Mécanique linéaire élastique de la rupture (rupture fragile) tridimensionnelle: de la théorie à la pratique

Three-dimensional linear elastic fracture mechanics (brittle fracture): from theory to practice

Habilitation à Diriger des Recherches

Véronique Lazarus

Laboratoire FAST, UPMC Univ Paris 6

Tuesday 6th of July 2010

CV

1991-1994	**Engineering degree**. Ecole Nationale Supérieure de Techniques Avancées.
1994	**DEA (Postgraduate degree)** in Mechanical Engineering (University of Paris 6).
1994-1997	**PhD** in Mechanical engineering at the Laboratoire de Modélisation en Mécanique (Paris 6). Advisor: J.-B. Leblond. “Some three-dimensional problems of brittle fracture mechanics”.
1997–98	**ATER** at University of Paris 6.
1998–	**Associate professor** at University Paris 6.

Teaching

Almost 2200 teaching hours.
60% practical (TD), 20% numerical work (TP num), 20% Lab work (TP exp), 20% courses.

- Continuum thermomechanics.
- Solid mechanics.
- Fluid mechanics.
- Mathematics.
- History of sciences
- Computer science.

Research

1994–2006	Laboratoire de Modélisation en Mécanique (**LMM**), incorporated in 2007 into the **Institut Jean Le Rond d'Alembert (IJLRDA, UMR 7190)**. Mechanics of Solids and Structures Team.
2006–2008	**Institut Jean Le Rond d'Alembert and FAST laboratory** during teaching sabbatical leave (3 X 6 months, CRCT, Delegation CNRS).
2008–	**Laboratoire Fluides, Automatique et Systèmes Thermiques (FAST, UMR 7108)**. Porous and Fractured Media Team.

Publications

- 21 articles in peer-reviewed journals
  (7 J Mech Phys Solids, 4 Int J Solids Struct, 3 Cr Acad Sci II B, 2 Int J Fracture, 1 Phys Rev E, 1 EPL-Europhys Lett, 1 Langmuir, 1 J Appl Mech-T ASME, 1 Reflets de la Physique).
- 13 articles in International Conference proceedings.
- 2 successful ANR and 2 successful Triangle de la Physique funding requests.

PhD students

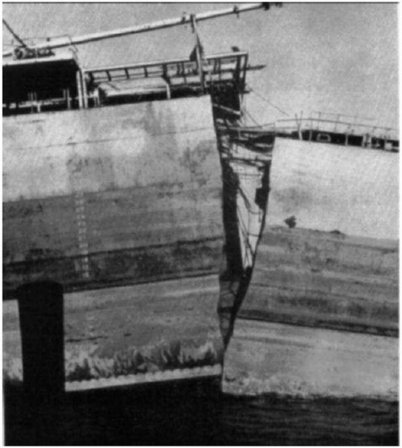
- On 3D LEFM theory with J.B. Leblond:

09/2001-10/2005	Elie FAVIER.
09/2005-07/2009	Nadjime PINDRA .
09/2008–	Laurène LEGRAND.

- On drying of colloidal suspensions with L. Pauchard:

2007	Cécile BOUSQUET. Industrial PhD, CEA Marcoule.
10/2007–	Mourad CHEKCHAKI.

Brittle Fracture mechanics (LEFM): History



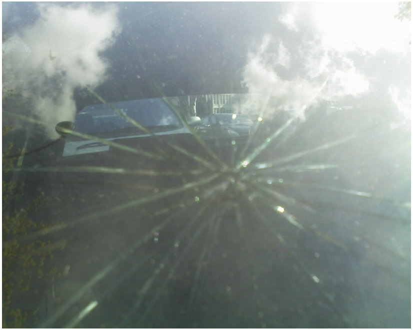
- Starting point: Cracked Liberty ship (second World War).
- Griffith 20's → Irwin 50's (Naval research Laboratory).
- Failure occurs for **small strains** and **negligible plasticity**. (glass, metal at low temperatures, rocks...).

Brittle Fracture mechanics: the aims

Take an elastic loaded body:

1. Will a crack appear?
2. If yes, can we predict its shape? Can we predict the number of radial cracks?
3. If cracks are present, will they propagate, over which distance? until the total breakdown of the body?
4. Can we predict the crack front shape?

Brittle Fracture mechanics: the aims



JL Prensier

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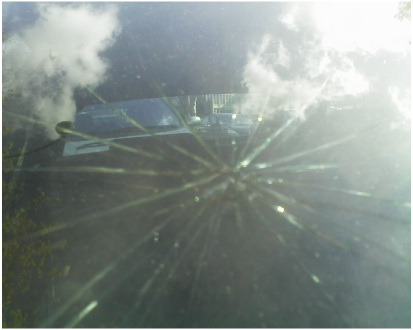

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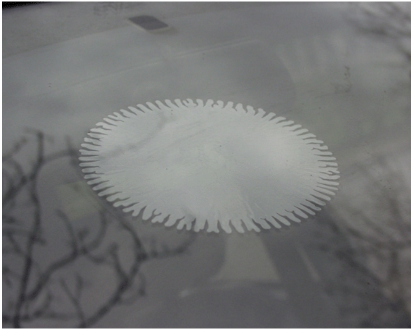
Brittle Fracture mechanics: the aims



J Tignon

Take an elastic loaded body:

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Outline

1. **Bases (Irwin 1950-)**

   If cracks are present, will they propagate, over which distance? until the total breakdown of the body?

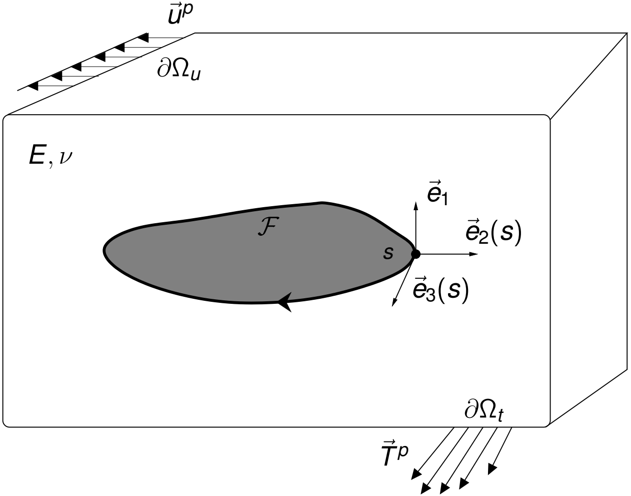
2. **Deformation of the crack front (Rice 1985-)**

   Can we predict the crack front shape?

3. **Crack initiation (2000's)**

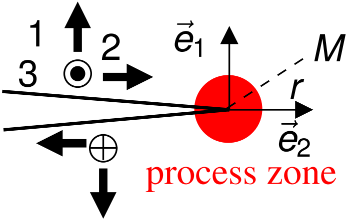
   Will a crack appear?
   If yes, can we predict its shape? Can we predict the number of radial cracks?

Traditional LEFM approach



Linear Elastic material. Condition of crack propagation?

Definition of the Stress Intensity Factors



Westergaard (1938), Williams (1952), Leblond and Torlai (1992):

$$\sigma_{1p}(M) \propto \frac{K_p(s)}{\sqrt{r}} \text{ for } r \to 0.$$

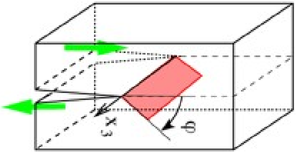
Stress Intensity Factors (SIFs): $K_1(s)$, $K_2(s)$, $K_3(s)$

Energy release rate $\mathcal{G}$:

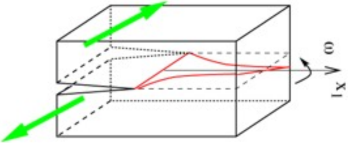
$$\begin{aligned} \mathcal{G}(s) &\equiv -\frac{\mathrm{d}E_{elast}}{\mathrm{d}S} \\ &= \frac{1-\nu^2}{E}(K_1(s)^2 + K_2(s)^2) + \frac{1+\nu}{E}K_3(s)^2 \quad \text{Irwin's formula} \end{aligned}$$

Crack propagation direction criterions

Whatever the loading, in an homogeneous brittle material, the crack propagates in order to reach a situation of pure tension loading (Hull, 1993).



Mode 2: local kink φ



Mode 3: rotation along x_1

PLS: Goldstein and Salganik (1974)
MTS: Erdogan and Sih (1963)
Review: Qian and Fatemi (EFM, 1996)

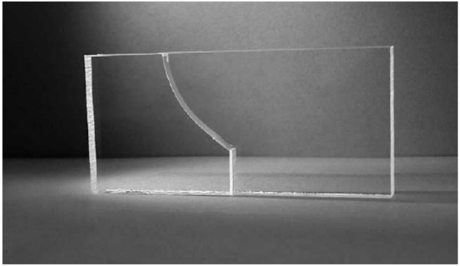
Lazarus et al. (JMPS 2001-I,II, IJF 2008)
Lin et al. (IJF 2010)
Pons et Karma (Nature 2010)

Crack propagation direction criterions

In memory of F. Buchholz (univ. Paderborn, Germany) who made the following experiments in PMMA:

Mode 1+2: in-plane 4PS

F.-G. Buchholz et al. / Engineering Fracture Mechanics 71 (2004) 455–468



Mode 1+2+3: in-plane 3PB

Now, coplanar propagation (weak interface) even in mode 1+2+3

Crack advance versus loading criterions

- **Brittle fracture**: Griffith (1920)' criterion

$$\mathcal{G} < \mathcal{G}_c \Rightarrow \text{ no propagation,}$$
$$\mathcal{G} = \mathcal{G}_c \Rightarrow \text{propagation.}$$

In mode 1, it is equivalent to Irwin (1958)'s criterion:

$$K_1 < K_c \Rightarrow \text{ no propagation,}$$
$$K_1 = K_c \Rightarrow \text{propagation.}$$

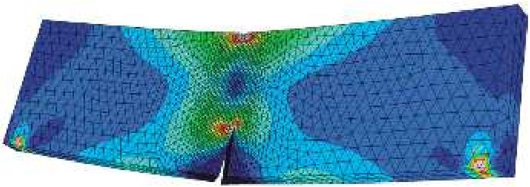
- **Fatigue, subcritical fracture**: Paris (1961)' type law

$$\frac{\partial a(t)}{\partial t} = C\mathcal{G}^\beta$$

For $\beta \gg 1$, regularization of Griffith (1920)' threshold criterion.

Determination of the SIF

- Engineering: Finite Elements simulations, XFEM (Belytschko et al. 2000's).



From Lazarus, Buchholz, Fulland, Wiebesiek (IJF, 2008).

- Research analytical approach:
  Crack front perturbation approaches initiated by Rice (1985).

Outline

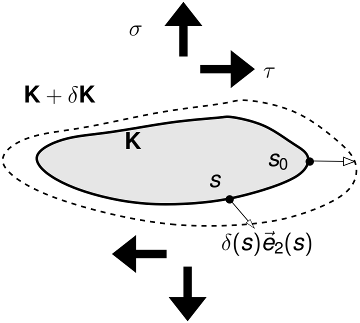
Bases of the LEFM approach

Deformation of the crack front shape

Crack initiation

Conclusion

Crack front perturbation approach: $\delta\mathbf{K}(s)$ knowing their initial values $\mathbf{K}(s)$



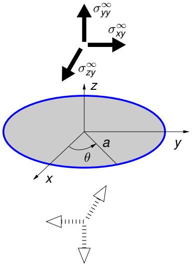
- Mode 1: Rice (1989)
- Mode 2+3 : Favier, Lazarus and Leblond (IJSS, 2006)

$$\begin{aligned}\delta K_i(s_0) &= N_{ij}(\nu)\cdot K_j(s_0)\delta'(s_0)\\ &+\frac{1}{2\pi}\,\mathrm{PV}\int_{\mathcal{F}}\frac{W_{ij}(s_0,s)}{D^2(s_0,s)}\cdot K_j(s)\left[\delta(s)-\delta(s_0)\vec{e}_2(s_0).\vec{e}_2(s)\right]\mathrm{d}s.\end{aligned}$$

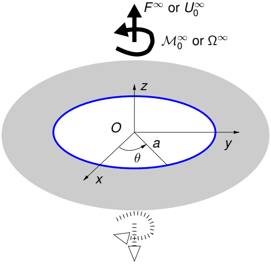
+Similar, but more complex, formula for $\delta W_{ij}(s_0, s_1)$.

Initialisation: geometry for which $W_{ij}(s, s_0)$ are known...

Initialisation: Circular cracks



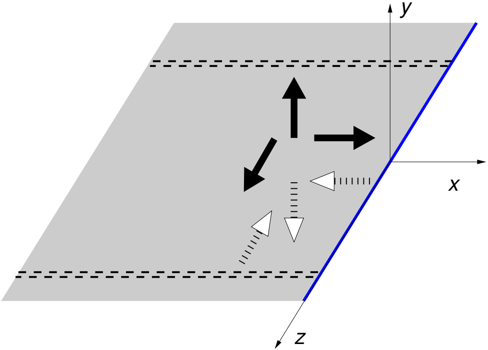
An *internal circular* crack



An *external circular* crack

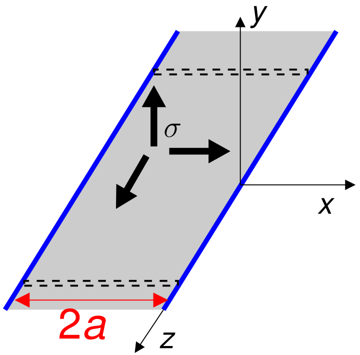
- Internal: Kassir and Sih (1975), Tada et al. (1973), Gao et Rice (1987), Gao (1988).
- External: Stallybrass (1981), Gao et Rice (1987), Rice (1989)

Initialisation: Half-plane cracks



- Homogeneous case: Meade and Keer (1984), Bueckner (1987), Rice (1985), Gao and Rice (1986).
- Interfacial crack: Lazarus et Leblond (1998), Bercial-Velez et al. (2005), Piccolroaz et al. (2007).

Initialisation: Tunnel cracks



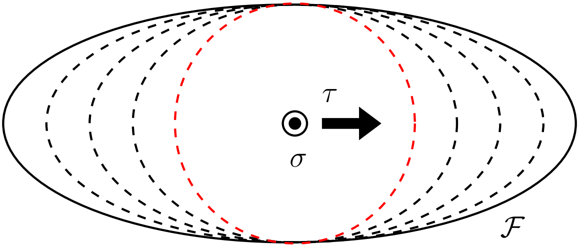
A *tunnel-crack* loaded by:

- remote tensile: Leblond, Mouchrif et Perrin, 1996;
- shear stresses: Lazarus and Leblond, 2002

Examples of application of the perturbation approach

1. Largescale propagation simulations
2. Stability of the straight crack front shape in an homogeneous media
3. Crack propagation in heterogeneous media

Largescale propagation simulations: PlaneCracks

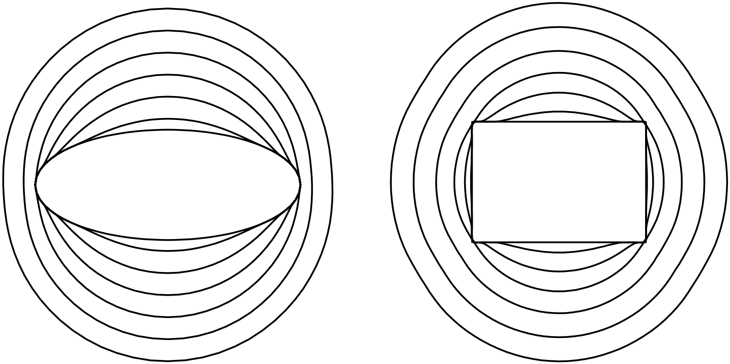


- Initialisation of **K**, **W**: internal circle under remote tensile or shear loading
- Step 1: Determination of **K** and **W** along the front $\mathcal{F}$ by successive small perturbations of the circle
- Step 2: Determination of the crack advance by Paris' law

$$\frac{\partial a(t)}{\partial t} = C\mathcal{G}^{\beta}$$

Rice (1989), Bower and Ortiz (1990), Lazarus (2003), Favier, Lazarus, Leblond (2006)

Examples in mode 1

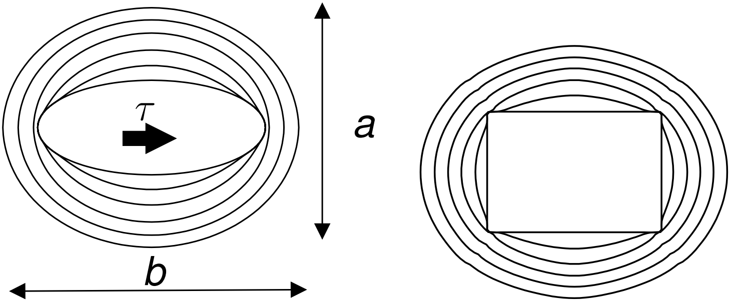


Brittle fracture $\beta = 50$

The stationnary shape is circular.

Lazarus, 2003

Examples in mode 2+3 (coplanar propagation case)



Brittle fracture $\beta = 50$

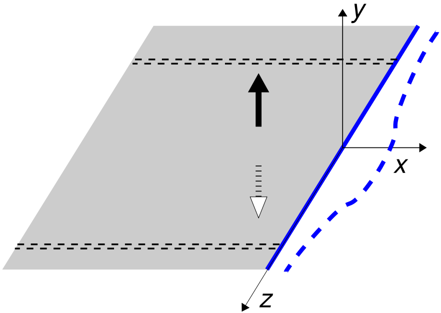
The stationnary shape is nearly elliptical with:

$$\frac{a}{b} = (1-\nu)^{\frac{\beta}{\beta+1}}$$

$$\frac{a}{b} = (1-\nu) \qquad \text{if } \beta \gg 1 \quad (\mathcal{G} = \mathcal{G}_c)$$

Favier, Lazarus, Leblond (IJSS, 2006)

The configuration stability problem

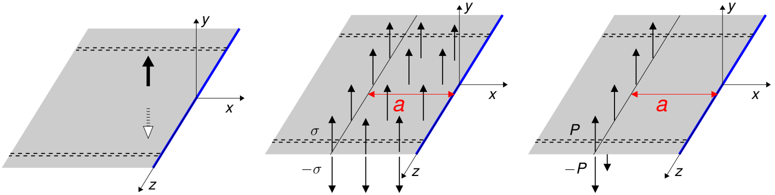


- Problem: circular, straight cracks are often used in engineering, is it safe?
- If the crack front is perturbed, will the perturbation increases (instable) or decreases (stable) in time?

This stability problem has been studied by

- Rice and Gao (1985-1990) for circular and half-plane cracks
- Lazarus and Leblond (1998) for the interfacial half-plane crack
- Leblond, Favier, Pindra, Lazarus, Mouchrif, Perrin (1996-) for tunnel-cracks

Straight front stability in mode 1: unperturbed problem

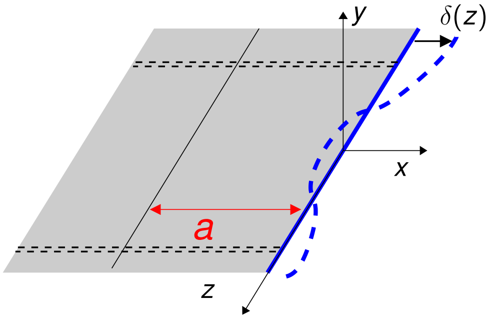


$K = cst$, $\alpha = 0$ $\qquad$ $K = 2\sqrt{\frac{2}{\pi}}\sigma a^{1/2}$, $\alpha = 1/2$ $\qquad$ $K = \sqrt{\frac{2}{\pi}} P a^{-1/2}$, $\alpha = -1/2$

$$K(a) = ka^{\alpha}$$

- if $\alpha > 0$, $\dfrac{dK(a)}{da} > 0$: instable propagation at constant loading.
- if $\alpha < 0$, $\dfrac{dK(a)}{da} < 0$: stable propagation at constant loading.

Model problems: SIF along the perturbed configuration



$$\frac{\delta K(z)}{K(a)} = \alpha \frac{\delta(z)}{a} + \frac{1}{2\pi}\, PV \int_{-\infty}^{\infty} \frac{\delta(z') - \delta(z)}{(z'-z)^2} \mathrm{d}z'$$

In Fourier transform along $z-$axis:

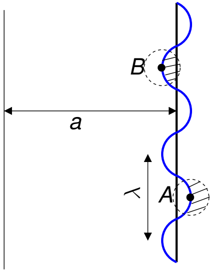
$$\frac{\delta \widehat{K}(k)}{K(a)} = \left(\alpha - \frac{ka}{2}\right) \frac{\widehat{\delta}(k)}{a} \quad k \text{ wavenumber}$$

Stability of the crack shape. Case $\alpha < 0$, $\dfrac{dK(a)}{da} < 0$.

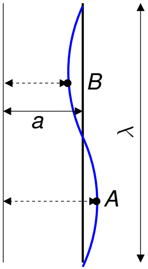
Then $\alpha - \dfrac{ka}{2} < 0$ whatever the value of k.
Since,

$$\frac{\delta \widehat{K}(k)}{K(a)} = \left(\alpha - \frac{ka}{2}\right) \frac{\widehat{\delta}(k)}{a}$$

it implies that any perturbation disappears.

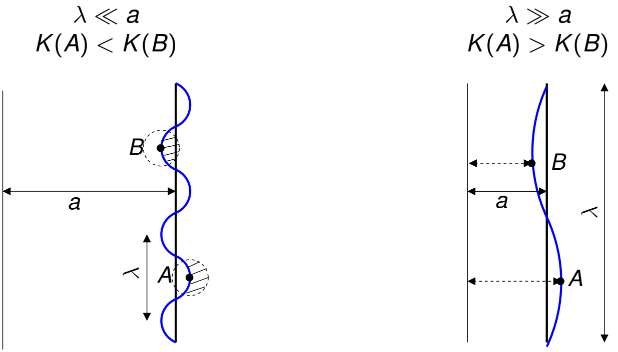


$\lambda \ll a$
$K(A) < K(B)$



$\lambda \gg a$
$K(A) < K(B)$

Stability of the crack shape. Case $\alpha > 0$, $\dfrac{dK(a)}{da} > 0$.

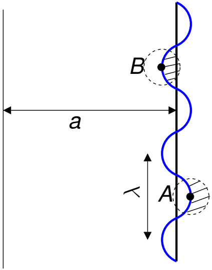


Stability of the crack shape. Case $\alpha > 0$.

Stability

$\lambda < \lambda_c$

$K(A) < K(B)$



Bifurcation

$\lambda = \lambda_c = \lambda_c^* a$

$K(A) = K(B)$

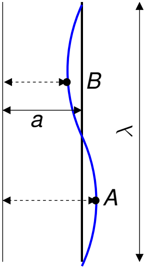
where

$$\lambda_c^* = \frac{\pi}{\alpha}$$

Instability

$\lambda > \lambda_c$

$K(A) > K(B)$



Since $\lambda_c = \lambda_c^* a \nearrow$ with a, ultimately any perturbation tends to disappear.
If :

- The first order approach remains valid
- The fracture properties are homogeneous...

Crack propagation in heterogeneous media

Crack front shape such as $K(x,z) = K_c(x,z)$, $\forall (x,z) \in \mathcal{F}$?
For slightly toughness heterogeneities:
$K_c(z,x) = K_c(1 + \Delta K_c(z,x)), \quad |\Delta K_c| \ll 1$:

$$\hat{\delta}(k,a) = -\frac{a\widehat{\Delta K_c}(k,a)}{\frac{ka}{2} - \alpha}$$

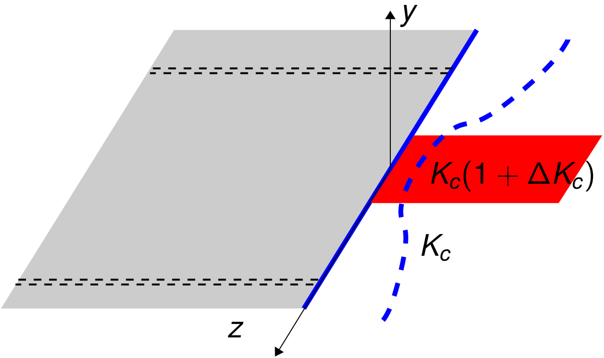
Meaningless if $\alpha \geq 0$ due to the existence of a bifurcation

$\Rightarrow \alpha < 0$ in the sequel.

Application to

1. Crack trapping by obstacles
2. Crack propagation in disordered medium

Application to the crack trapping



1. Gao and Rice (1989, 1991)
2. Dalmas, Barthel, Vandembroucq (2009)
3. ANR MEPHYSTAR
   Theory: S. Patinet, postdoc with V. Lazarus D. Vandembroucq.
   Experiments: L. Alzate (PhD D. Dalmas, St Gobain)

Application to disordered medium.

$$\hat{\delta}(k,a) = -\frac{a\widehat{\Delta K_c}(k,a)}{\frac{ka}{2} - \alpha}$$

- Power spectrum of the crack front fluctuations:

$$|\hat{\delta}(k,a)|^2 = a^2 \frac{|\widehat{\Delta K_c}(k)|^2}{\left(|\alpha| + \frac{ka}{2}\right)^2}$$

  as a function of $|\widehat{\Delta K_c}(k)|^2$ power spectrum of the toughness fluctuations ΔK_c.
- If $|\widehat{\Delta K_c}(k)|^2 = cst = \widehat{\mathcal{K}}_0$ (white noise), one obtains:

$$\frac{|\hat{\delta}(k,a)|^2}{\widehat{\mathcal{K}}_0 a^2} = \frac{1}{\left(|\alpha| + \frac{ka}{2}\right)^2}$$

  which corresponds to a Family-Viscek (1985) scaling $\zeta = 0.5$ and $\tau = 1$.

Application to disordered media. Other results

- **Interfacial/homogeneous**: minor influence.

  Pindra, Lazarus, Leblond, JMPS 2008.
- **Mode mixity**: minor influence

  Pindra, Lazarus, Leblond, JMPS 2010
- **Tunnel-crack/half-plane** crack: minor influence
- **Loading** α: MAJOR influence
- **Irwin/Paris law**: MAJOR influence with a memory effect

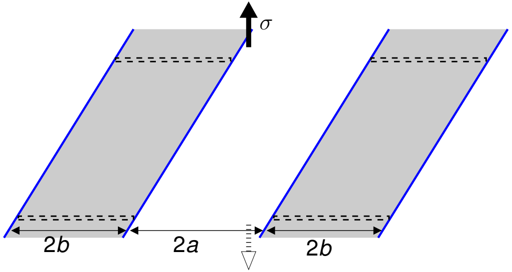
$$\widehat{\delta}(k,a) = \int_{a_0}^{a} \left[\frac{\exp(-ka/2)}{\exp(-ka'/2)}\right]^{2\beta} \left(\frac{a}{a'}\right)^{\beta} \widehat{\delta c}(k,a')\, da',$$

Lazarus and Leblond (2002); Favier, Lazarus and Leblond (JMPS, 2006).

Statistical physics approach:
Daguier, Bouchaud and Lapasset (1995), Schmittbuhl, Maloy et al. (1995-2010), Ramanatha, Fisher, Ertas (1997), Krauth and Rosso (2002), Hansen et al. (2003), Roux, Vandembroucq, Hild, (2003), Katzav et Adda-Bedia (2006), Ponson (2007), Bonamy (2009)...

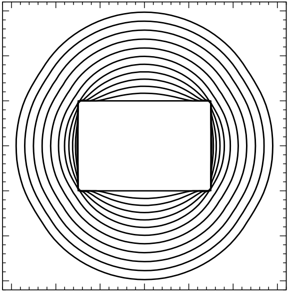
Interaction between several cracks

- Interaction between two tunnel-cracks:
  - Determination of W_{ij} for two-tunnel cracks (Pindra, Lazarus, Leblond, 2010)
  - Bifurcation wavelength $\lambda_c = \lambda^* a \searrow$ when $a \nearrow$ (Legrand, Leblond, 2010).

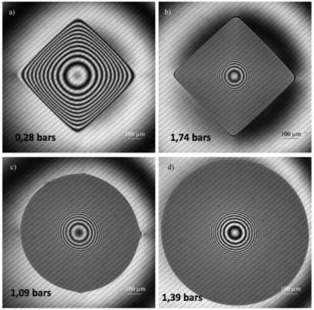


- Extension of the code PlaneCracks for two cracks (PhD of L. Legrand).

Comparison of PlaneCracks with experiments



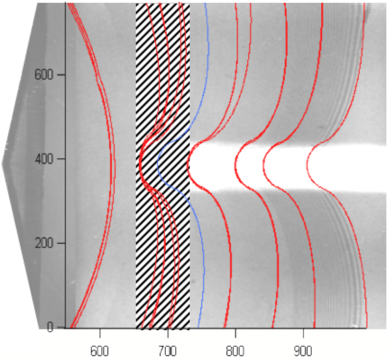
Lazarus (2003)



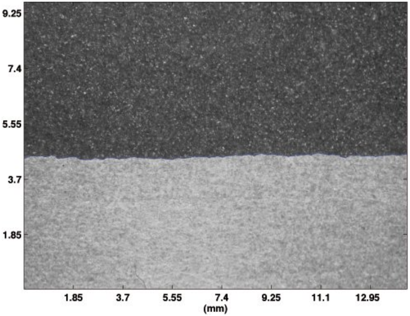
Dupeux et al. (1998)

Collaboration envisaged with Muriel Braccini (SIMAP, Grenoble).
One crack, interaction of two cracks (L. Legrand).

Heterogeneous media: comparison with experiments



D. Dalmas and D. Vandembroucq
ANR Mephystar



J. Schmittbuhl et al.

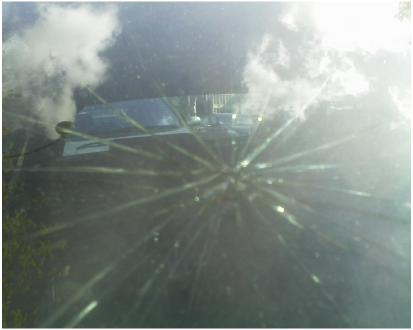
Outline

Bases of the LEFM approach

Deformation of the crack front shape

Crack initiation

Conclusion



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1. Will a crack appear?
2. If yes, can we predict its shape?

Variational approach to fracture.
Bourdin, Francfort and Marigo (1998-2008)

Principle

$\mathcal{F}$ cracks such as $E_{tot}(\mathcal{F}) \equiv E_{elastic}(\mathcal{F}) + \mathcal{G}_c$ length$(\mathcal{F})$ is minimum.

- Identical to the traditional approach if a crack is still present.
- Applicable only if an "idea" of the crack shape.

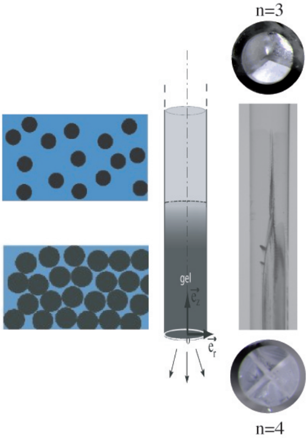
Regularized form: Non-local damage model

$\alpha(M)$ damage field such as $E_{tot}(\alpha) \equiv E_{elastic}(\alpha) + \mathcal{G}_c f(\alpha, \ell)$ is minimum.

where $f(\alpha, \ell)$ chosen such as $\lim_{\ell \to 0} f(\alpha, \ell) =$ length$(\mathcal{F})$.

- Convergence toward the initial principle for $\ell \to 0$.
- Suitable for numerical purposes.

Directional drying in capillary cells



Gauthier, Lazarus and Pauchard (Langmuir 2007, EPL 2010)

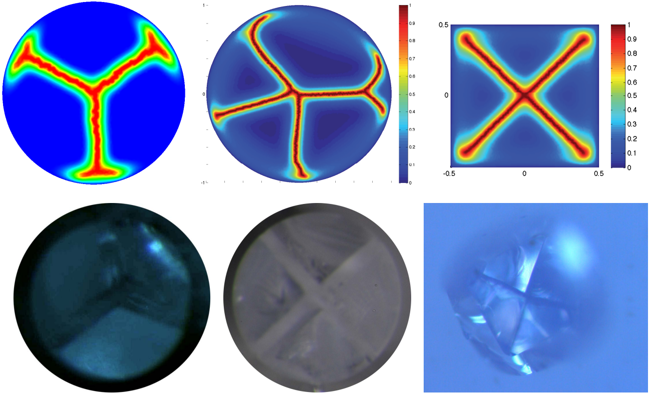
- Capillary tubes of diameter $\sim$ 1 mm.
- Ludox® colloidal aqueous suspension of silica particles (diameter $\sim$ 10 nm).
- Drying from the single bottom open edge.
- Contraction prevent by adhesion
  $\Rightarrow$ tensile stresses
  $\Rightarrow$ vertical star-shaped cracks.
- Variation of the drying conditions $\Rightarrow$ various number n of radial cracks

Model: Linear elastic cross-section 2D problem:

$$\boldsymbol{\sigma} = \frac{E\nu}{(1+\nu)(1-2\nu)} \mathrm{tr}\varepsilon \mathbf{1} + \frac{E}{(1+\nu)}\varepsilon + \sigma_0 \mathbf{1}$$

Use of the regularized approach.

VL, Gauthier, Pauchard, Maurini, Valdivia (ICF12, 2009):



Qualitative agreement.
Idea of the crack shape.

Use of the direct approach

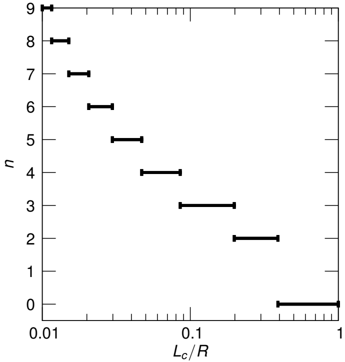
For dimensional reasons:

$$n = f(\mathcal{L})$$

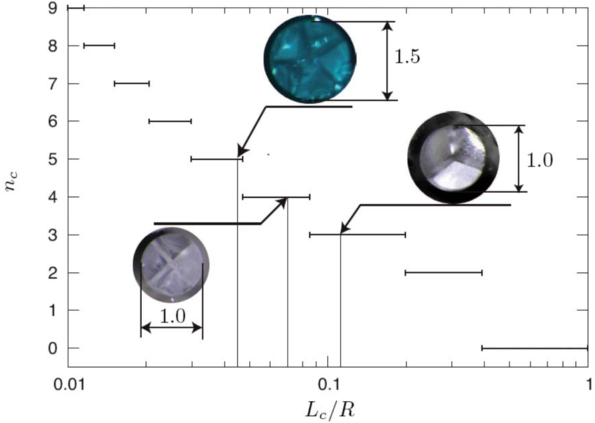
where $\mathcal{L} = \frac{L_c}{R} \propto \frac{\text{fracture energy}}{\text{elastic energy}}$

More precisely, L_c is the Griffith length defined by (K_c toughness):

$$L_c = \frac{E\mathcal{G}_c}{\sigma_0^2} = \frac{K_c^2}{\sigma_0^2}$$



Comparison theory/experiments:



Good agreement between theory and experiments.

Application: Geological shrinkage crack patterns

Septarias

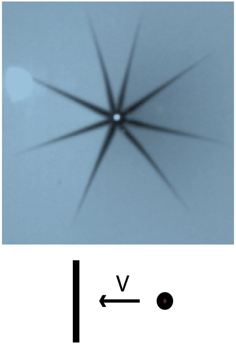
Giant's causeway, Ireland

Port Arthur, Tasmania

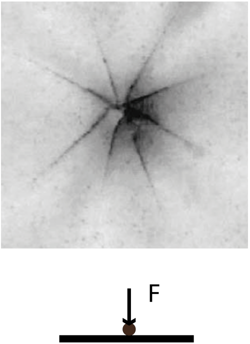
Minimum principle $\Rightarrow \frac{L_c}{R} = \frac{K_c^2}{\sigma_0^2} = f^{-1}(n) \Rightarrow \sigma_0 = \frac{K_c}{\sqrt{f^{-1}(n)}} \Rightarrow$ origin?

With A. Davaille (FAST), S. Morris (Univ Toronto), colleagues of IDES (Orsay)

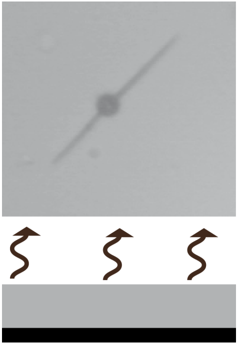
Application to crack nucleation



Impact (Vandenberghe, Vermorel, 2009)



Indentation (Rhee et al., 2001)



Drying of a thin colloidal suspension film (L. Pauchard)

Conclusion.

New theoretical developments in LEFM in the last 10-20 years:

- Traditional approach
- Statistical physics
- Variational approach
- Phase-field
- XFEM
- Experiments

Now:

- Collaborations
- Application several fields:
  - Engineering
  - Soft Matter
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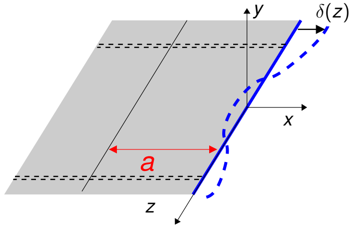
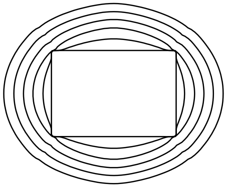


Gao, Rice, Leblond, Lazarus et al. (1985-)



Bower, Ortiz, Favier, Lazarus, Leblond (1990-)

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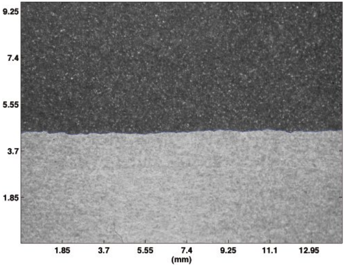

Bouchaud, Schmittbuhl, Maloy, Hansen, Roux, Vandembroucq, Katzav, Adda-Bedia, Ponson, Bonamy ...(1995-)

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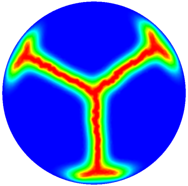

Marigo, Francfort, Bourdin, Maurini (1998-)

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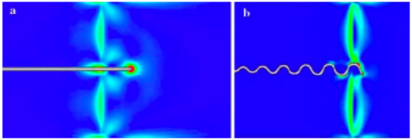
Corson et al. (PRL 2009)



Karma, Hakim, Henry, Levine...(2001-)

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Belytschko et al. (2000):

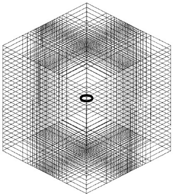

Figure 7. Mesh (surface) for the penny crack problem.

Belytschko, Moes, Gravouil, Sukumar,...
(1997-)

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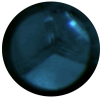
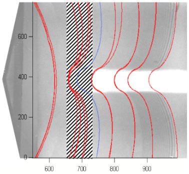


Pauchard, Gauthier



Dalmas, Barthel, Alzate, Teisseire (St Gobain)

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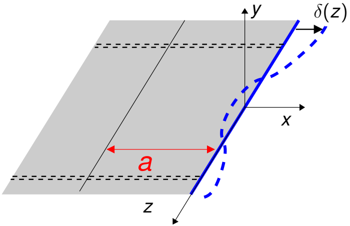
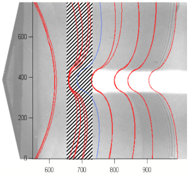


Lazarus



Dalmas, Vandembroucq (ANR MEphystar)
Schmittbuhl et al.

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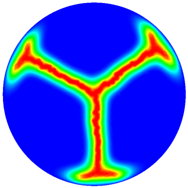
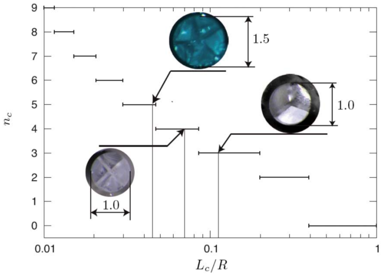


Bourdin, Maurini



Gauthier, Lazarus, Pauchard

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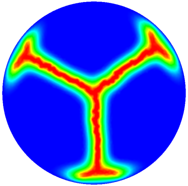

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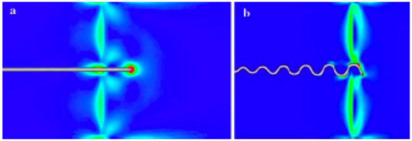

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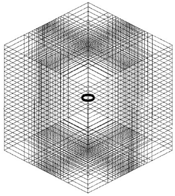

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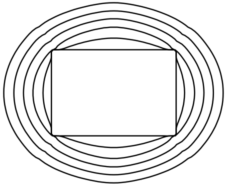

Lazarus

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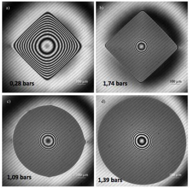

M. Braccini, SIMAP (Grenoble)

Now:

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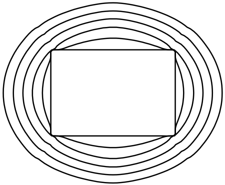

Lazarus

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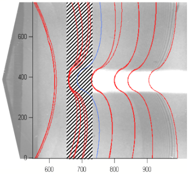

Inverse method: determination of ΔK_c
More tough materials

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$n = f(L_c/R)$

Shrinkage cracks in wood or concrete

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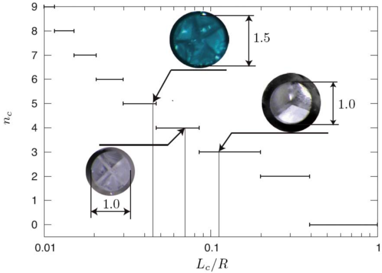

Better comprehension of the consolidation process

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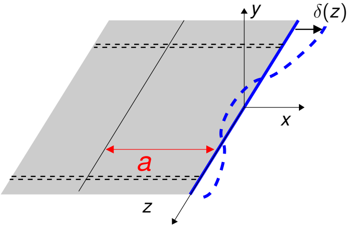
Origin?


y
$\delta(z)$
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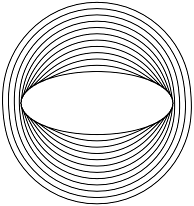

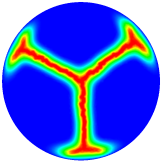

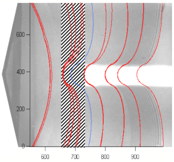
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