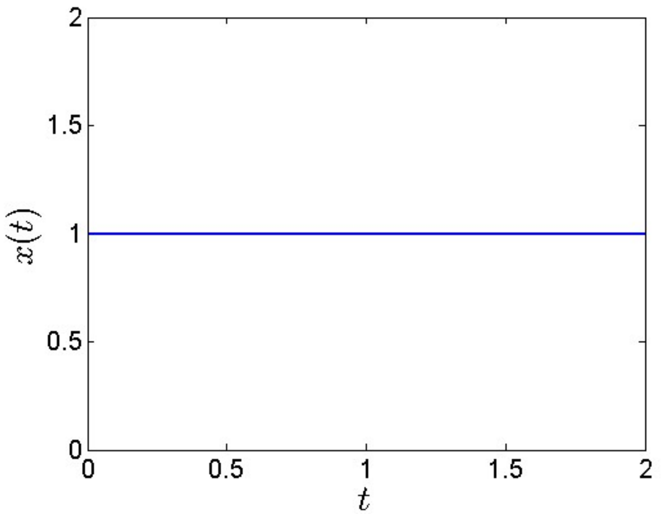
$$R_{fg}(\tau) = \int_{-\infty}^{+\infty} f(t)g(t-\tau)dt$$

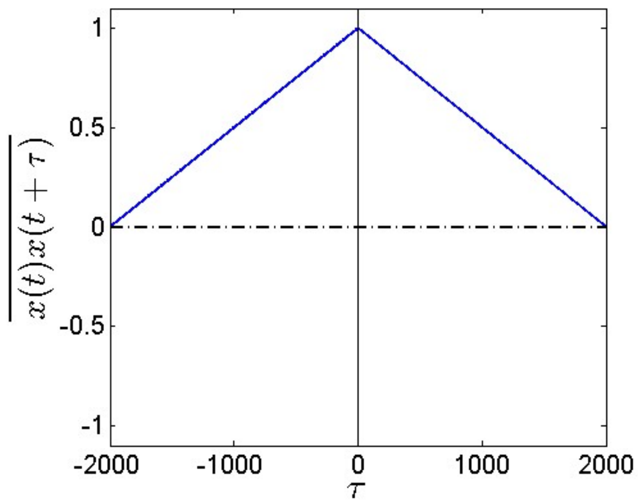
Cross-correlation

Signal processing: post-processing chain

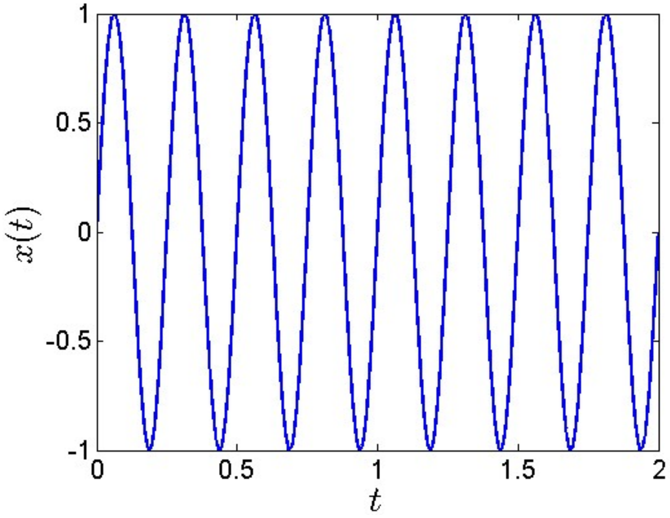
Correlations

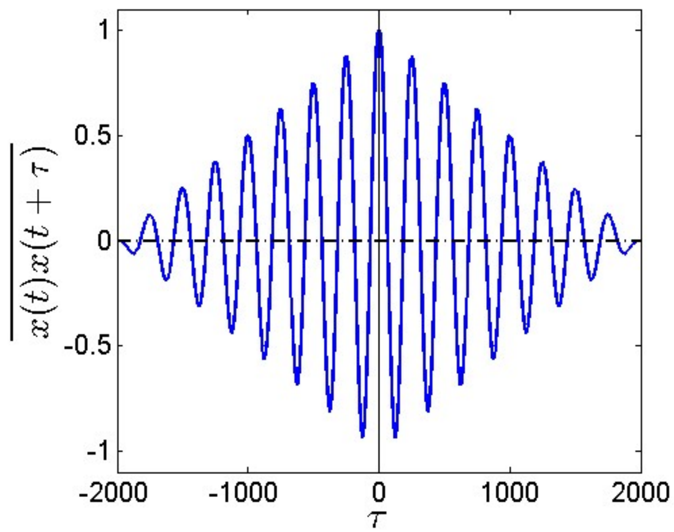
Constant





Sinusoïd

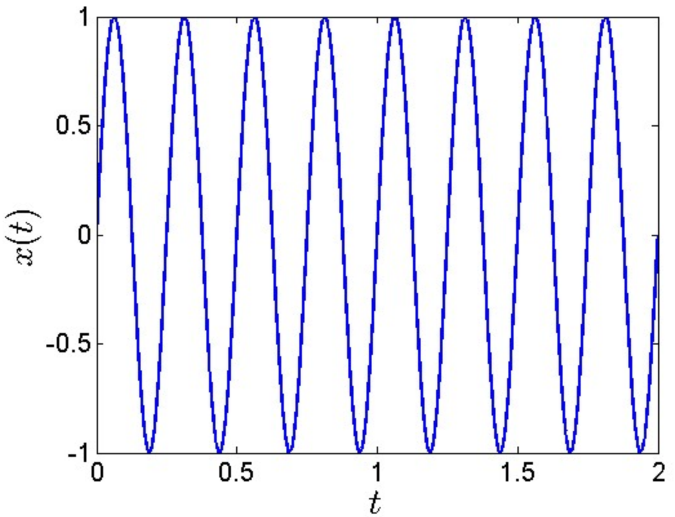


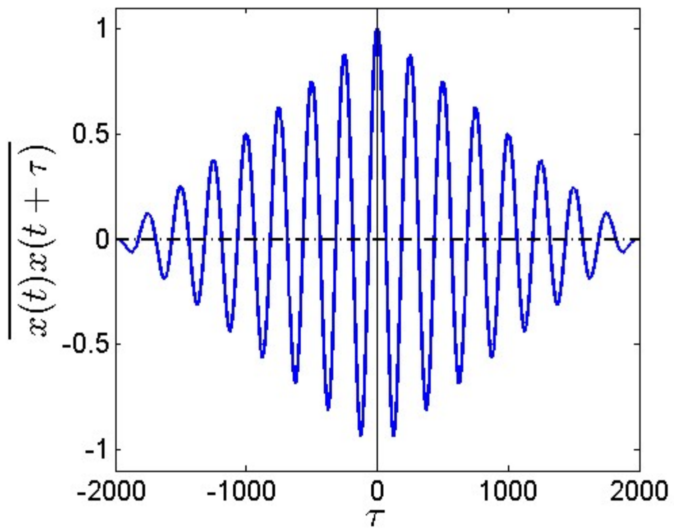


⟹ **Soustract mean value!**

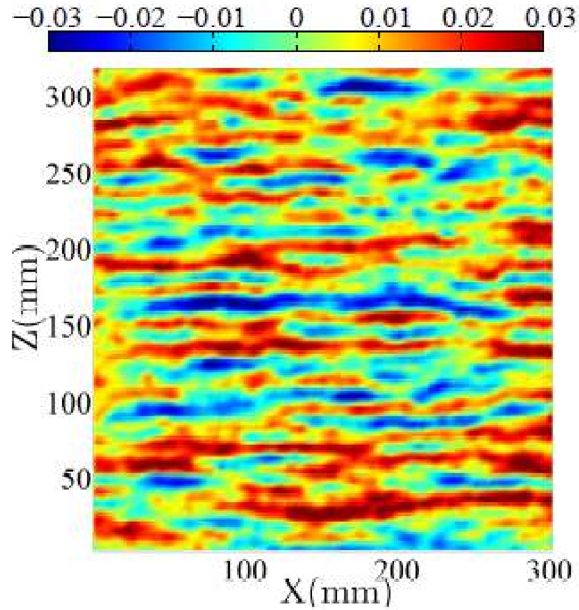
Correlations

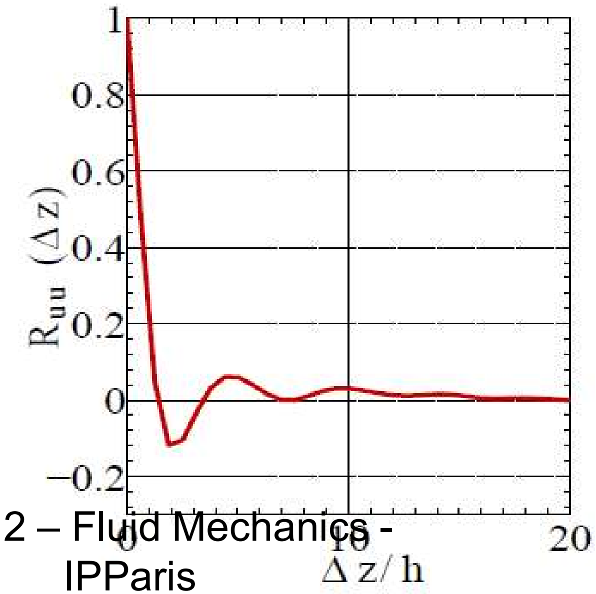
Sinusoïd





Plane Couette velocity

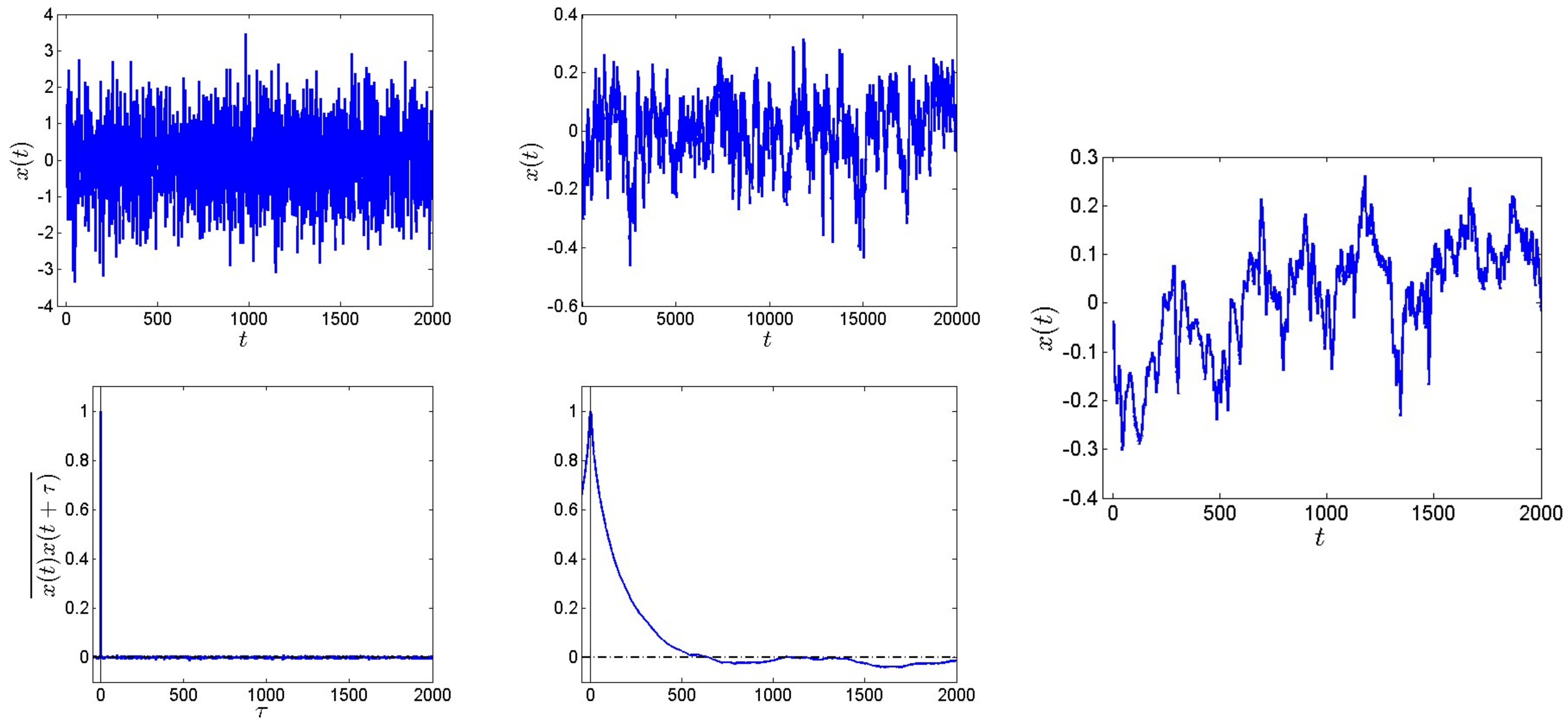




Correlations

Gaussian white noise

Turbulent jet



→ **Be careful for statistical estimations**