Improving the Efficiency of Multipole-Accelerated Method-of-Moments Solvers using Dual Grid Multipole Expansions *

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Abstract

Method-of-Moments (MOM) based 3-D electromagnetic analysis programs typically generate dense systems of equations which are extremely expensive to solve. In the last several years, very fast MOM solvers have been developed by sparsifying the dense system using a hierarchy of multipole expansions or grid projection plus the Fast Fourier Transform. The hierarchical multipole algorithms represented clusters of source distributions with an expansion in the center of the cluster, where as grid projection algorithms represent clusters using grid-locked point sources. In this paper we consider how to improve the efficiency of either algorithm by using grid-locked multipole expansions to represent clusters of sources.

1 Introduction

When an iterative algorithm is used to solve the Method-of-Moments matrices associated with integral formulations of electrostatic or magnetoquasistatic analysis, the major cost of the algorithm is computing the dense matrices and then solving the dense matrix system of equations [2]. If an iterative method is used to solve the system of equations, then only matrix-vector products are required. A variety of sparsification techniques have been applied to rapidly compute the matrix-vector products, such as the fast multipole algorithms [3, 4, 1, 5] and the precorrected-FFT methods [8]. As the example in figure (1) and the performance data given in Table (1) demonstrate, these methods are extremely effective.

Sparsification techniques like fast multipole or precorrected-FFT algorithms start by dividing the problem domain containing the discretized structure into cubes, where each cube contains a small number of discretization elements. These elements are typically referred to as "panels" or "roof-tops". Interactions between elements in neighboring cubes are computed directly, and interactions between elements in distant cubes are approximated. In the paper, we describe more efficient approaches to approximating interaction between elements in distant cubes.

2 Improving the Efficiency

In calculating the potential at distant points due to the charge distribution within a given cube, two methods of approximating long-range potentials have been used. In fast multipole algorithms,

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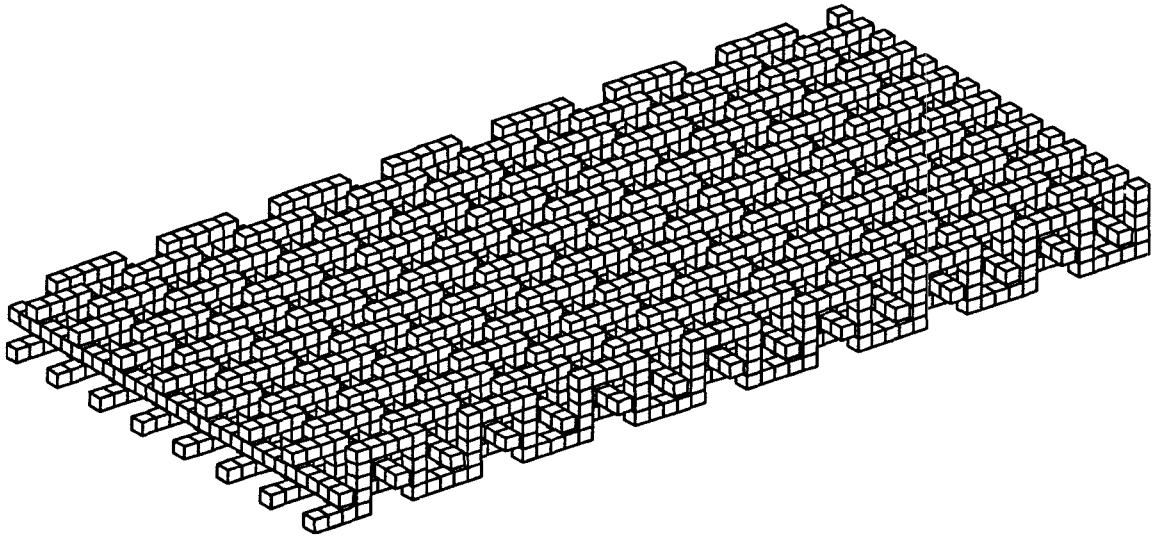

Figure 1: Woven buss example with 82,000 panels (panels shown not to scale).

Example		CPU Usage			Memory Usage	
Name	Panels[conductors]	P/FFT	Iterative	Direct	P/FFT	Direct
via	6120[4]	1.1 min	(5.6 min)	(1.9 hrs)	21 Mb	(286 Mb)
woven5x5	9360[10]	5.2 min	(42 min)	(6.9 hrs)	50 Mb	(668 Mb)
woven15	82080[30]	1.7 hrs	(11.5 days)	(194 days)	246 Mb	(50.2 Gb)

Table 1: Precorrected-FFT Performance

the charge distribution is approximated by a multipole expansion at the cube's center, the order of accuracy increasing with the number of terms taken in the expansion. In the Precorrected-FFT methods, the charge distribution is represented by point charges at grid points of the cube, the order of accuracy increasing with the number of grid points. The grid points method with 8 grid points is more efficient than a monopole expansion at the center because each grid point is shared by 8 neighbors, and there are more degrees of freedom to accurately model the charge distribution. However, this efficiency becomes less significant as the number of grid points increases, since the grid points would be shared by fewer neighboring cubes. Also, it is not certain that manipulating the number of grid points alone can achieve the arbitrary order of accuracy that can be achieved by taking more terms in the multipole expansion.

Given the different advantages of the two methods in approximating long-range potentials, sharing by grid points, and arbitrary order of accuracy by multipole expansion, it is possible that a charge distribution in a given cube can be even better represented by multipole expansions at the cube vertices. The set of cube vertex points are a dual grid of the set of points at the cube centers, and one scheme for computing the coefficients of the dual grid expansions is based on forcing the grid expansion to reproduce the exact potential on a set of collocation points (for an alternative see [6]). These collocation points are usually chosen to be the quadrature points on a sphere containing the cube [9].

In order to test the new method, a point charge, and later a dipole, were systematically placed inside a cube at various positions, and the maximum error in long-range potential due to the point charge or dipole is found at the second-nearest neighbor cube, presumely the place with the largest error in the potential calculation. Different methods are then compared in terms of this largest possible error that can occur in any potential calculation due to a point charge or dipole.

	center	monopoles at grid	multipoles at grid
	$p=1,l=2,WU=8$	$p=2,l=0,WU=1$	
error	5.78×10^{-2}	9.80×10^{-2}	
		$p=3,l=0,WU=8$	
error		1.38×10^{-2}	
	$p=1,l=7,WU=64$	$p=4,l=0,WU=27$	$p=2,l=1,WU=4$
error	5.51×10^{-3}	7.39×10^{-3}	3.08×10^{-2}
			$p=2,l=2,WU=9$
error			5.39×10^{-3}
	$p=1,l=13,WU=196$	$p=6,l=0,WU=125$	$p=2,l=4,WU=25$
error	5.52×10^{-4}	1.52×10^{-4}	2.75×10^{-4}

	center	monopoles at grid	multipoles at grid
	$p=1,l=2,WU=8$	$p=2,l=0,WU=1$	
error	9.25×10^{-2}		
		$p=3,l=0,WU=8$	
error		3.34×10^{-2}	
	$p=1,l=7,WU=64$	$p=4,l=0,WU=27$	$p=2,l=1,WU=4$
error	6.51×10^{-3}	3.19×10^{-2}	4.11×10^{-2}
			$p=2,l=2,WU=9$
error			1.83×10^{-2}
	$p=1,l=13,WU=196$	$p=6,l=0,WU=125$	$p=2,l=4,WU=25$
error	3.85×10^{-3}	1.04×10^{-3}	1.67×10^{-3}

3 Results

The tables above show absolute errors at the second-nearest neighbor for a point charge and for a dipole inside a cube. p^3 is the number of grid points per cube, l is the order of the multipole expansion. WU is a work unit which is defined as the number of coefficients to be calculated per cell. Relative errors are twice the absolute errors.

The advantages of using the mixed method of multipole expansion at grid points are clearly shown by the results in the tables. For a point charge inside the cube, with comparable work units of $8(p=3,l=0)$ or $9(p=2,l=2)$, multipoles is more than twice as accurate as grid point charges, and is over ten times as accurate as multipole expansion at the center. With a comparable work of $27(p=4,l=0)$ and $25(p=2,l=4)$, multipoles is over twenty-six times as accuarate. For a dipole inside the cube, with comparable work units of $8(p=3,l=0)$ or $9(p=2,l=2)$, multipoles is almost twice as accurate as the grid point charges, and is more than five times as accurate as multipole expansion at the center. With a comparable work of $27(p=4,l=0)$ and $25(p=2,l=4)$, multipoles is almost twenty times as accuarate. In the last row of the tables, multipoles$(p=2,l=4)$ can achieve no worse than twice the error as grid point charges$(p=6,l=0)$ with one-fifth reduction in work. Thus one could meet intermediate accuracy requirements by using multipoles with the added benefit of a larger work reduction than dictated by the accuracy reduction.

4 Conclusions

In this paper, we showed that using dual grid (cube vertex) multipole expansions are more efficient for a given required accuracy than either point-charge or standard (cube center) multipole expansions.

As is not surprising, the gain in efficiency can be as much as as a factor of eight. The authors wish to thank Joel Phillips for his help in understanding the precorrected-FFT method.

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