Experimental methods

One point techniques

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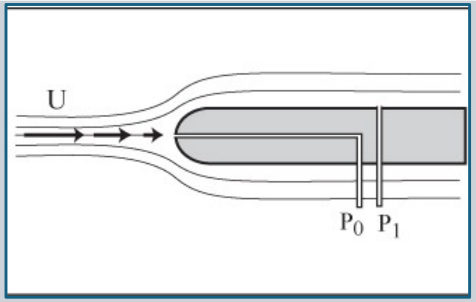
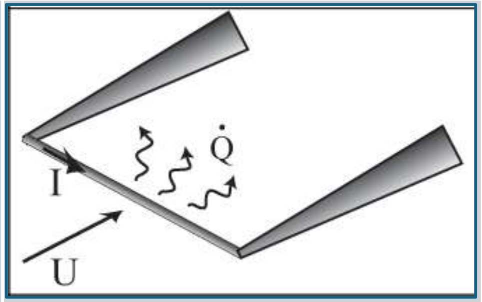
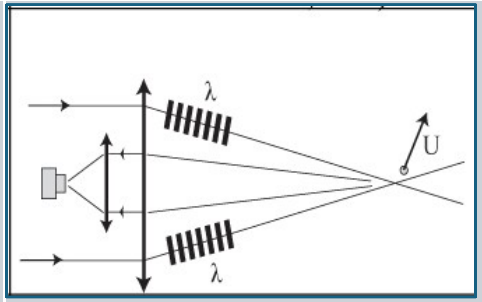
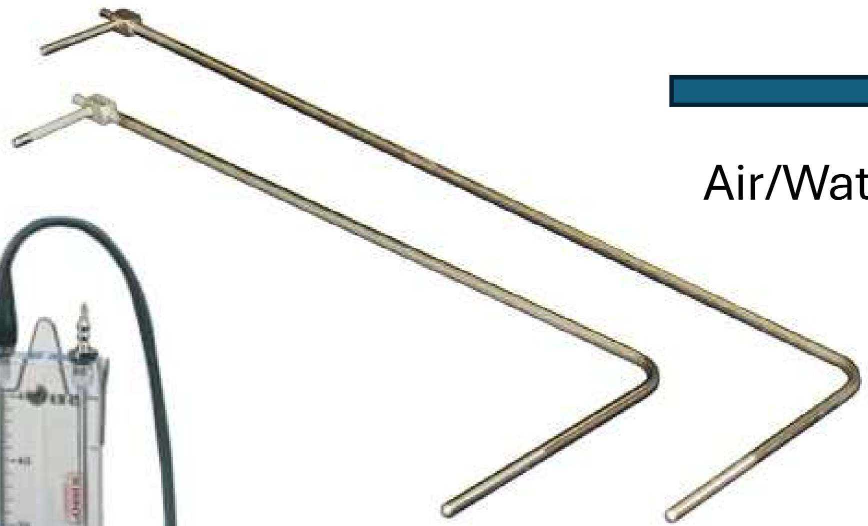
One point techniques

	Pitot Tube	Hot Wire Anemometry	Laser Doppler Anemometry
Sketch			

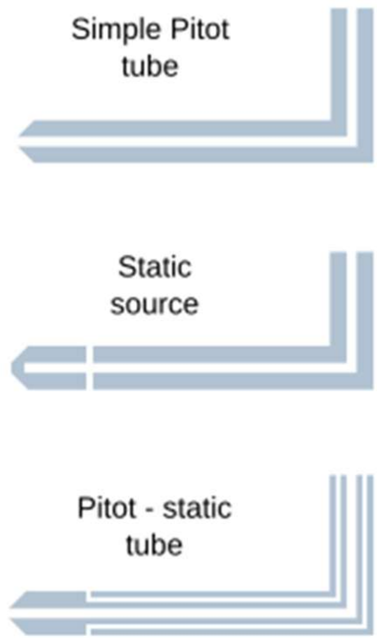
Pitot tube

Principle:

Stagnation pressure measurement
Static pressure measurement
Bernoulli theorem



Air/Water flow



Pitot tube

Principle:

Stagnation pressure measurement
Static pressure measurement
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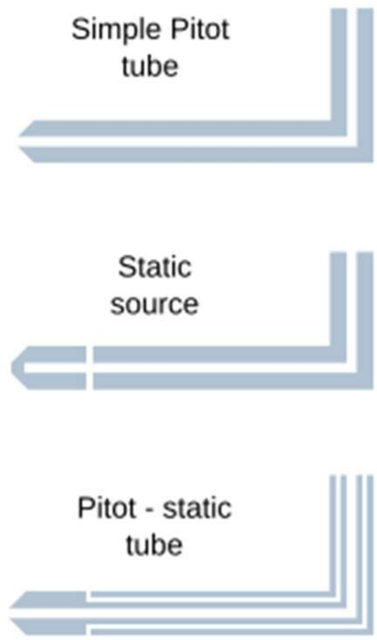
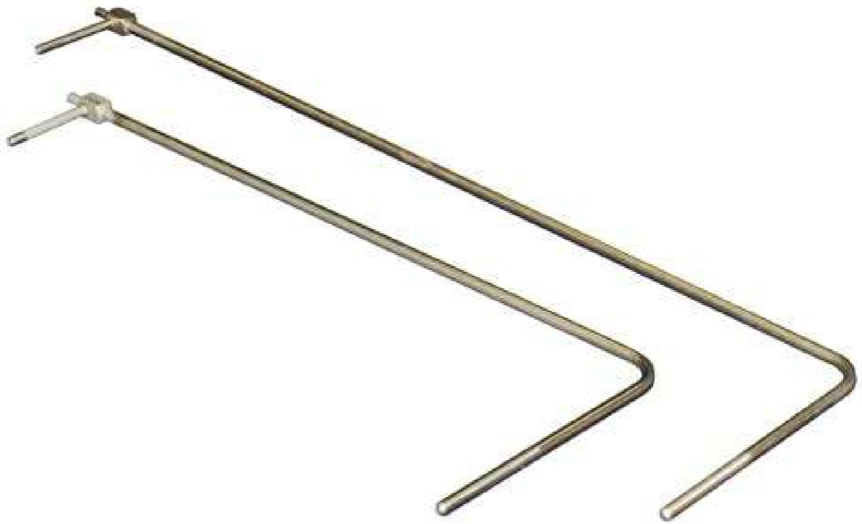
Air/Water flow



Pros / cons:

Cheap
Easy to use
Suited for time averages
Intrusive
Poor time resolution
Poor spatial resolution

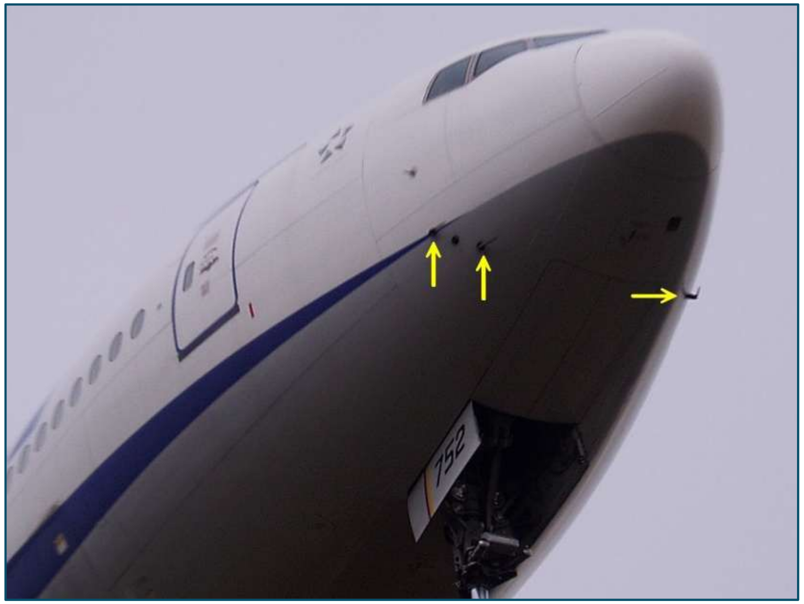


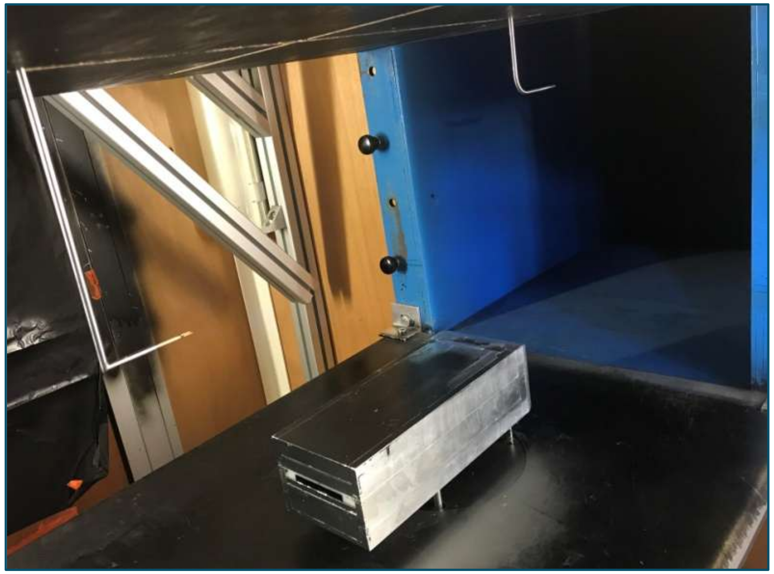
Uses:

Monitoring
Calibration
In situ
Large arrays

Illustrations: wikipedia

Pitot tube

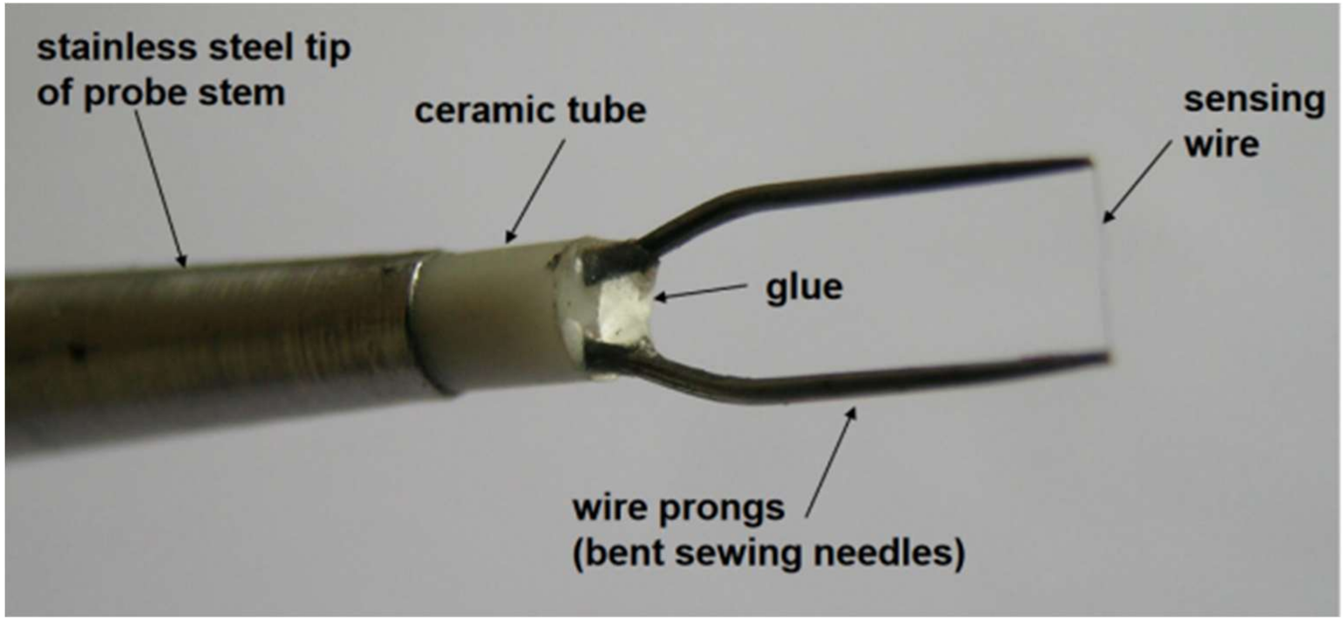


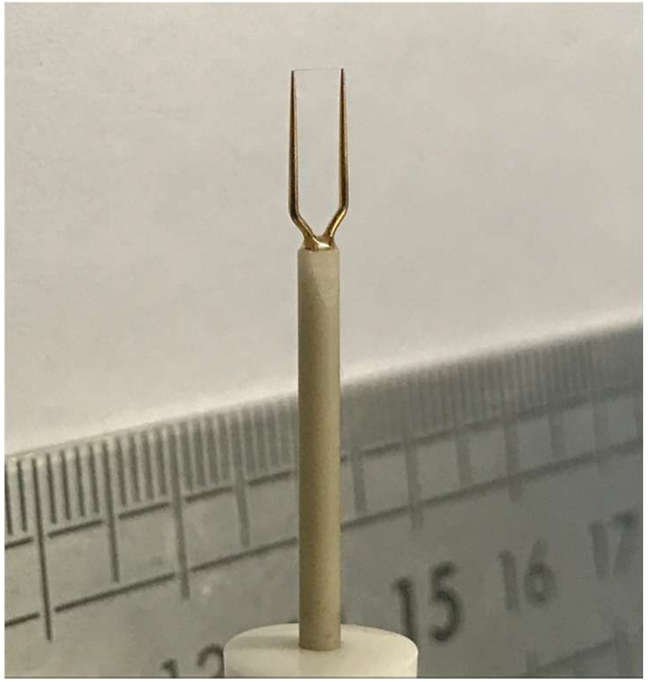


Illustrations: wikipedia, souffleries2a.com

Hot wire anemometry

How does it look like ?





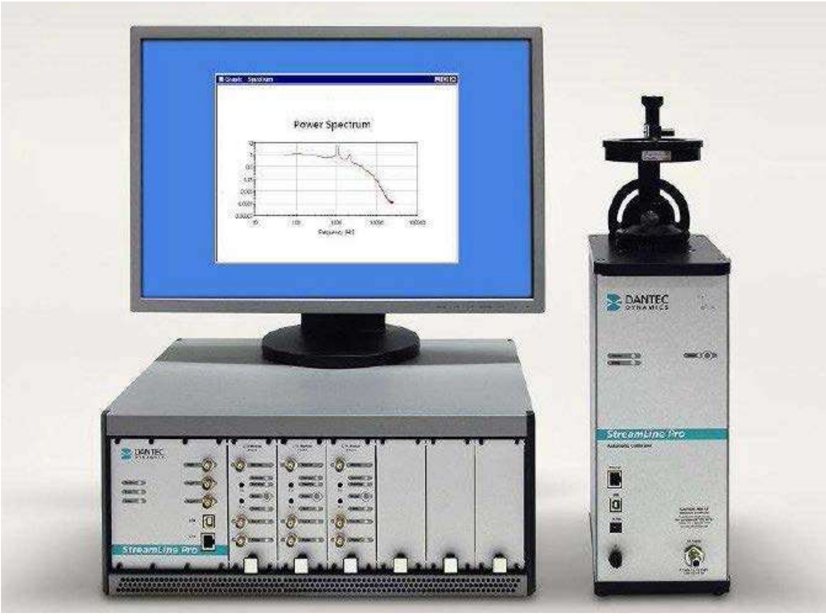
Platinium or tungsten thin wire
(few microns thick, few mm long)
welded to the prongs of the probe support

Illustrations: von Karman institute, ENSTA

Hot wire anemometry

How does it look like ?

Pocket size model
(mean velocity)
150 – 400 €



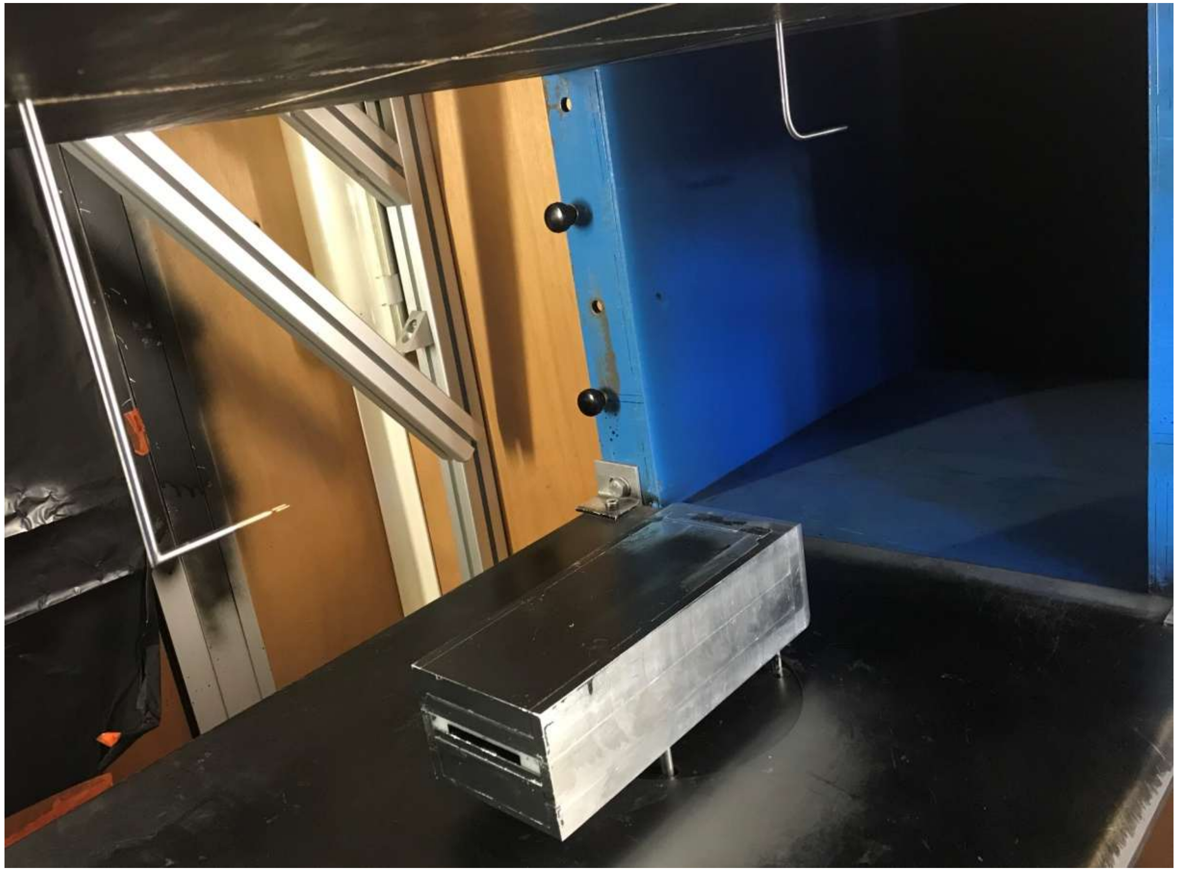
Lab model
(fluctuations, 10 probes)
30-40 k€

Mini CTA
(fluctuations, 1 probe)
2-4 k€

Illustrations: DANTEC dynamics

Hot wire anemometry

How does it look like ?

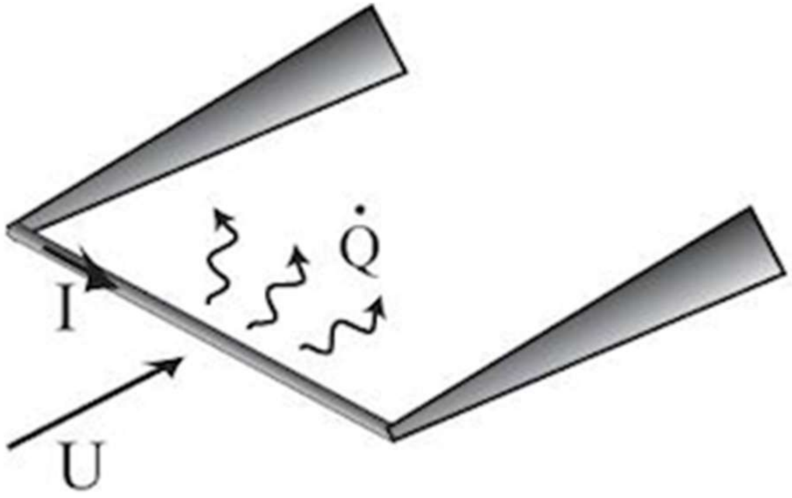


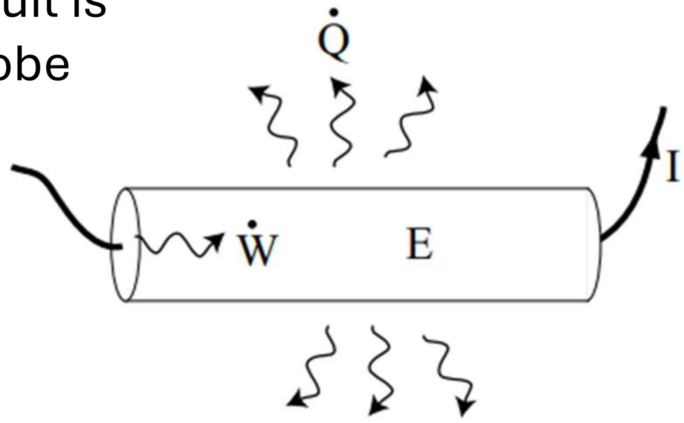
ENSTA wind tunnel – Ahmed body + hot wire

Hot wire anemometry

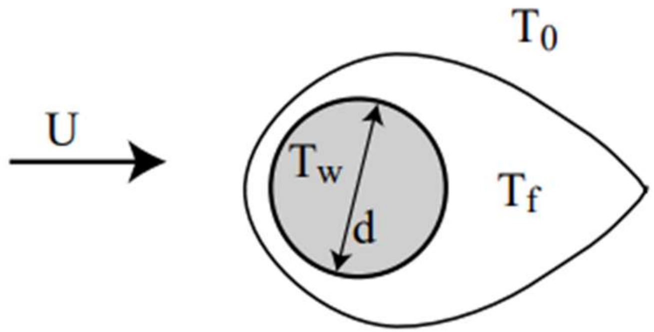
How does it work ?

Electronic circuit is heating the probe





Flow is cooling the probe: « chill factor »



Thermal boundary layer

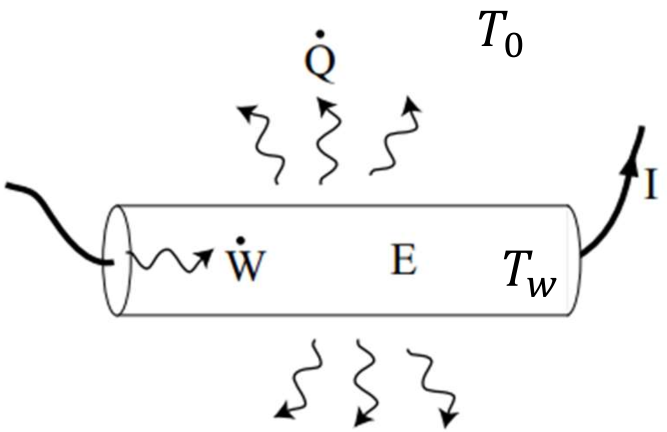
⟹ **Convection and Joule effect balance**

Hot wire anemometry

Governing equations

Energy stored in the wire:

$$\frac{dE}{dt} = \dot{W} - \dot{Q}$$



Wire area

$A = \pi d l$

$l/d \ll 1$

Electrical power $\dot{W} = R_w I^2 > 0$

Joule effect

Heat variation $\dot{Q} > 0$

Conduction to fluid
Convection to fluid
Conduction to support
Radiation

$$\left.\right\} \frac{\dot{Q}}{A} = h(T_w - T_0)$$

Nusselt number

$$Nu = hd/k_f$$

Convection efficiency

Heat flux

$$\vec{j_Q} = -k_f \vec{\nabla} T$$

$$\Longrightarrow \dot{Q} = k_f l \pi \, (T_w - T_0) \, Nu$$

Hot wire anemometry

Governing equations

Power budget
$$\frac{dE}{dt} = R_w I^2 - \pi l k (T_w - T_0) Nu$$

At equilibrium
$$R_w I^2 = \pi l k (T_w - T_0) Nu$$

Link between Nu and the velocity U ?

Hot wire anemometry

Governing equations

Heat transfer: $Nu = f(Re, Pr, Ma, l/d \dots)$

Wire Reynolds number: $Re_w = \dfrac{Ud}{\nu_f} \ll 1$

Fluid Prandtl number: $P_r = \dfrac{\nu_f}{\kappa_f} \simeq 1$

⟹ Laminar flow
Thermal and viscous boundary layers similar

⟹ $$Nu = 1 + \sqrt{2\pi Re_w}$$

King's laws

$$Nu = 0.42\, P_r^{\frac{1}{5}} + 0.57\, Re_w^{\frac{1}{2}} P_r^{\frac{1}{3}}$$

$$Nu = a_0 + b_0\sqrt{Re_w}$$

Hot wire anemometry

Governing equations

$$R_w I^2 = \pi l k (T_w - T_0)(a_0 + b_0 Re_w^{\frac{1}{2}})$$

What do we measure?

- Tension: $e = R_w I$
- No link with U a priori
- But …

$$\boldsymbol{R_w = f(T_w)}$$

$$R_w = R_0(1 + \beta(T_w - T_0))$$

$$\beta = \frac{1}{R_w}\frac{\partial R_w}{\partial T} \simeq cst$$

For Tu or Pt:

$$\beta \simeq 5 \times 10^{-3} K^{-1}$$

$$\Longrightarrow \quad \frac{R_w I^2}{R_w - R_0} = a + b\sqrt{U}$$

$$\Longrightarrow \quad \frac{e^2}{R_w(R_w - R_0)} = a + b\sqrt{U}$$

Hot wire anemometry

Two strategies

$$\frac{e^2}{R_w(R_w - R_0)} = a + b\sqrt{U}$$

King's law

$$e = R_w I$$

Ohm's law

Constant current anemometry CCA

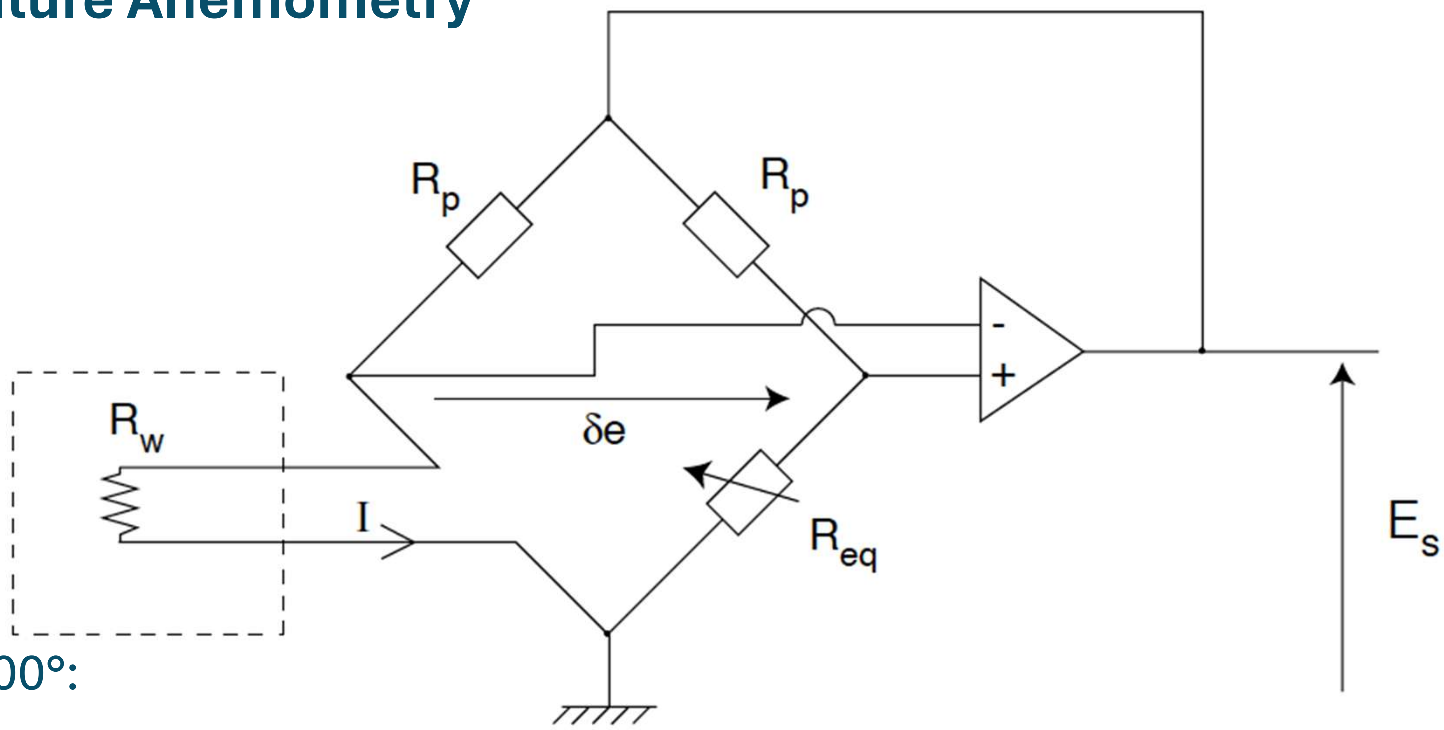
- I is constant
- U through fluctuations of R_w
- Obsolete

Constant temperature anemometry CTA

- R_w i.e. T_w is constant
- U through fluctuation of I i.e. e

Hot wire anemometry

Constant Temperature Anemometry



Overheat 100° – 200°:

$$R_{eq} < R_w$$

Negative feedback: ⟹ $$E_s = \sqrt{R_w(R_w - R_0)\left(a + b\sqrt{U}\right)}$$

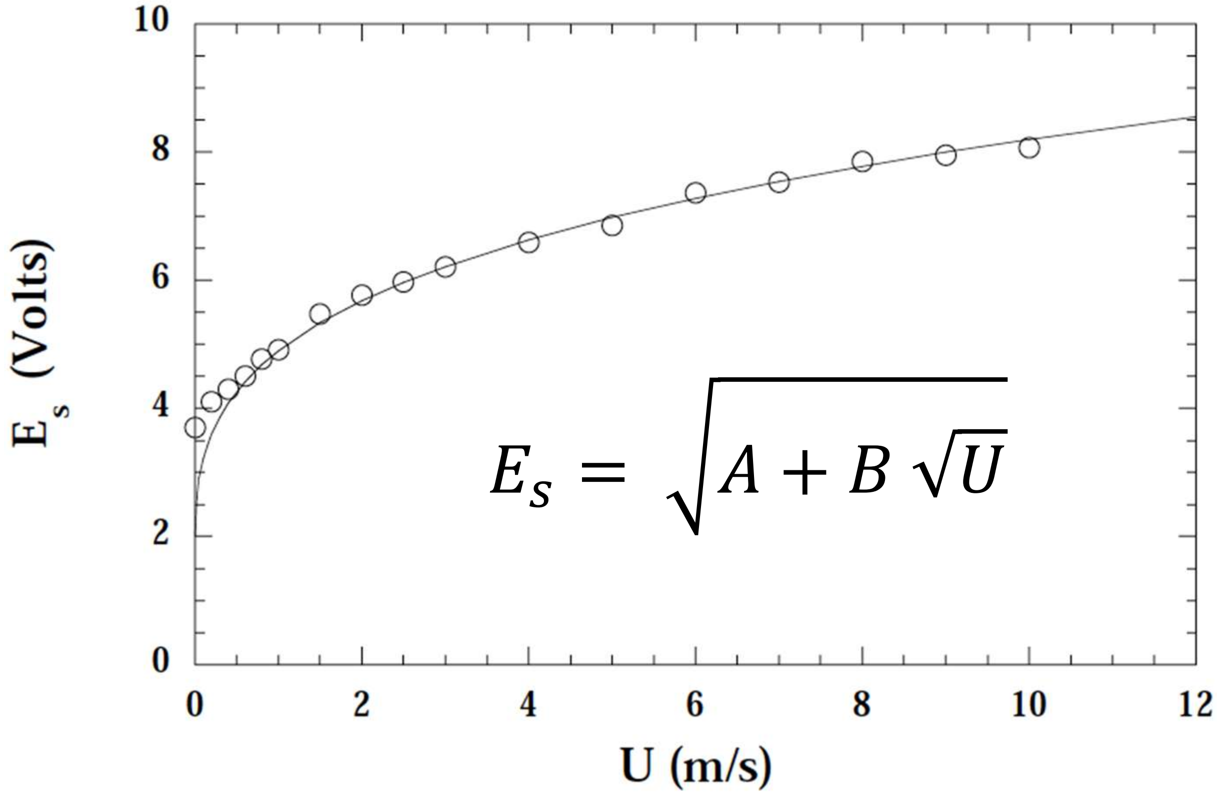
$$\delta e = I(R_w - R_0)$$

$$E_s = G\,\delta e$$

$$\boldsymbol{E_s = \sqrt{A + B\sqrt{U}}}$$

Hot wire anemometry

Calibration



Modified King's law

$$E_s^2 = A + B U^n$$

n between 0.4 and 0.6

Fitting strategy

$$A = E_s^2(0)$$

$$n \log U + \log B = \log\left(E_s^{2(U)} - A\right)$$

Hot wire anemometry

Empirical relations

Collis & Williams (1959)'s empirical relation

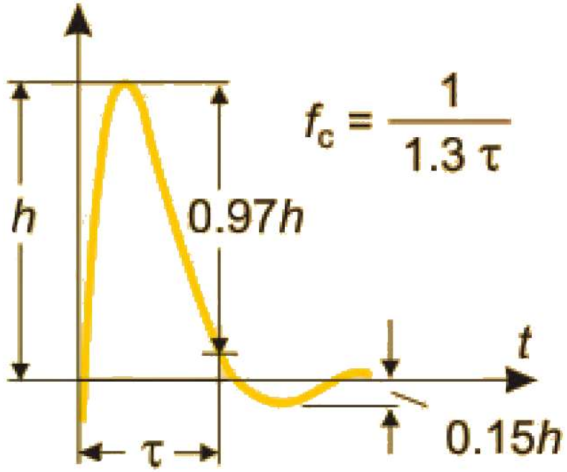
$$n = 0.45 \text{ for } 0.02 < Re_w < 44$$
$$n = 0.51 \text{ for } 44 < Re_w < 140$$

The wire is a cylinder in a cross flow:

- $Re_w < 44$: the wake is steady and symmetric
- $Re_w > 44$: the wake is unsteady and non symmetric with periodic vortex shedding: heat transfer is enhanced
- $Re_w > 140$: the wake becomes disorderd, heat transfer is further enhanced

Hot wire anemometry

Frequency response

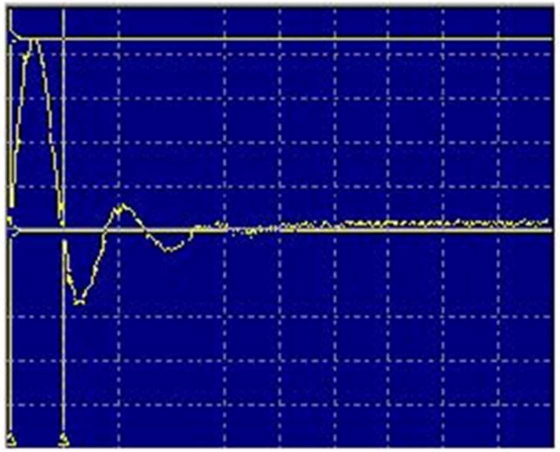


Square excitation

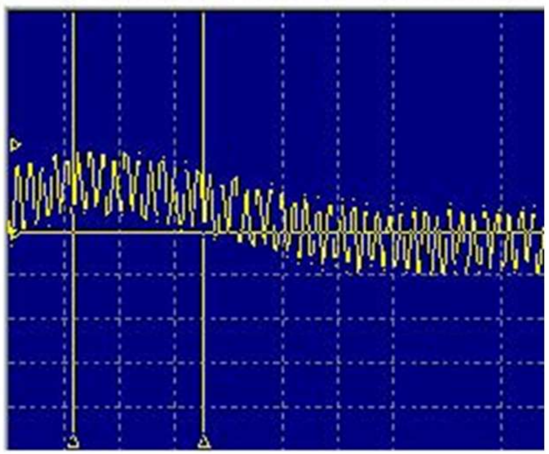
τ: empirical response time with undershoot (15%)

f_c: cut-off frequency

Freymuth P (1977) Frequency response and electronic testing for constant-temperature hot-wire anemometers. J Phys E Sci Instrum 10(7):705–710



Underdamped



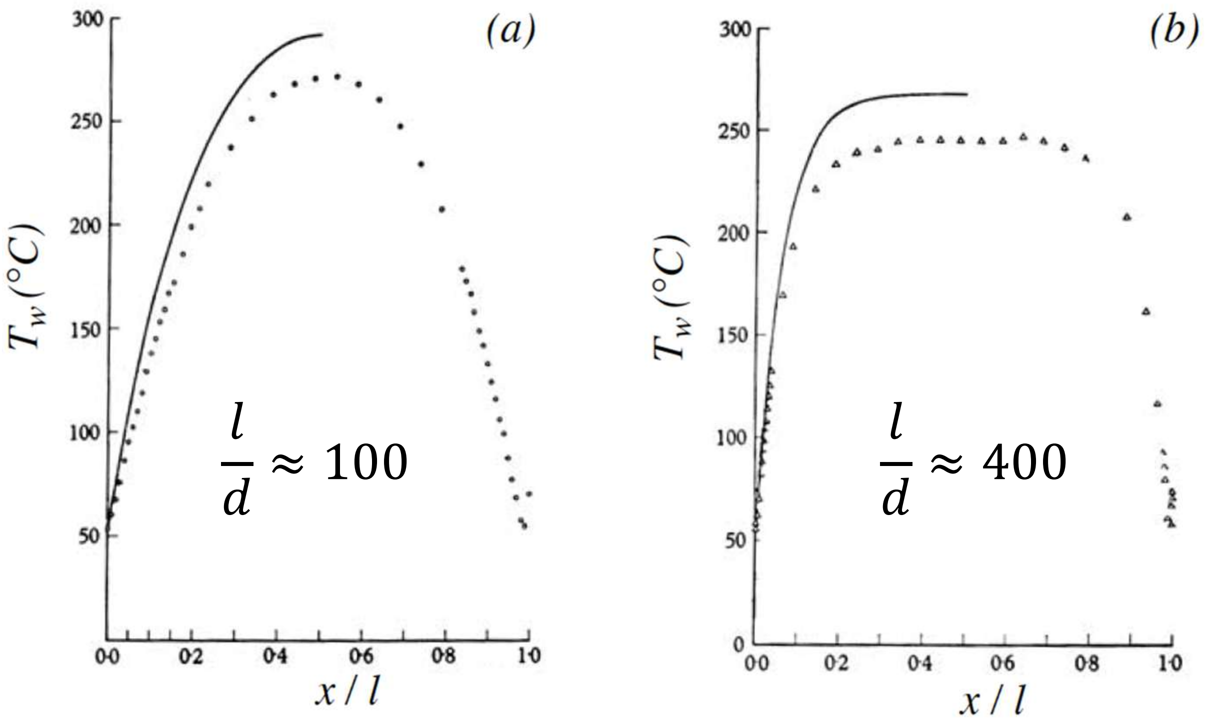
Too long cable

Hot wire anemometry

Finite length effect

From Champagne et al (1967) reproduced by Lomas (1986)

Non uniform temperature in wire



Minimise k_w

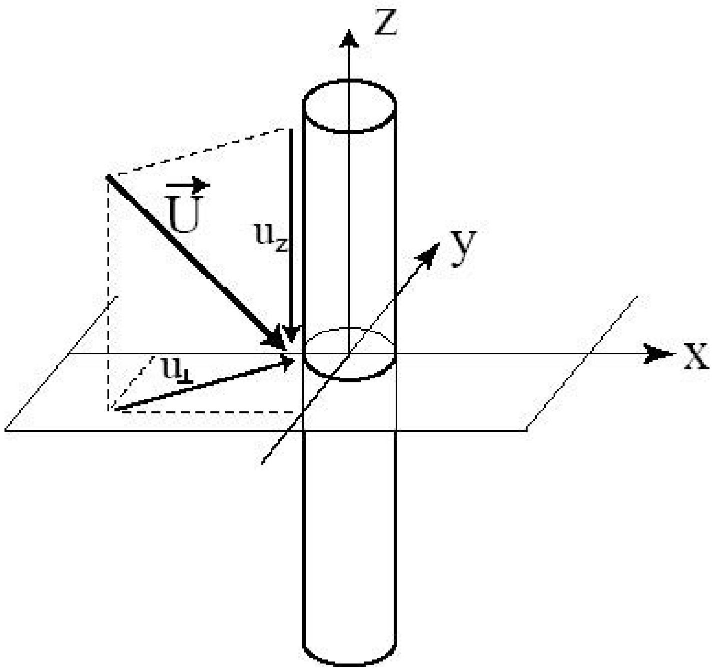
Maximise $\frac{l}{d}$

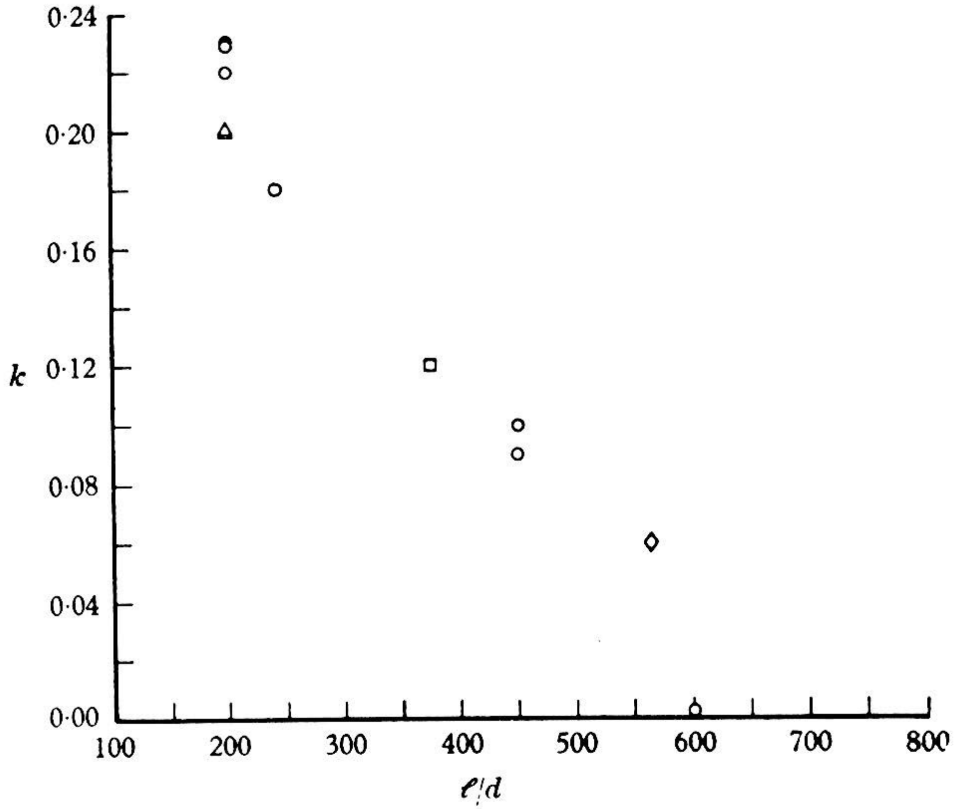
Keep l not too long

⟹ Usual compromise: $l \approx 1$ mm, d ≈ 5 µm

Hot wire anemometry

Directional sensitivity





Measured velocity:

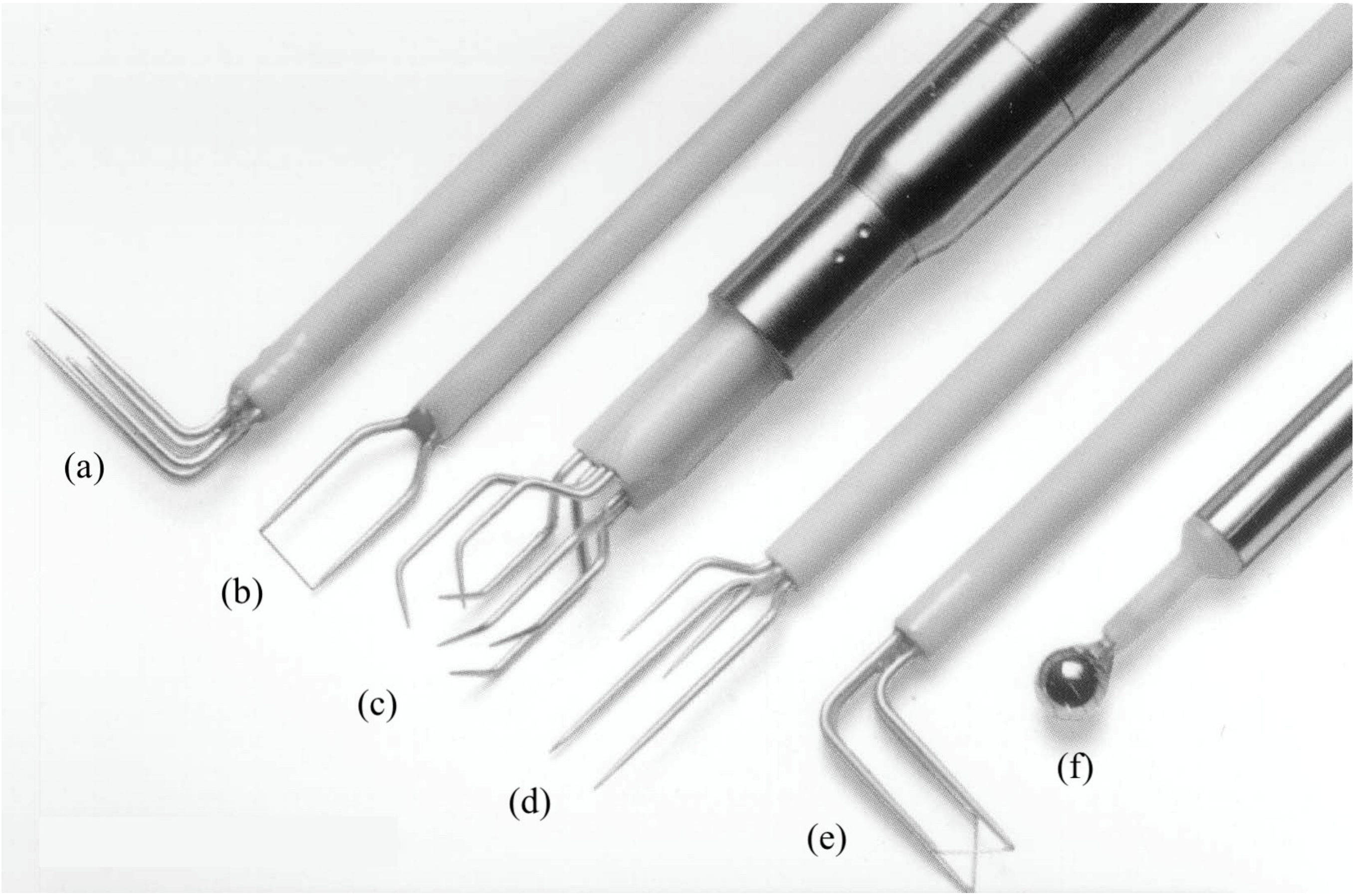
$$U_{eff}^2 = \sqrt{u_\perp^2 + k^2 u_z^2}$$

When $\frac{l}{d} \rightarrow \infty : k \rightarrow 0$

When $\frac{l}{d} \rightarrow 1 : k \rightarrow 1$

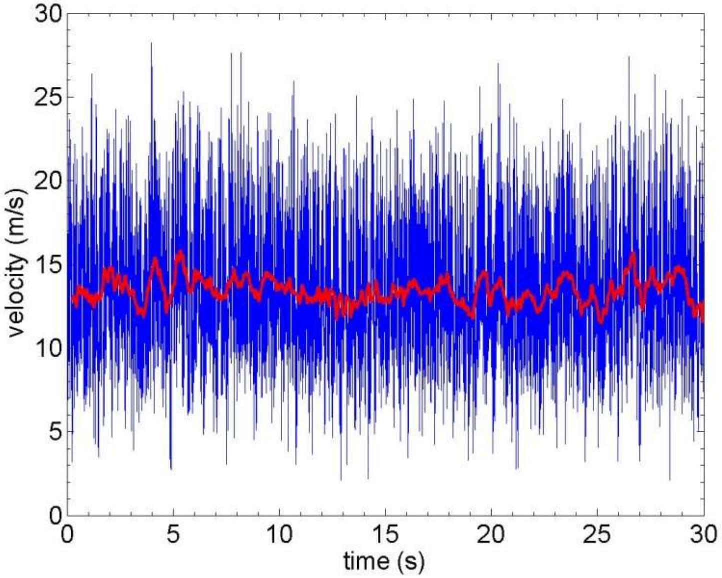
Hot wire anemometry

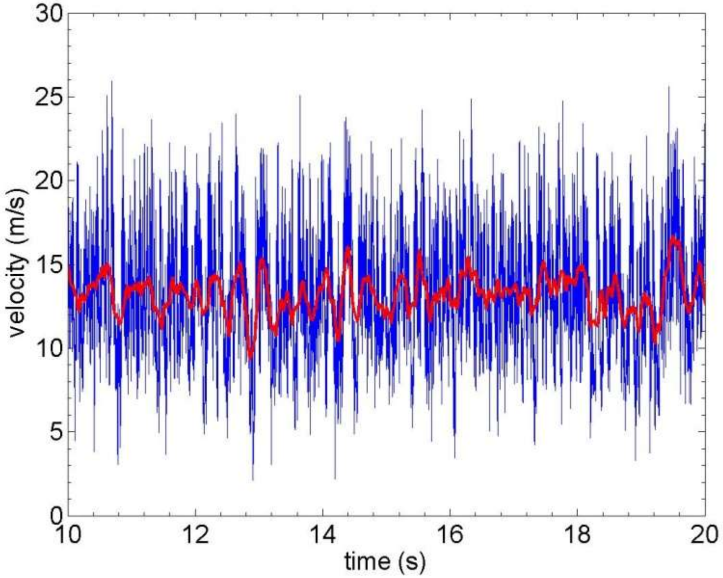
Different probes

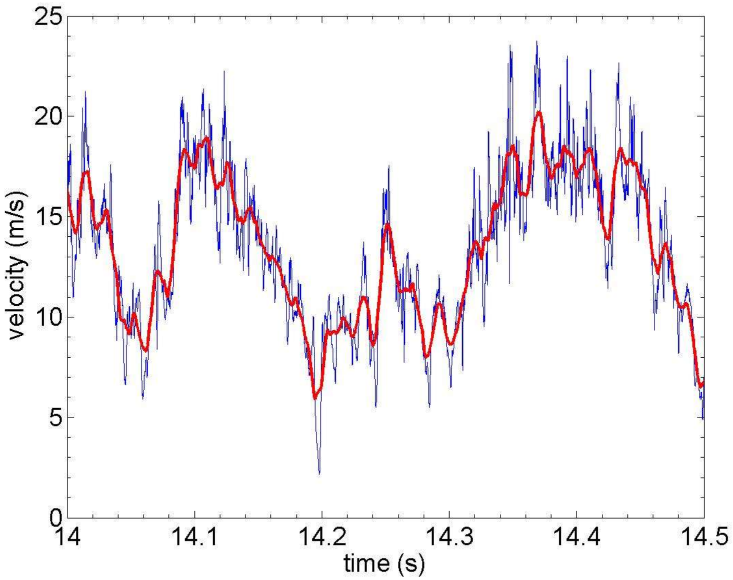


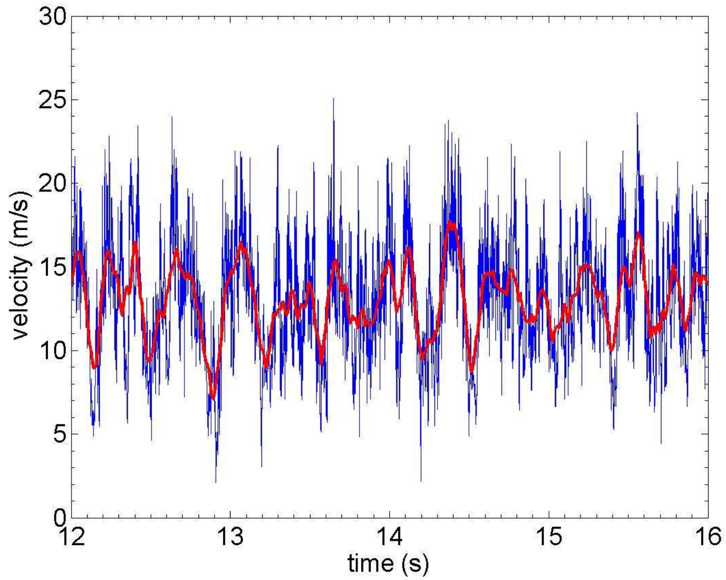
- Several probes (a,c,e): to measure several components
- With cold probes (d): to compensate temperature
- Hot films (f): water measurements

Hot wire anemometry





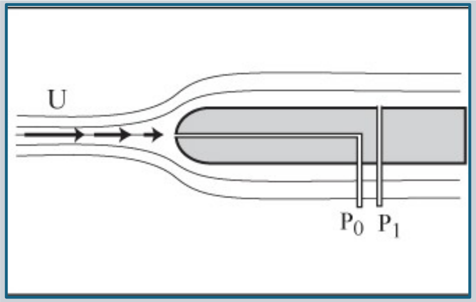
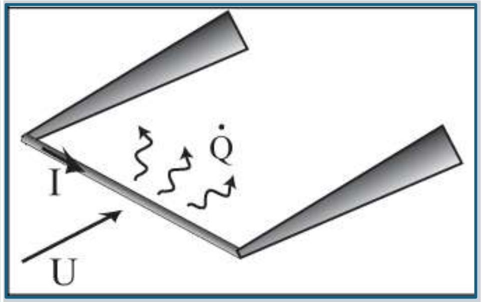




Hot wire anemometry

Main features

- Time resolved
- Point measurement (0D)
- Mainstream flow
- Non linear calibration
- Directional ambiguity
- Emperature sensitivity

	Pitot Tube	Hot Wire Anemometry
Sketch		
Principle	Two pressure measurements: static and dynamics Bernoulli	Measure of dissipated Joule power in a wire
Pros	Easy to use Cheap (1 k€) Suited for time average	Very high time and space resolution Suited for fluctuation measurements Easy to use Medium price (10 k€)
Cons	Highly intrusive Very poor time & space resolution	Intrusive, fragile Non linear calibration Sensitive to temperature

Laser Doppler anemometry

Invented in 1964 by Yeh & Cummins

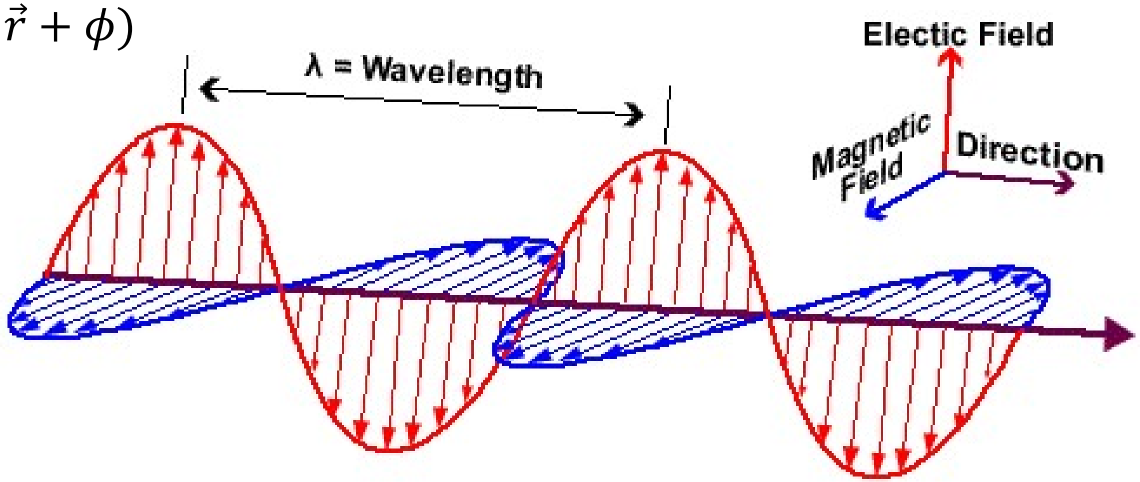
Laser Doppler anemometry

Laser

- Electromagnetic waves

$$\vec{E}(t,\vec{r}) = \vec{p}\,E_0 \cos(\omega t - \vec{k}.\vec{r} + \phi)$$

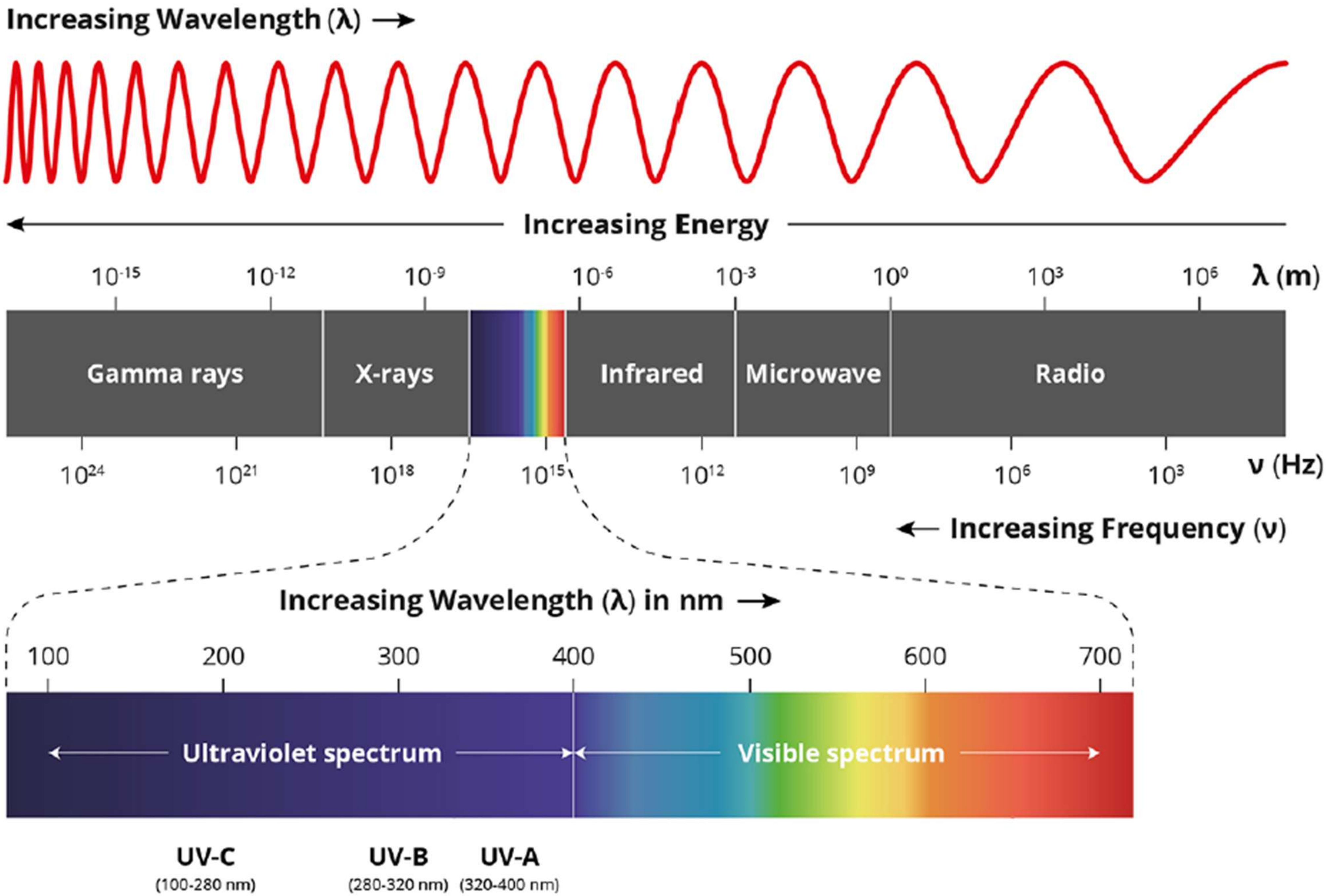
- Wavelength $\lambda = \frac{2\pi}{k}$
- Celerity $c = \frac{\omega}{k}$



Properties

- Monochromatic
- Coherent
- Linearly polarized
- Collimated
- Gaussian intensity distribution

Laser Doppler anemometry



Laser Doppler anemometry

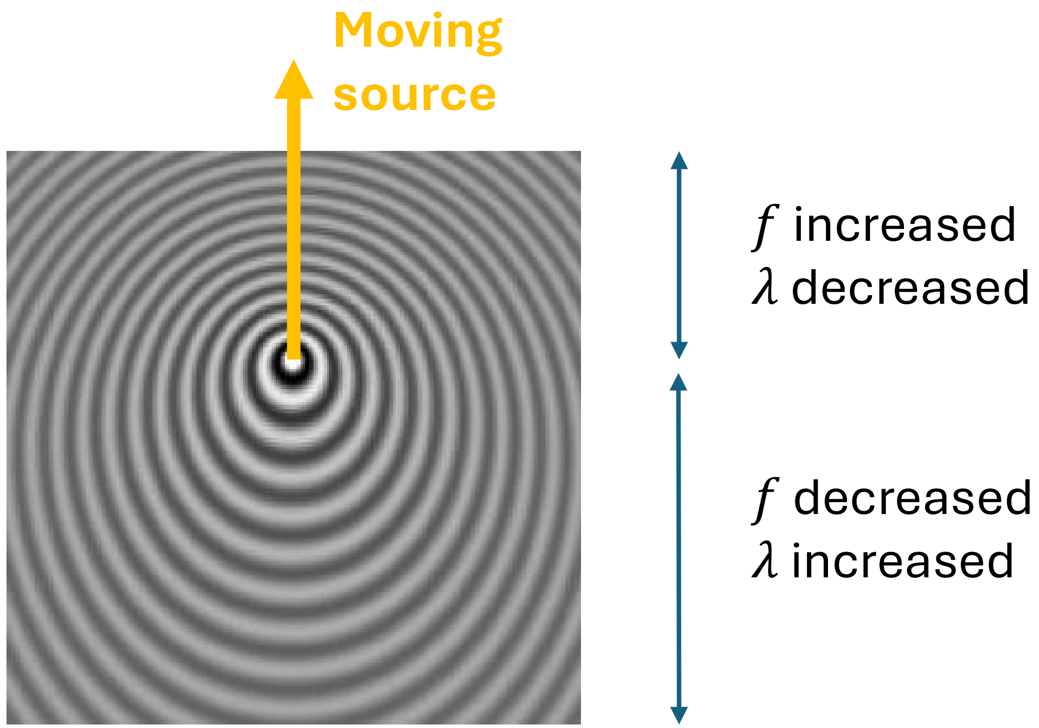
Usual Lasers

- Argon: continuous power in green-blue
- Helium: continuous power in red
- Nd:Yag: pulsed in infrared shifted to green

LASER	λ (nm)	color	power (mW)	diameter (mm)
He-Ne (gas)	632.8	red	1-15	0.65
Ar^{2+} (gas)	476.5	violet	1-600	1.5
	488	blue	1-1500	1.5
	514.5	green	1-2000	1.5
doubled YAG (solid)	532	green	20-2000	1

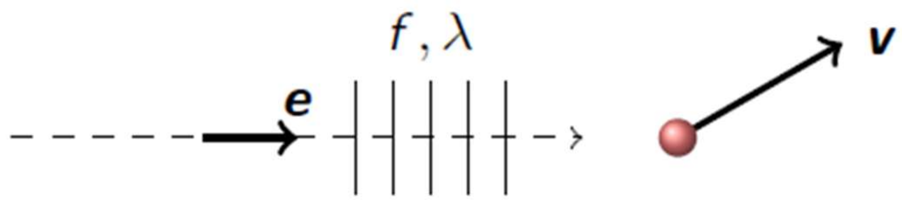
Laser Doppler anemometry

Doppler effect



Doppler effect

Still source, moving receiver

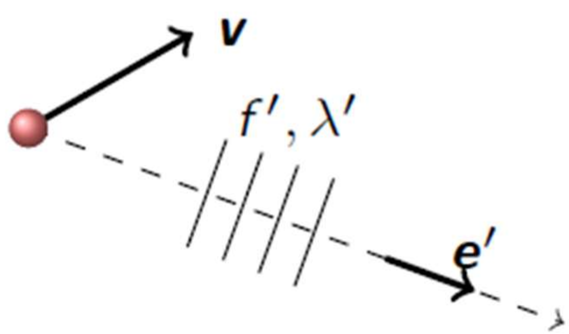


$$T = 1/f$$

$$\lambda' = cT + \vec{v}.\vec{e}T$$

$$f' = f\frac{1}{1+\frac{\vec{v}}{c}.\vec{e}}$$

Moving source, still receiver



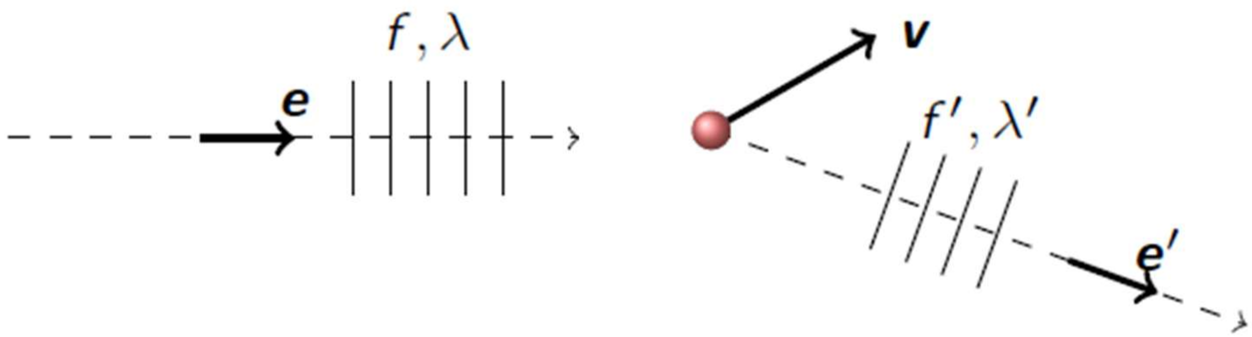
$$T' = 1/f'$$

$$\lambda'' = cT' + \vec{v}.\overrightarrow{e'}T'$$

$$f'' = f'\frac{1}{1-\frac{\vec{v}}{c}.\overrightarrow{e'}}$$

Doppler effect

Application to velocimetry



$$f' = f\frac{1}{1+\frac{\vec{v}}{c}.\vec{e}}$$

$$f'' = f'\frac{1}{1-\frac{\vec{v}}{c}.\overrightarrow{e'}}$$

Doppler frequency

$$f'' \simeq f + f_D$$

$$f_D = f\frac{\vec{v}}{c}.\left(\overrightarrow{e'} - \vec{e}\right)$$

Measurement

$$f \simeq 10^{14}\ \text{Hz}$$

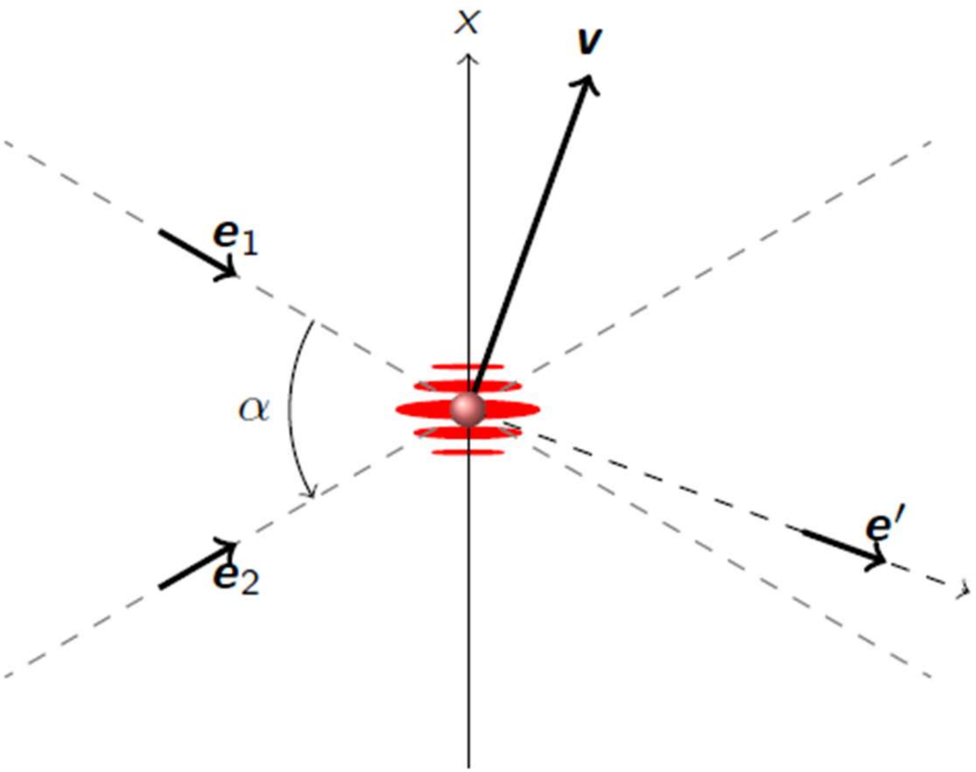
$$f_D \simeq 10^{6} - 10^{7}\ \text{Hz}$$

- Requires a 10^{-8} resolution for 10% accuracy
- Depends on observation direction

⟹ Measurement of f_D is preferred through interferometric system

Laser Doppler anemometry

Doppler effect on crossed beams



Frequency along e':

$$f_1 \simeq f + f\frac{\vec{v}}{c}.\left(\overrightarrow{e'} - \overrightarrow{e_1}\right)$$

$$f_2 \simeq f + f\frac{\vec{v}}{c}.\left(\overrightarrow{e'} - \overrightarrow{e_2}\right)$$

Intensity along e':

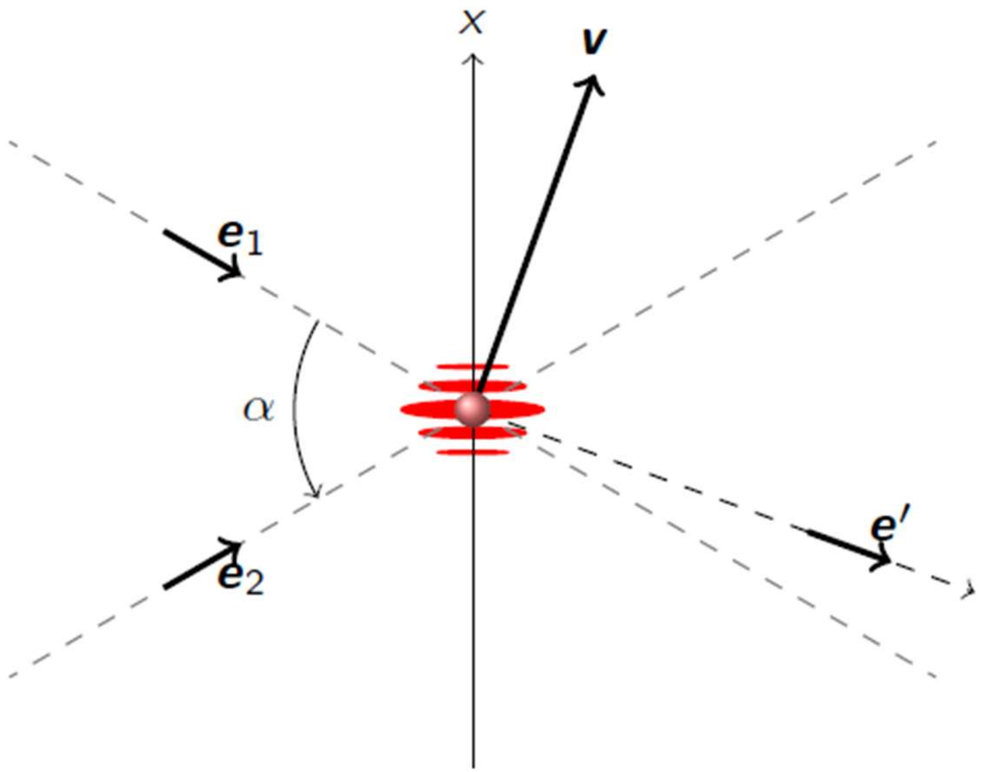
$$I = E_1^2 + E_2^2 + 2E_1E_2\cos\left(2\pi f\frac{\vec{v}}{c}.\left(\overrightarrow{e_1} - \overrightarrow{e_2}\right)\right)$$

Measurement

- One component detected: $f_D = f\frac{\vec{v}}{c}.\left(\overrightarrow{e_1} - \overrightarrow{e_2}\right) = 2f\frac{v_x}{c}\sin\left(\frac{\alpha}{2}\right)$
- No Doppler shift if $\vec{v} \perp \overrightarrow{e_x}$
- Independent of observation direction

Laser Doppler anemometry

Doppler effect on crossed beams



Problem

- Insensitive to direction
- Zero velocity not measurable

Solution

- Scrolling fringes
- Shift frequency Δf

$$f_D = 2f\frac{v_x}{c}\sin\left(\frac{\alpha}{2}\right) + \Delta f$$

Measurement

- When $V_x = 0$, $f_D = \Delta f$
- When $V_x < 0$, $f_D < \Delta f$
- When $V_x > 0$, $f_D > \Delta f$

Laser Doppler anemometry

Interferometry

- Amplitudes sum up
- Intensity is measured

$$I(\vec{r},t) = \left| E_1 \overrightarrow{p_1} \cos\left(\omega_1 t - \overrightarrow{k_1}.\vec{r}\right) + E_2 \overrightarrow{p_2} \cos\left(\omega_2 t - \overrightarrow{k_2}.\vec{r}\right) \right|^2$$

At the sensor time scale

$$\Longrightarrow \quad I(\vec{r},t) = I_0 + \gamma \cos((\overrightarrow{k_2} - \overrightarrow{k_1}).\vec{r})$$

interference network

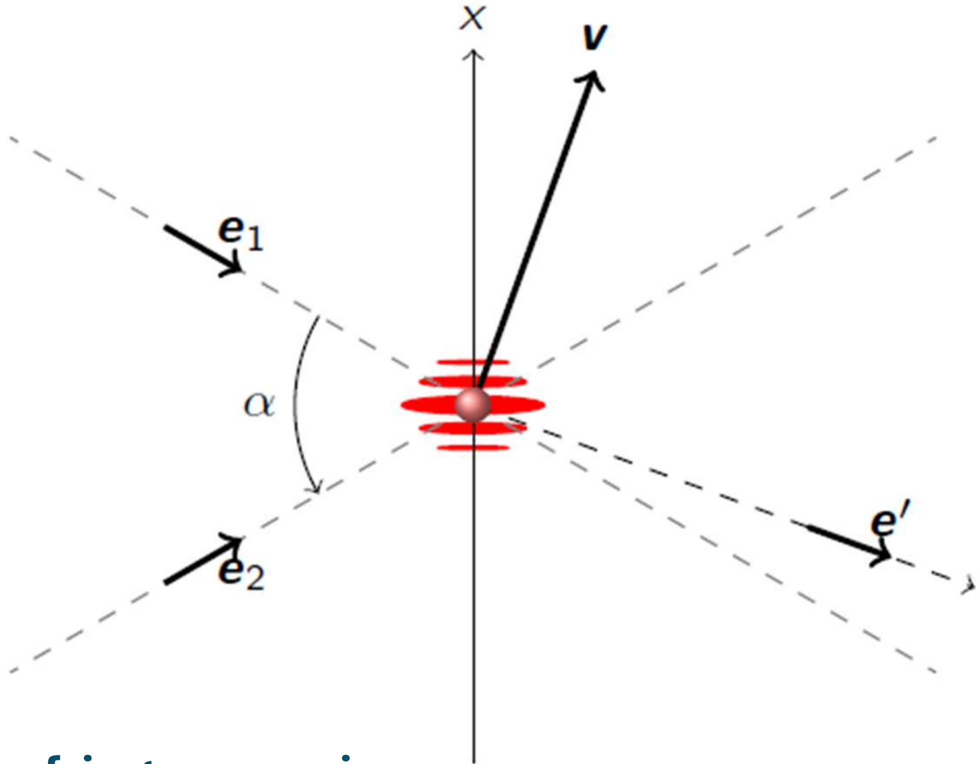
Interference pattern

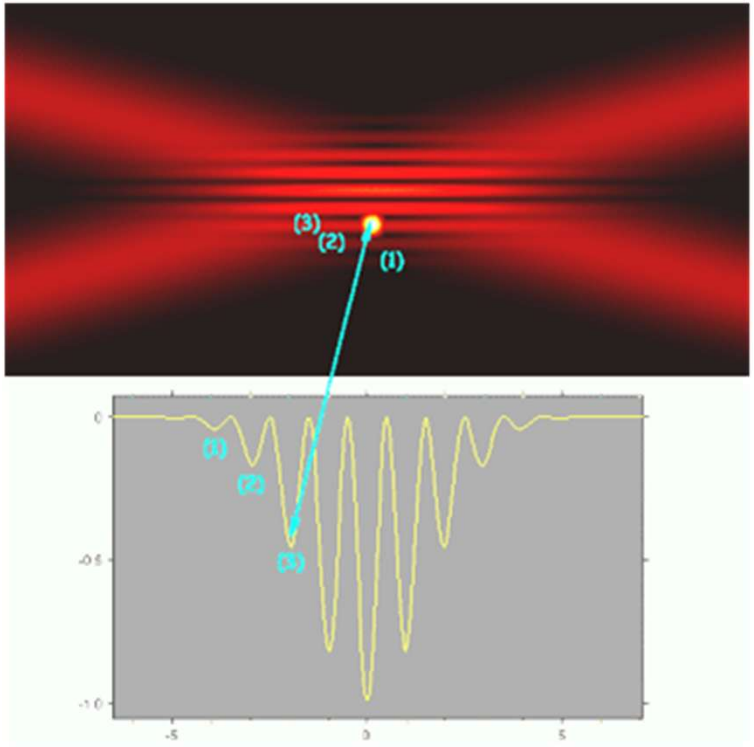
- Spatially structured
- Independent of time
- Max when both have the same polarisation, i.e. $\overrightarrow{p_1} = \pm \overrightarrow{p_2}$

Laser Doppler anemometry

Interferometry

$$I\,(\vec{r}\,,t\,) = I_0 + \gamma\cos((\overrightarrow{k_2} - \overrightarrow{k_1})\,.\vec{r})$$





Interfringe: exercise

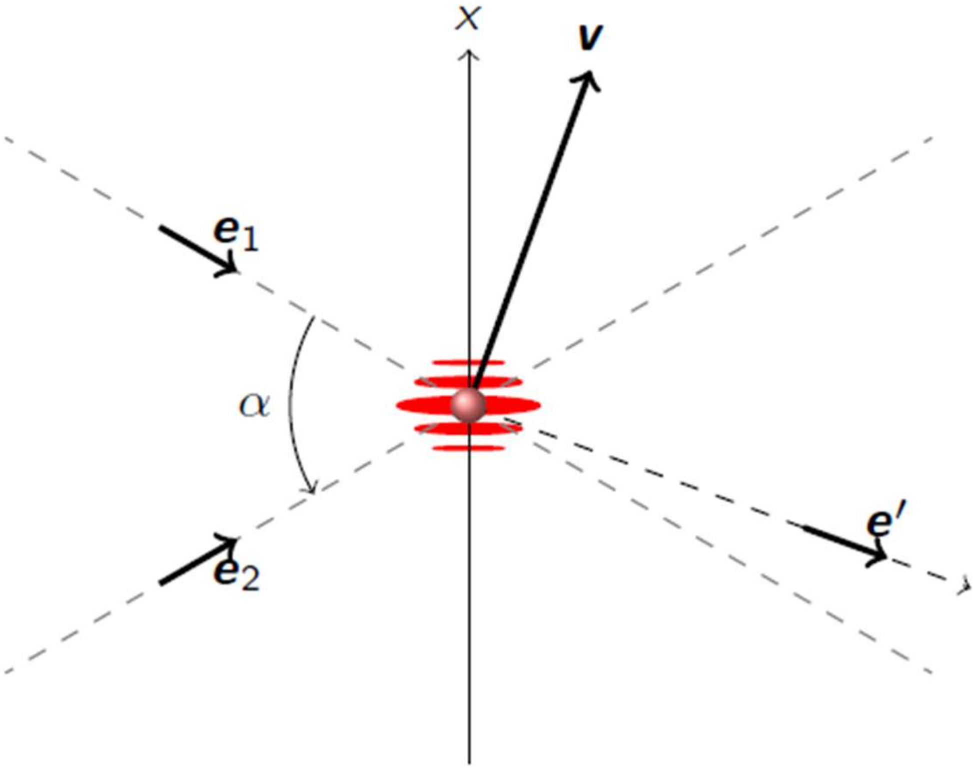
$$(\overrightarrow{k_2} - \overrightarrow{k_1})\,.\vec{r} = 2\pi$$

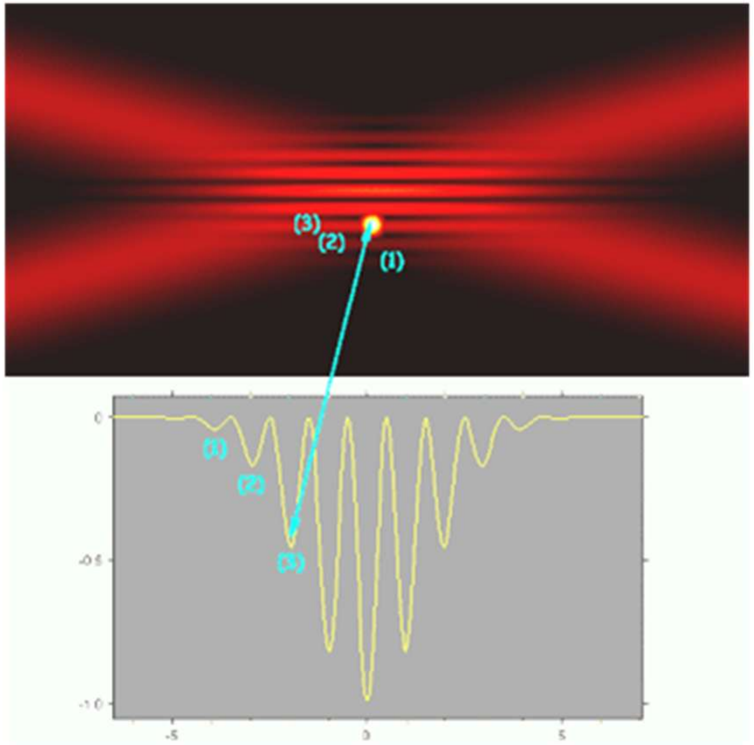
$$d = \frac{\lambda}{2\sin\frac{\alpha}{2}}$$

Laser Doppler anemometry

Interferometry

Seeding particles pass through the fringe at velocity $\vec{v}$





Scattered light is modulated in time at period T_D or frequency f_D

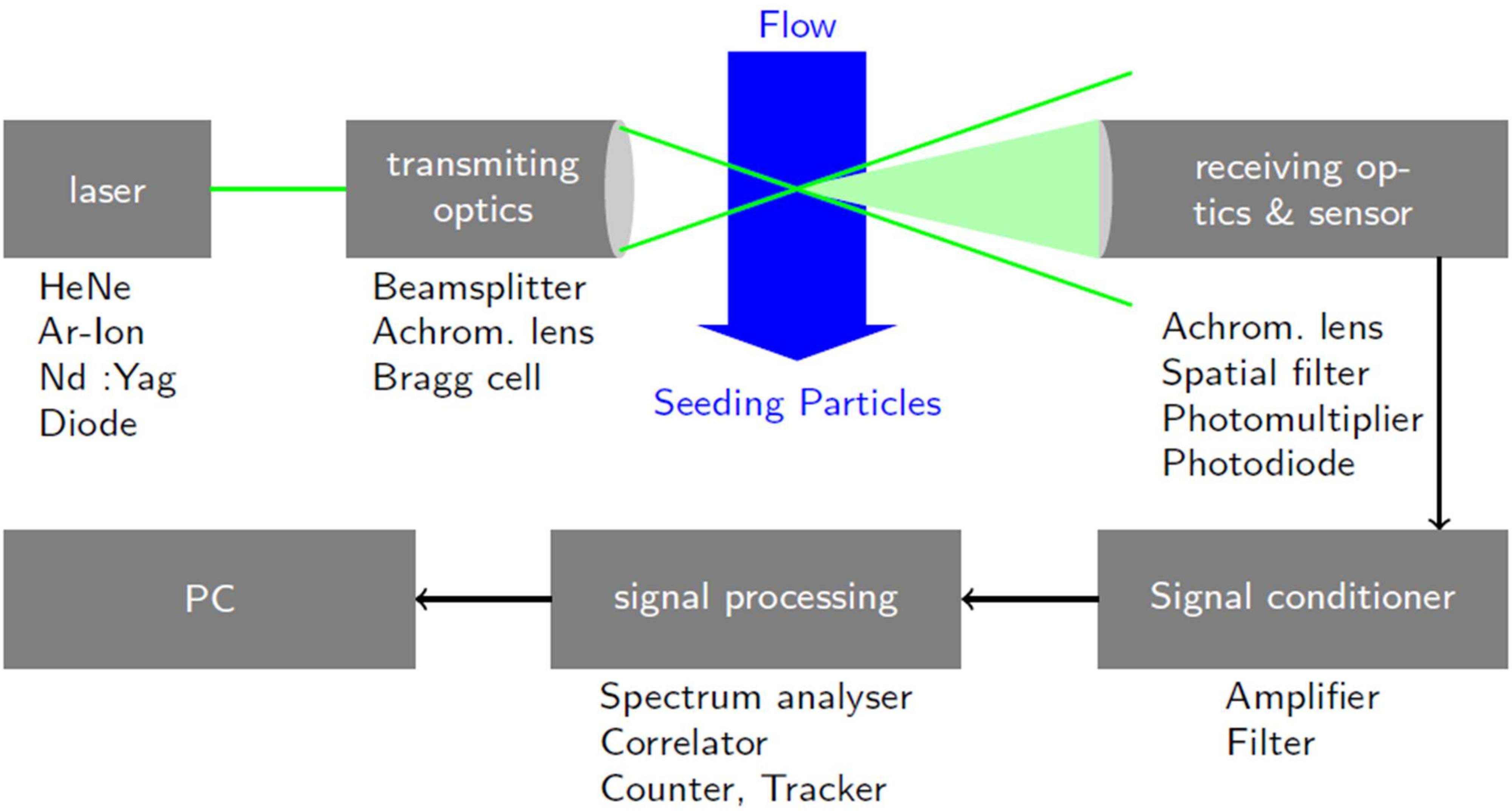
$$T_D = \frac{d}{\vec{v}.\overrightarrow{e_x}}$$

$$f_D = \frac{2v_x}{\lambda}\sin\frac{\alpha}{2}$$

⟹ **We recover the Doppler shift !**

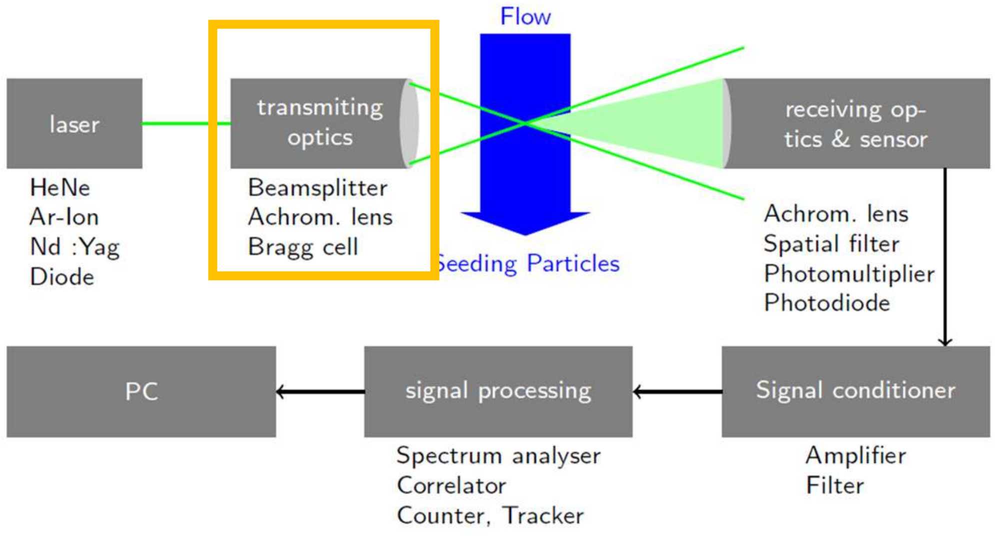
Laser Doppler anemometry

Implementation

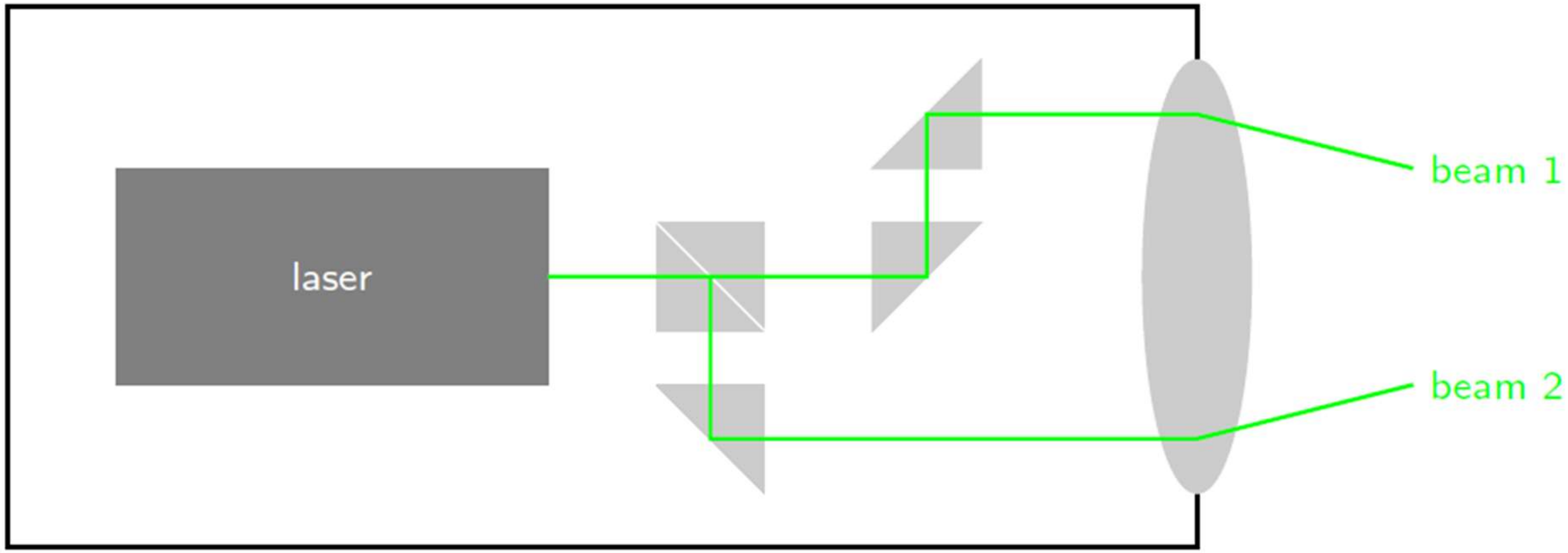


Laser Doppler anemometry

Beam generator

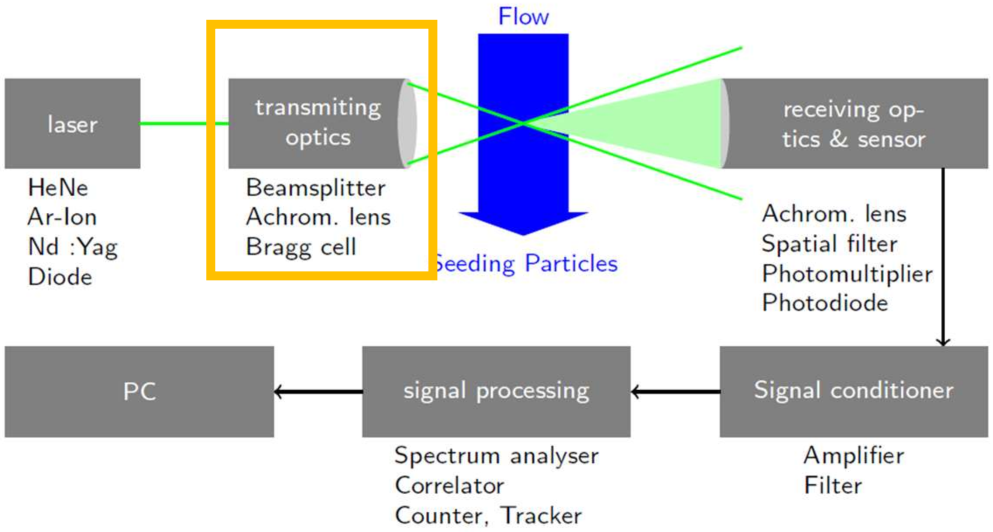


Generating two interfering beams

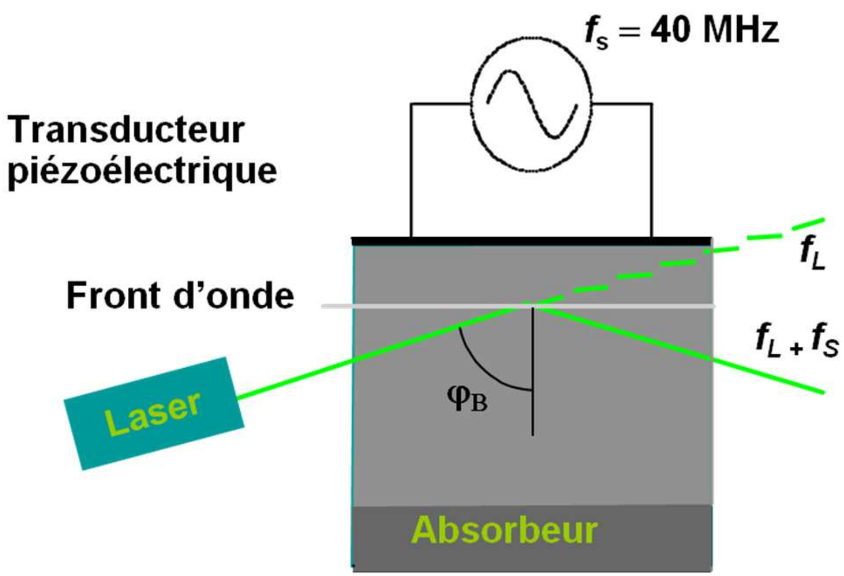


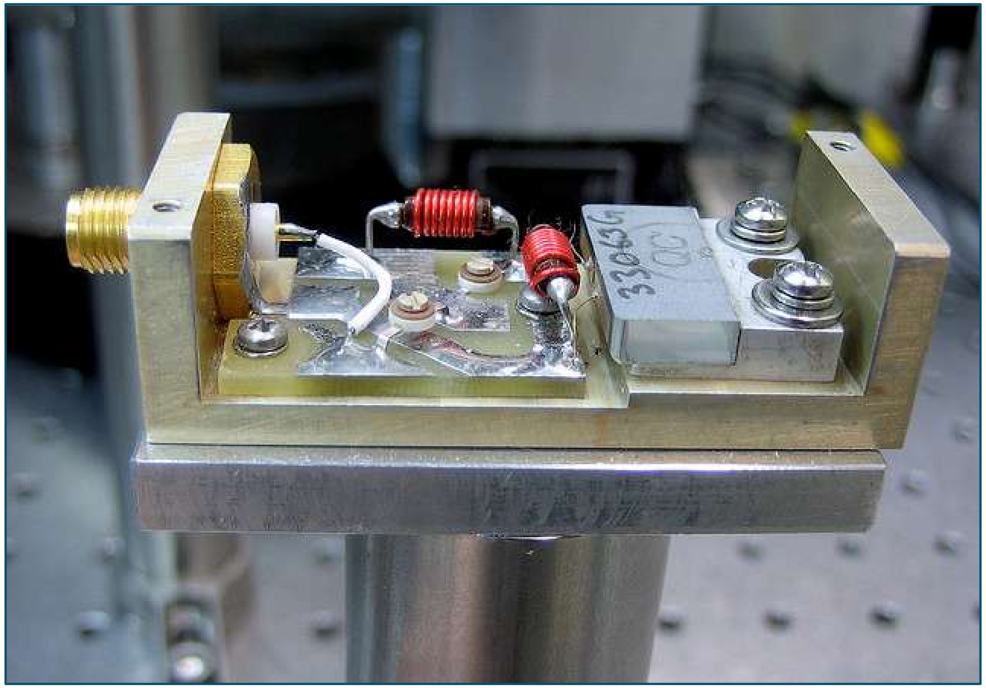
Laser Doppler anemometry

Beam generator



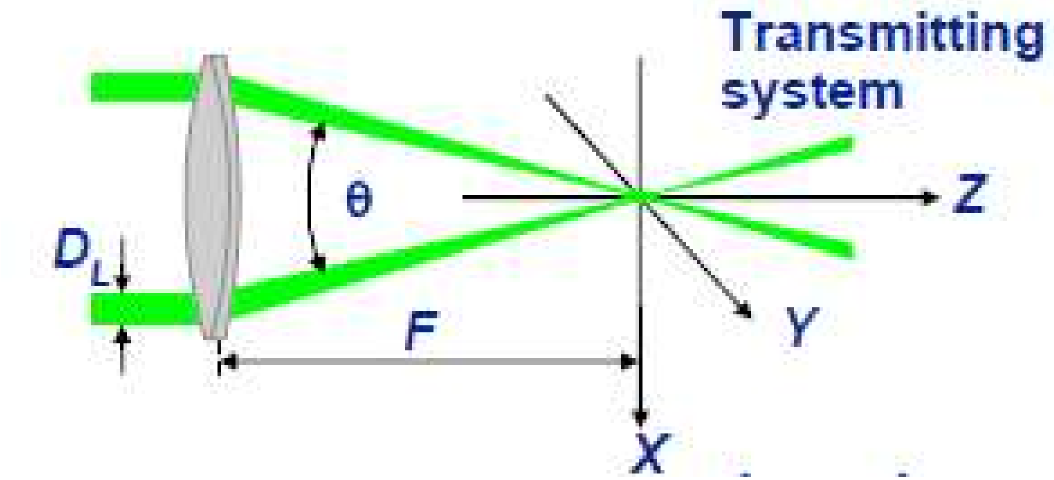
Bragg cell

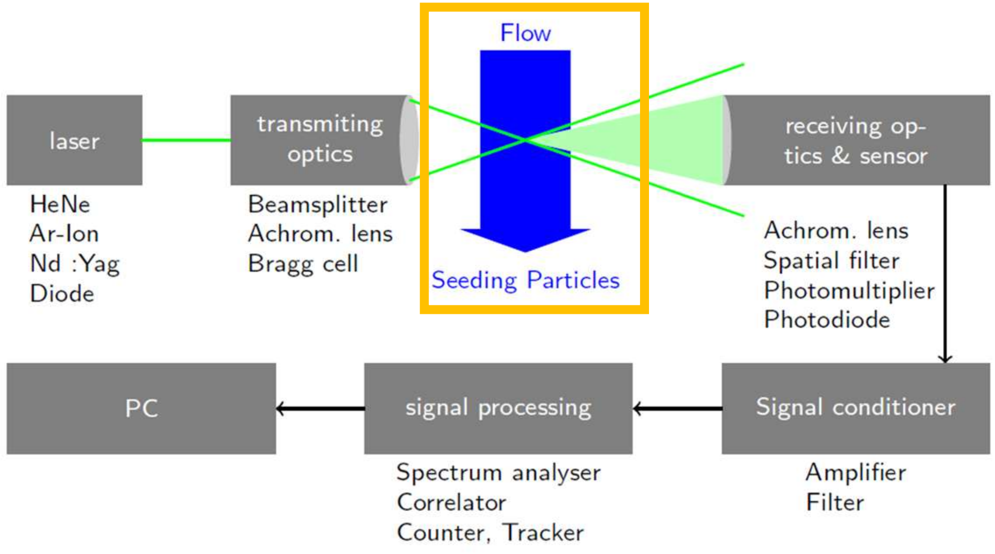




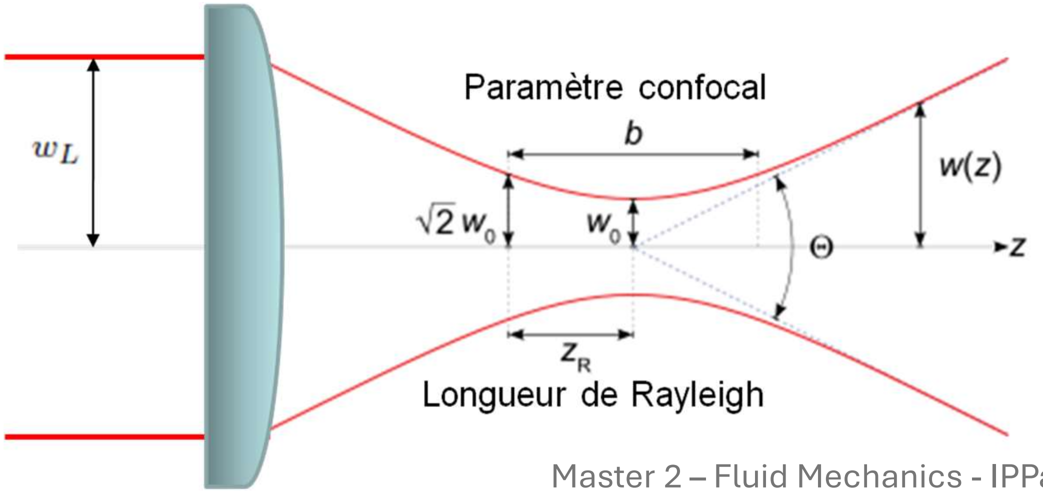
Laser Doppler anemometry

Measurement volume





Diffraction limit for gaussian beams



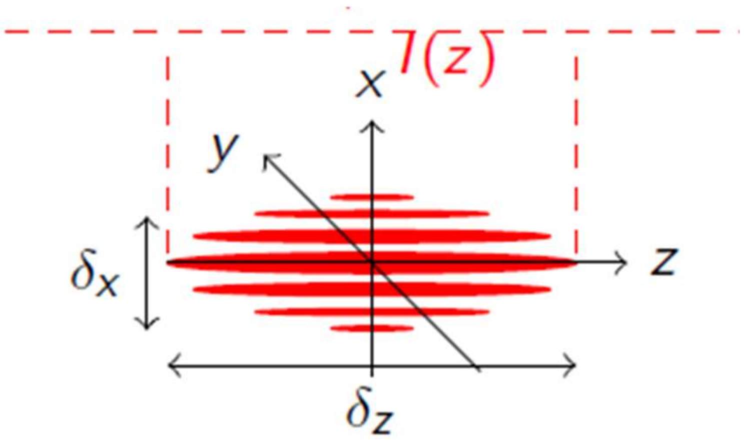
$$w_0 = \frac{\lambda f}{\pi w_L}$$

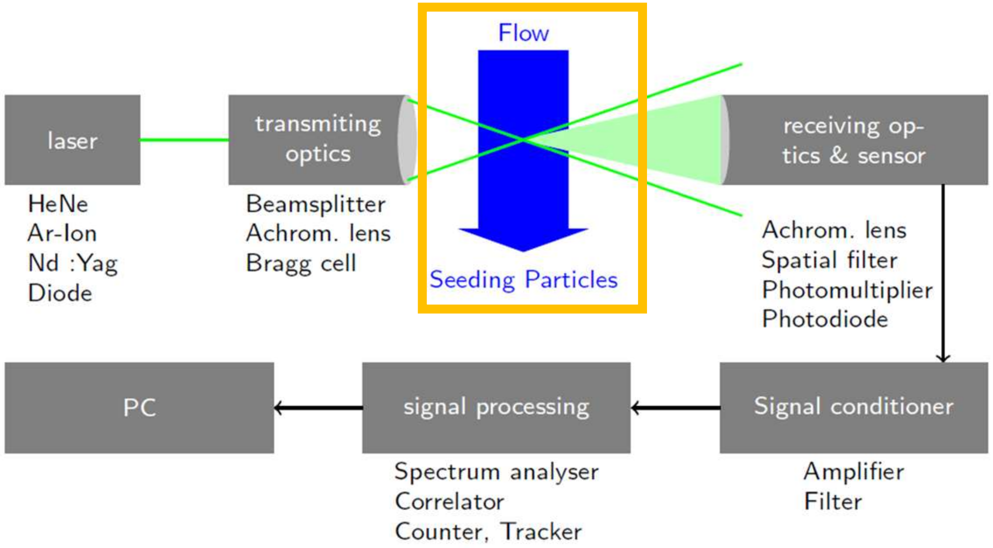
$$z_R = \frac{\pi w_0^2}{\lambda}$$

$$\Theta = 2\theta \simeq \frac{2\lambda}{\pi w_0}$$

Laser Doppler anemometry

Measurement volume





Volume dimensions

$$\delta_z = \frac{4f\lambda}{\pi D_L \sin\frac{\alpha}{2}}, \qquad \delta_y = \frac{4f\lambda}{\pi D_L}, \qquad \delta_x = \frac{4f\lambda}{\pi D_L \cos\frac{\alpha}{2}}$$

Interfringe

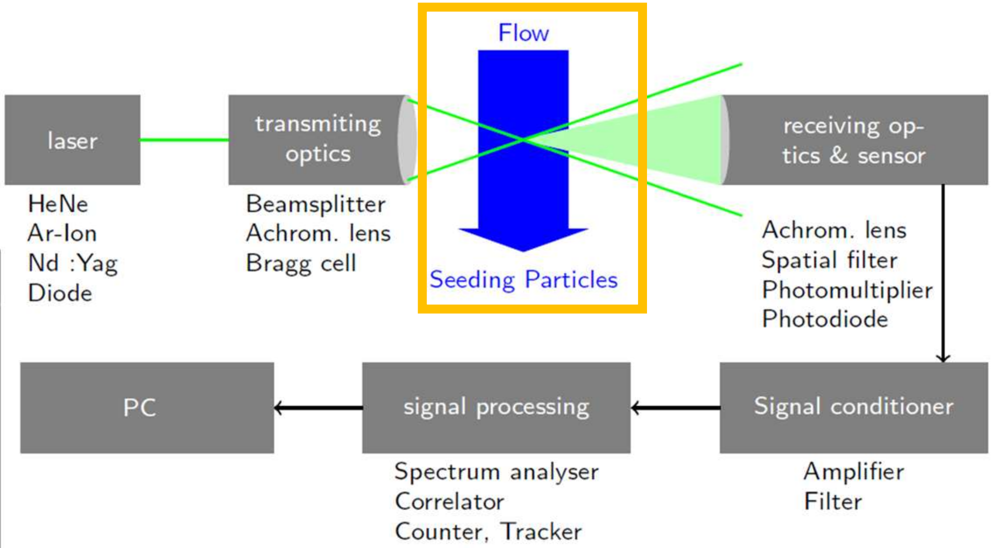
$$d = \frac{\lambda}{2\sin\frac{\alpha}{2}}$$

Number of fringes

$$N = \frac{8f\tan\frac{\alpha}{2}}{\pi D_L}$$

Laser Doppler anemometry

Particles



For liquids

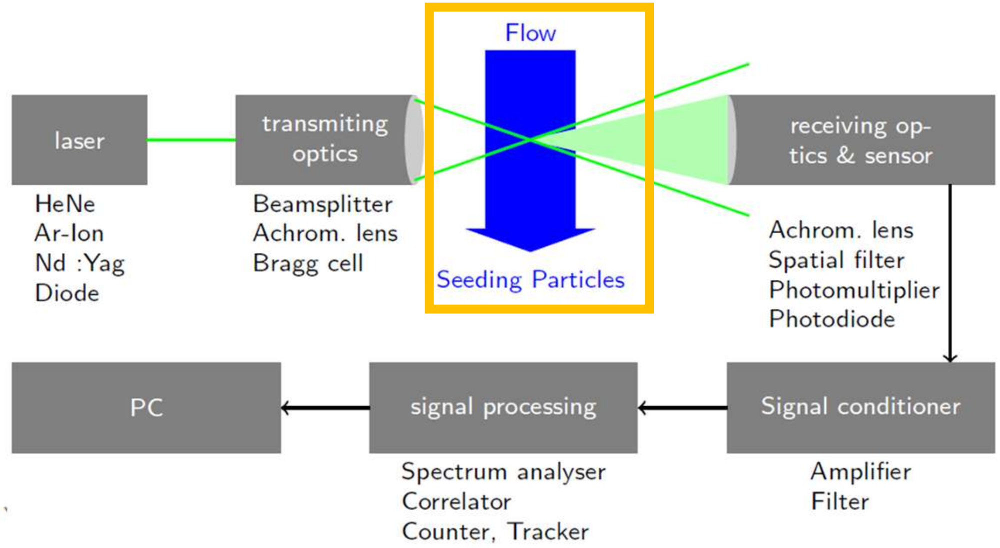
State	Material	Mean diameter (μm)
Solid	polystyrène	0.5-10
	Aluminium	2-7
	Magnesium	2-5
	synthetic granules	1-10
	glass microbeads	30-100
Liquid	oils	0.5-10
	dioctylphathalate	< 1

For gas

State	Material	Mean diameter (μm)
Solid	polystyrene	10-100
	Aluminium	2-7
	hollow glass sphere	10-100
	granules for synthetic coating	10-500
Liquid	oils	50-500
Gas	bubles of O_2, H_2, etc	50-1000

Laser Doppler anemometry

Particles



Seeking tracers

$$\underbrace{\frac{\pi}{6}\phi^3\rho_p\frac{d_p}{dt}\mathbf{v}_p}_{\text{inertial force}} = \underbrace{-3\pi\mu\phi(\mathbf{v}_p - \mathbf{v}_f)}_{\text{Stokes force}} - \frac{\pi\phi^3}{6}\underbrace{\nabla P}_{\rho\frac{d\mathbf{v}_f}{dt}}$$

$$- \underbrace{\frac{1}{2}\frac{\pi}{6}\phi^3\rho_f\left(\frac{d\mathbf{v}_f}{dt} - \frac{d_p\mathbf{v}_p}{dt}\right)}_{\text{fluid resistance to sphere acceleration}}$$

$$+ \underbrace{\frac{3}{2}\phi^2\sqrt{\pi\rho_f\mu}\int_0^t \frac{1}{\sqrt{t-t'}}\left(\frac{d\mathbf{v}_f}{dt} - \frac{d_p\mathbf{v}_p}{dt}\right)dt'}_{\text{drag force due to an unsteady flow}} + \mathbf{f}_{ext}$$

Maxey, Riley, Gattignol, 1983

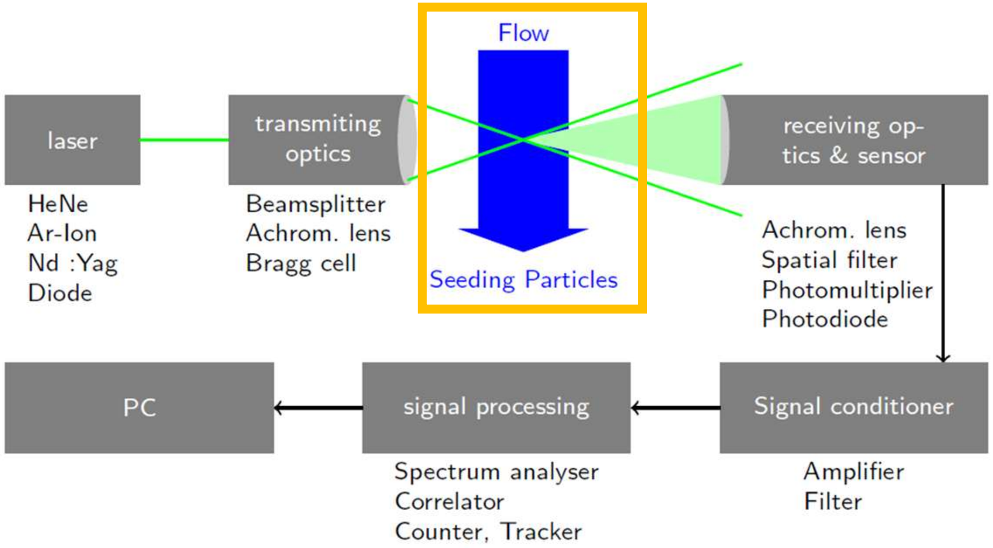
Laser Doppler anemometry

Particles

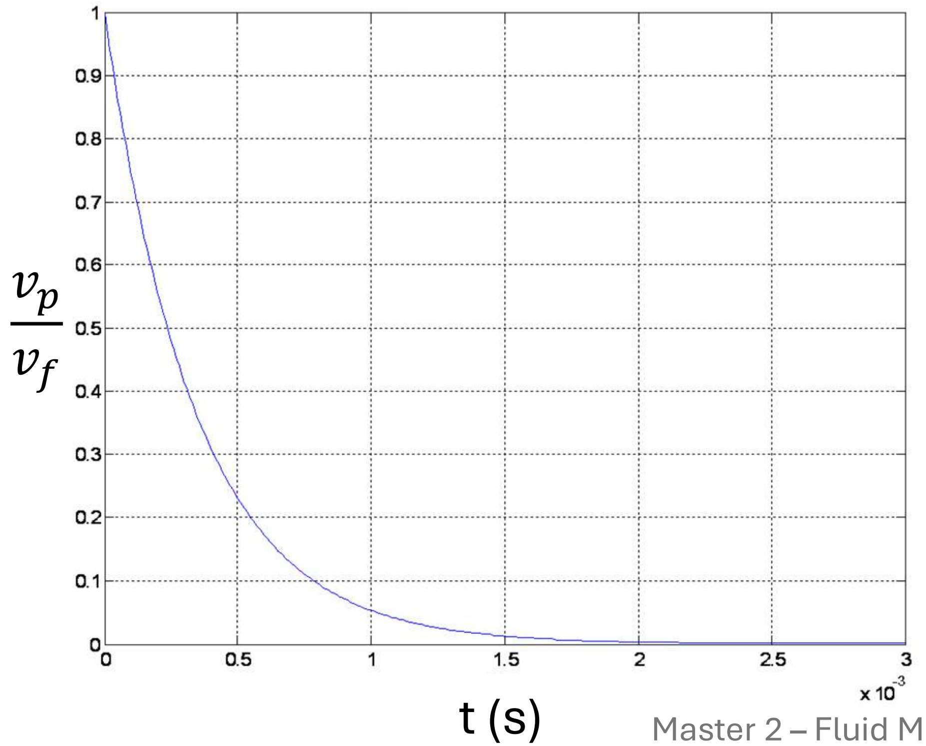
Seeking tracers

$$\frac{dv_p}{dt} = 18\frac{\mu}{\phi^2\rho_p}\left(v_f - v_p\right)$$

$$\Longrightarrow \quad v_p = v_f\left(1 - e^{-t/\tau\_p}\right)$$



Oil in air



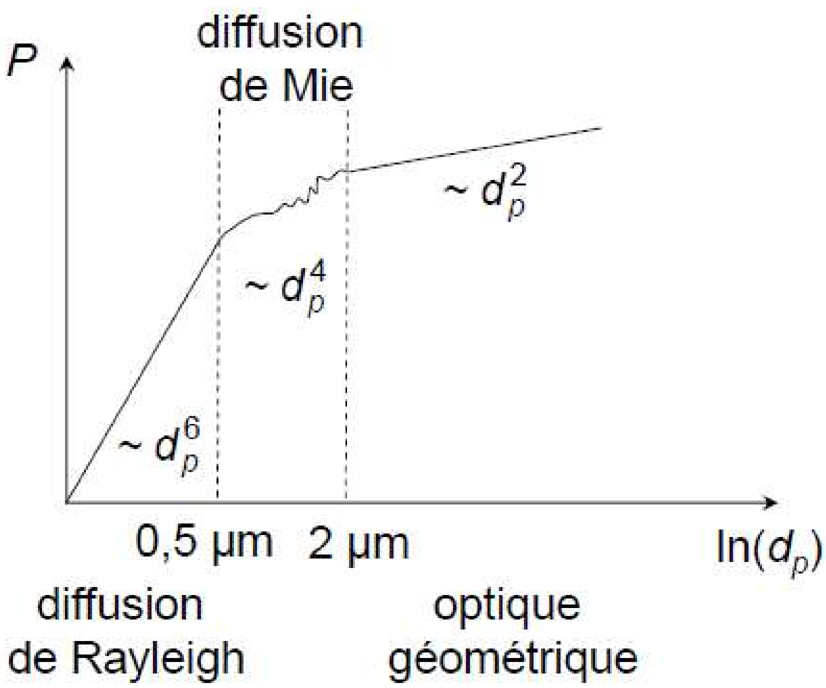
Relaxation time

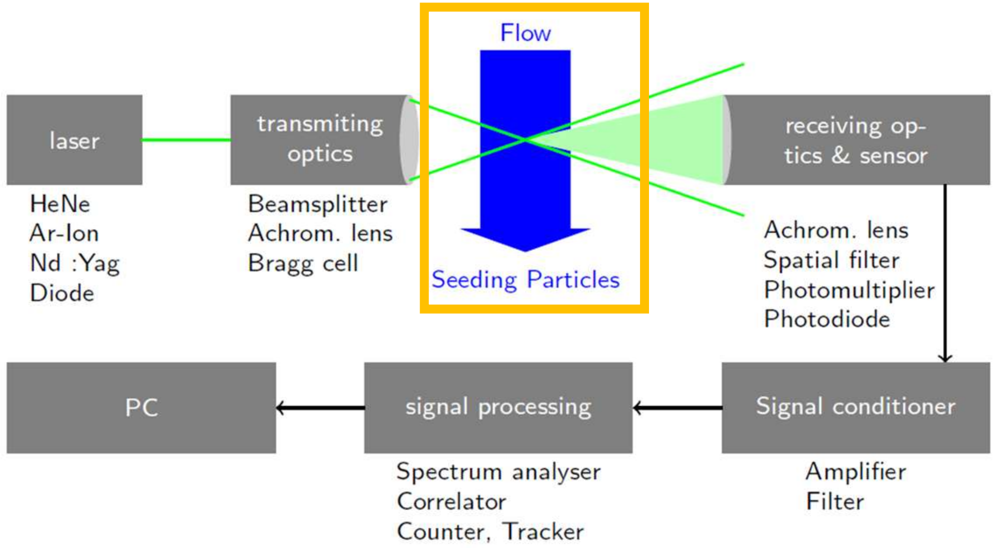
$$\tau_p = \frac{\rho_p\phi^2}{18\mu}$$

Laser Doppler anemometry

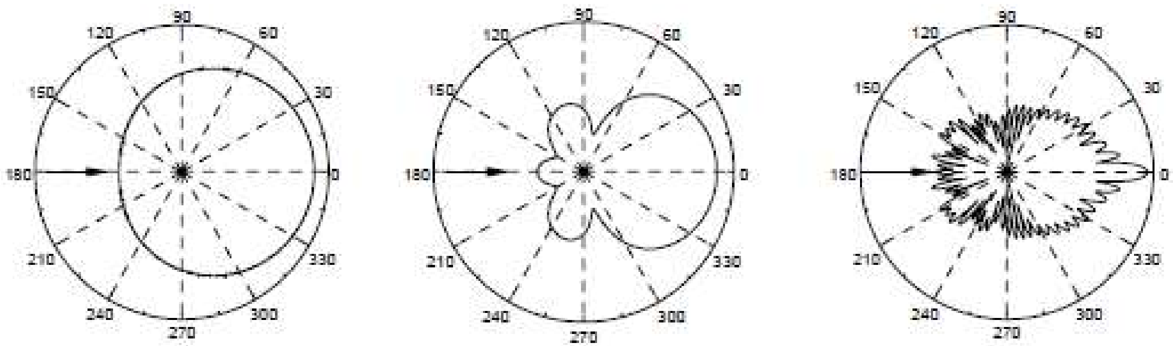
Particles

Looking for light



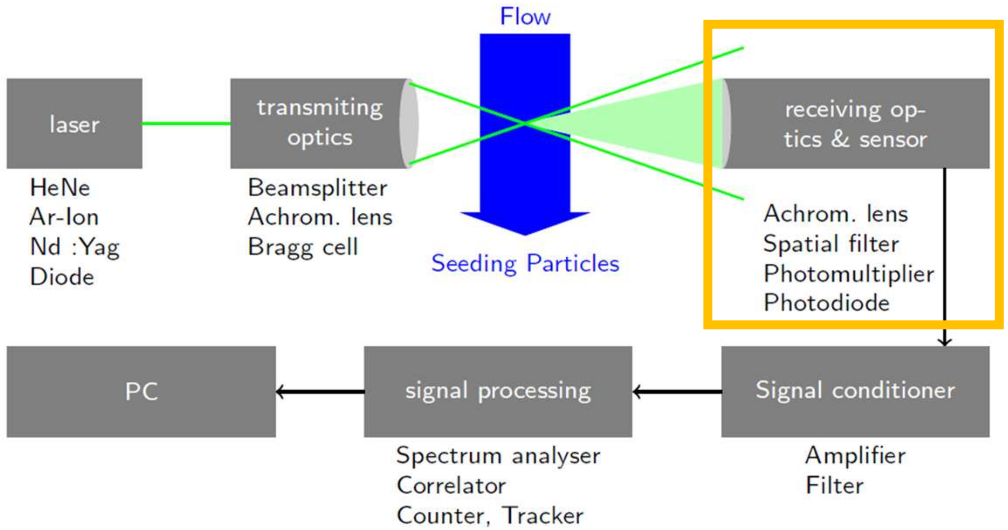


Diffracted light in all directions

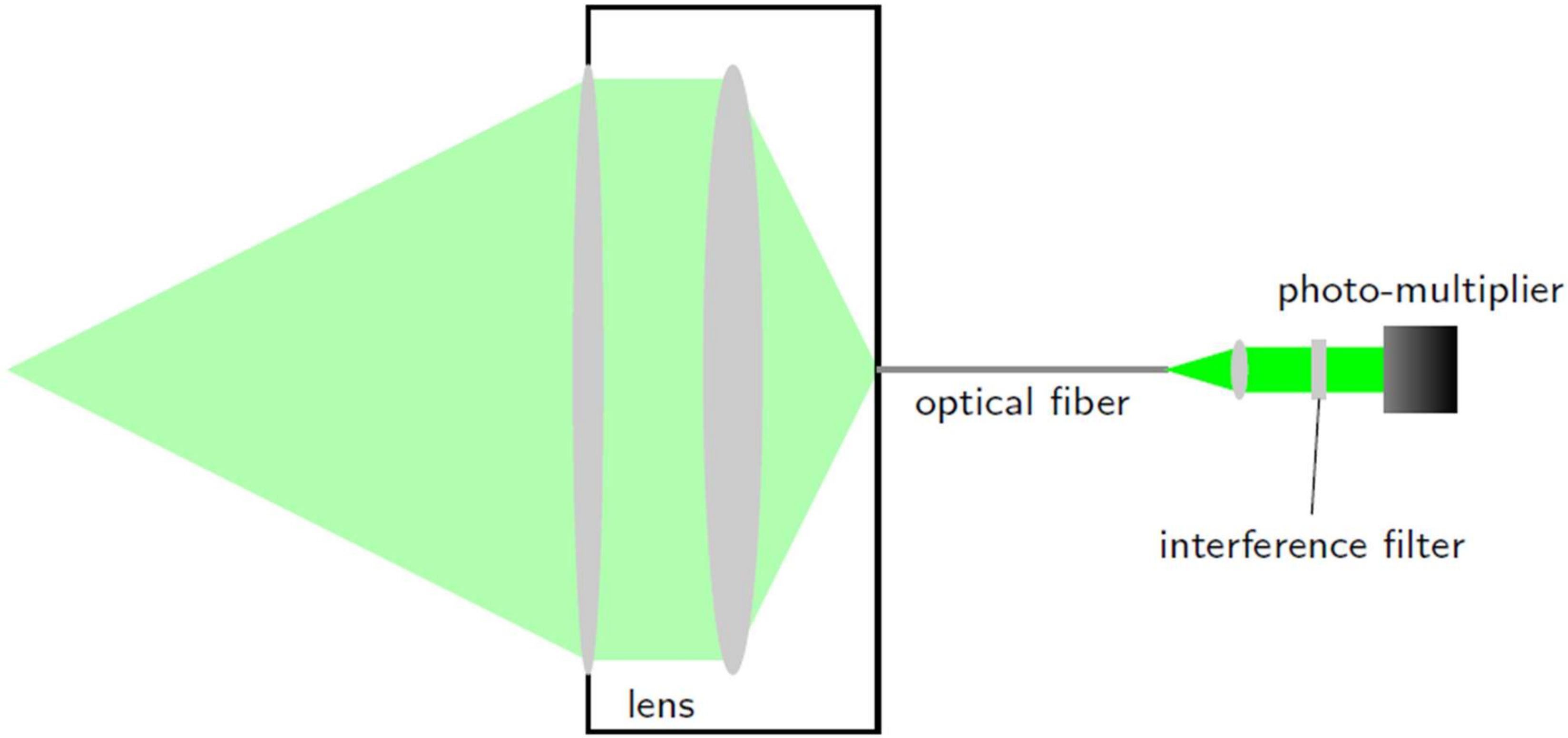


Laser Doppler anemometry

Implementation

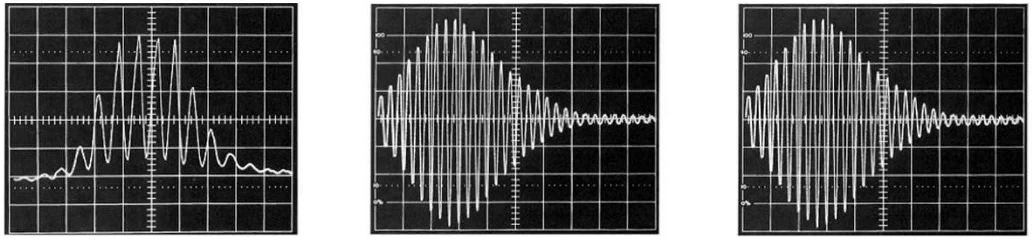


Detection system

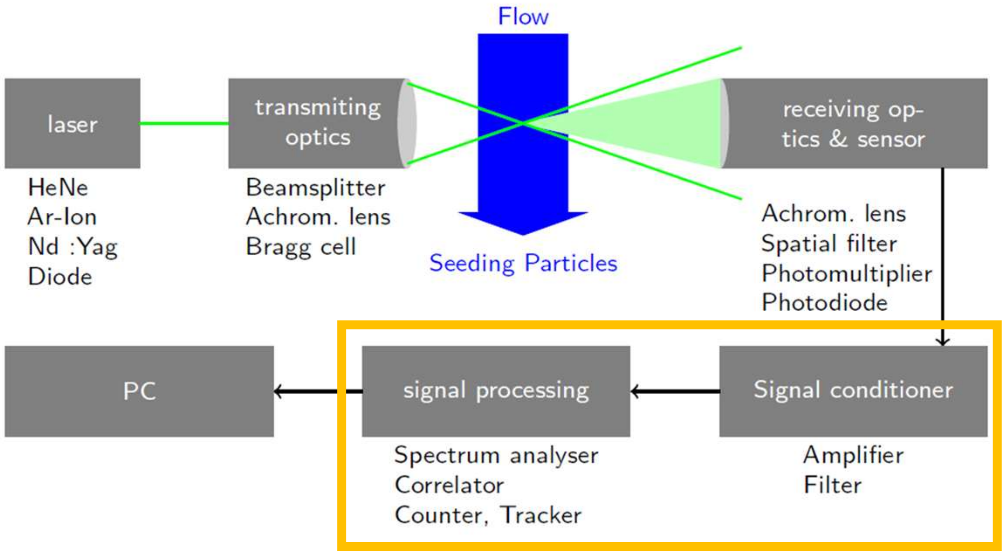


Laser Doppler anemometry

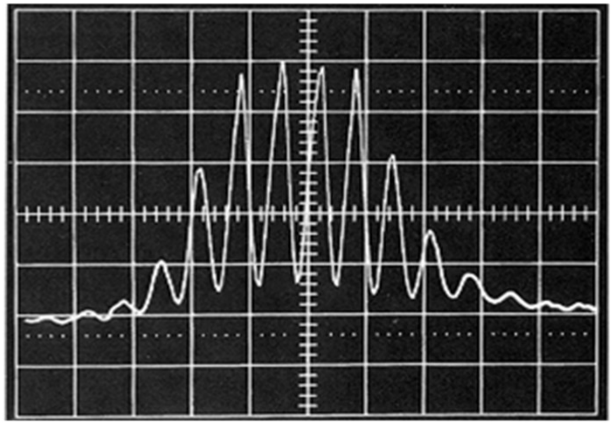
Implementation



Burst acquisition



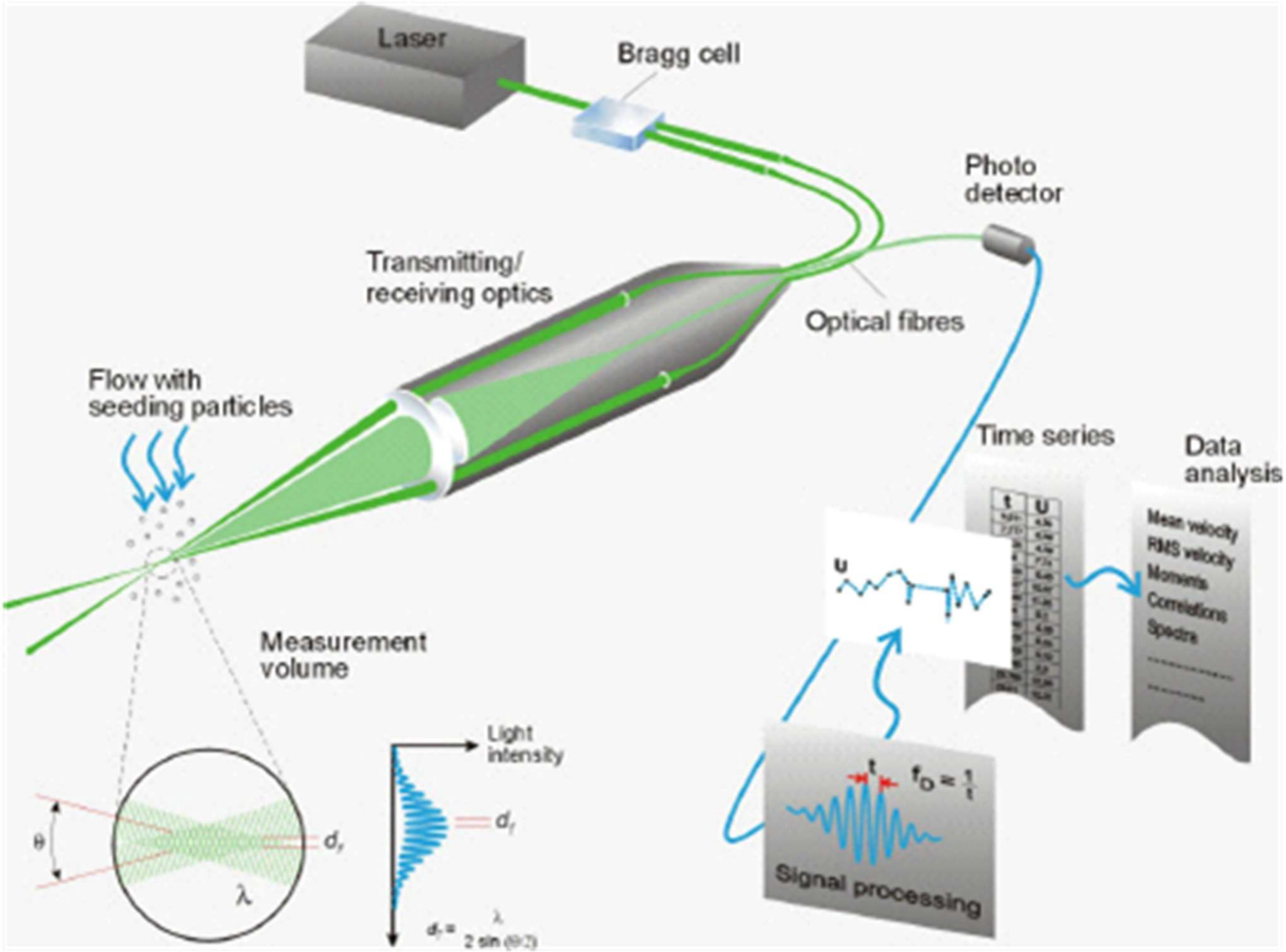
Fourier analysis



$$I(t) = a\,(1 + \sin(2\pi f_D t))\,G(t)$$

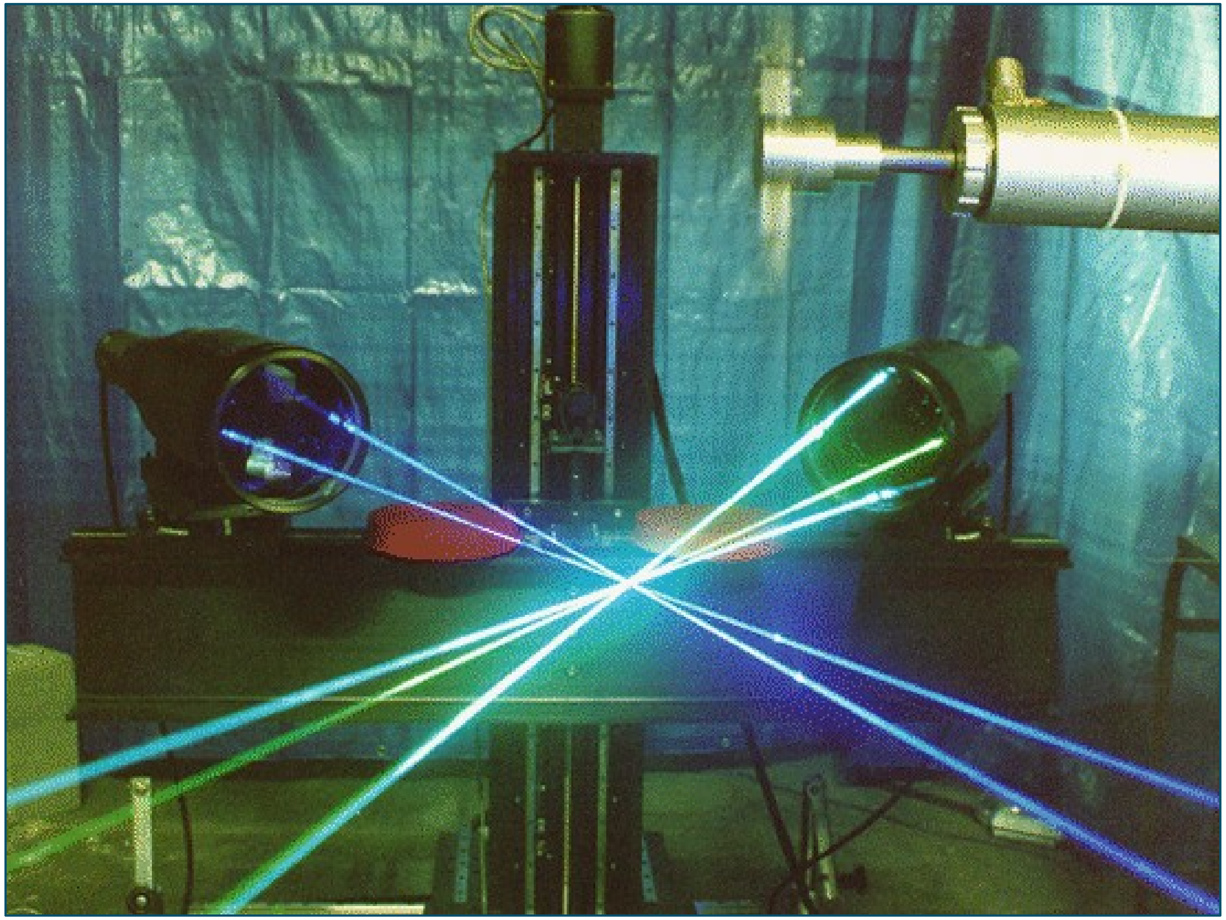
$$\Longrightarrow \quad \left|\hat{I}(f)\right| = a\,\hat{G}(f) + a\,\delta(f - f_D) * \hat{G}(f)$$

Laser Doppler anemometry



Laser Doppler anemometry

2D system



3D system

Laser Doppler anemometry

Standard full system

Small integrated 3D FiberFlow probe

Fiber Flow probes (60 & 85 mm)

Laser Doppler anemometry

Sources of noise

- Detection noise
- Electronic and thermal noises of the pre-amplifying line.
- High order laser modes (optical noise)
- Diffused light out of the control volume, dirts, damaged window, ambient light,
- Multiple particles, etc.
- Stray reflections (windows, lens, mirrors, …)

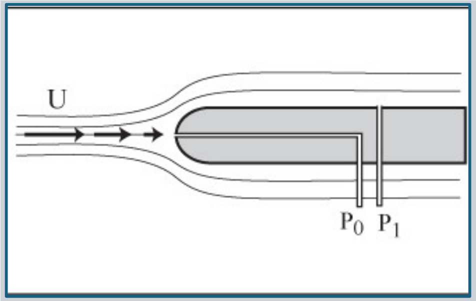
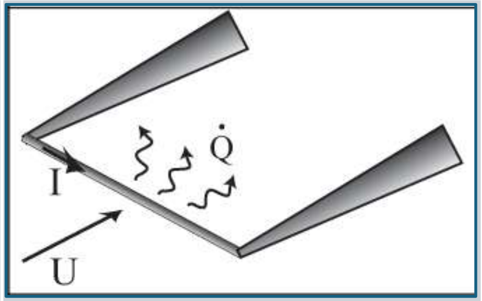
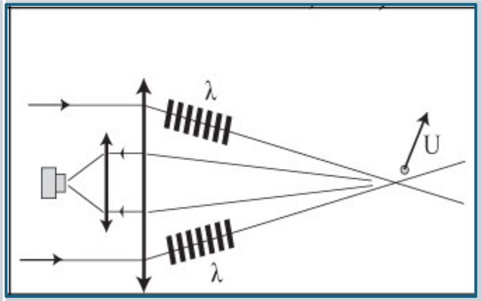
⇒ Laser power selection, seeding, optical parameters, … to optimise the signal over noise ratio

Laser Doppler anemometry

Main features

- Time-resolved point measurement
- Point measurement is a small ellipsoid (spatial coarsening)
- Non-uniform sampling frequency
- The flow must be seeded
- Linear law
- Expensive

Comparison

	Pitot Tube	Hot Wire Anemometry	Laser Doppler Anemometry
Sketch			
Principle	Two pressure measurements: static and dynamics Bernoulli	Measure of dissipated Joule power in a wire	Interferometric measurement of a Doppler shift on scattering particle
Pros	Easy to use Cheap (1 k€) Suited for time average	Very high time and space resolution Suited for fluctuation measurements Easy to use Medium price (10 k€)	Non intrusive High time and space resolution Suited for fluctuations Suited for several components
Cons	Highly intrusive Very poor time & space resolution	Intrusive, fragile Non linear calibration Sensitive to temperature	Non regular sampling High price (50-100 k€) Seeding required Difficult settings