Piezoelectric energy harvesting from flag flutter instability

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Summary. The piezoelectric energy harvesting from a flutter instability is investigated. A long plate equipped with adjacent pairs of piezoelectric elements shunted with independent resistive circuits is considered. When the length of the piezoelectric elements is low compared to the wavelengths of waves propagating in the system, governing equations are derived in the form of coupled fluid-solid-electrical wave equations. These equations are used to perform an optimization of the energy transfer between the fluid-solid system to the electrical system.

Introduction

Drawing profit from ambient vibrations to create electrical power has received a growing attention over the last decades. Among the various methods to convert mechanical energy to electrical energy, we focus here on method using piezoelectric elements, which generally investigated when distributed low power is necessary (of the order of the mW) [1]. We investigate here the energy harvesting potential of a plate in an axial flow equipped with a distributed series of small piezoelectric elements which size is much lower that typical wavelengths of deformations of the plate. The fully coupled fluid-solid-electrical wave equations are derived. These equations are used to address the optimization of energy conversion efficiency.

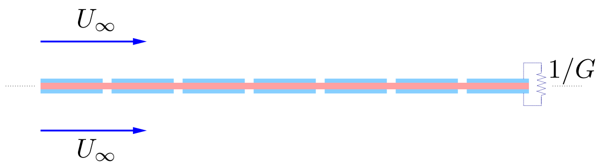

Figure 1: Schematic view of a plate in an homogeneous axial flow, equipped with small length piezoelectric patches on both sides. Each piezoelectric pair is shunted with a resistance like the one sketched on the left, modeling the electrical energy absorption.

Linear model and optimal conversion efficiency

Consider the three layers laminate of Fig.1. Its bending rigidity is B and its surface density μ. The problem is considered infinite in the spanwise direction. The displacement of the plate is noted $w(x,t)$. The surrounding fluid at both sides of the plate flows at a constant velocity U_∞. When the length of the piezoelectric elements are small compared to the typical lengths of deformation, the charge surface density q and the tension v at the outlets of the piezoelectric elements can be considered as continuous functions of x and the coupled wave equations governing the mechanical and electrical displacements take the following form:

$$\left(B+\frac{\chi^2}{c}\right)w'''' + \mu\ddot{w} - \frac{\chi}{c}q'' = -[P] \tag{1}$$

$$\frac{c}{g}\dot{q} + q - \chi w'' = 0, \tag{2}$$

where c is the capacity per unit surface of the piezoelectric elements, g is the conductivity per unit surface of the resistances, $\chi = e_{31}(h_0+h_p)/2$ is the moment arm of the piezoelectric laminate and $[P]$ is the pressure jump between both sides of the plate. For a given deformation $w(x,t)$, the mechanical to electrical conversion efficiency is defined as,

$$r = \int_0^T \langle \mathscr{P}_{el} \rangle dt \bigg/ \frac{1}{T}\int_0^T \langle \mathscr{E} \rangle dt \ , \tag{3}$$

where $\mathscr{P}_{el} = v\dot{q}$ is the power dissipated in the electrical networks and $\mathscr{E}$ is the total energy density of the system, sum of the solid kinetic and elastic energy as well as the electrical energy stored in the capacity of the piezoelectric material. In this last expression $\langle . \rangle$ stands for the spatial mean value for the considered mode, taken over either a wavelength in the local analysis or the entire plate in the global analysis. Note that since r is just a normalized energy output and not a thermodynamic efficiency, $r > 1$ is allowed.

Energy conversion through bending waves (local analysis)

Considering mechanical and electrical displacements in the form of an harmonic propagative wave of wavenumber k and frequency ω and introducing this form in the coupled wave equations (1) and (2), one obtain the dispersion relation of the system that links the wavenumbers and frequency. Stability of the infinite medium is ensured if frequencies have a zero or negative imaginary part for any real value of the wavenumber k. It is found that the medium is unstable for any flow velocity different than 0. Moreover, it is found that piezoelectric coupling destabilize a range of wavenumbers that is a range of neutral stable waves without coupling. These waves are referred to as *Negative Energy Waves*, after Cairns [2], who studied the effect of damping on stability of Kelvin-Helmotlz waves. It is then found that for any values of the

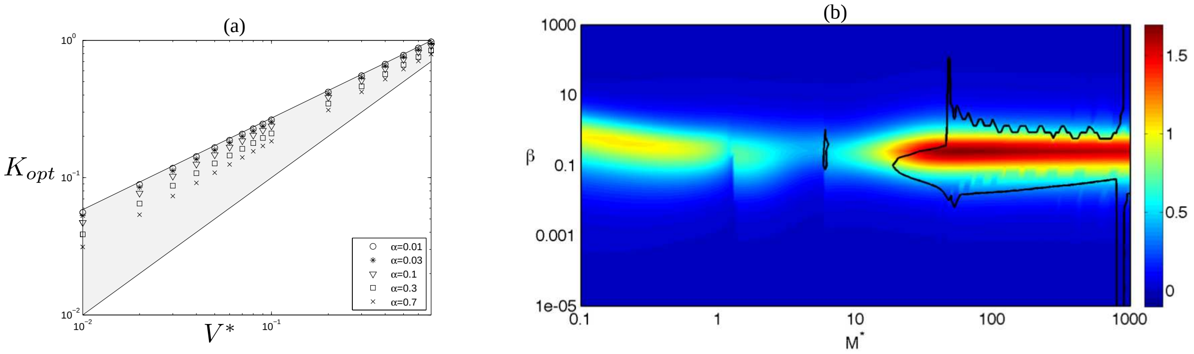

Figure 2: (a) Wavenumber of the unstable wave that maximizes the conversion efficiency; grayed region indicates the range of wavenumbers that are destabilized by piezoelectric coupling. (b) Contour levels of the optimal efficiency in the (M^*, β) plane; black line indicates the region where piezoelectric coupling has a destabilizing effect.

system's parameters, the unstable wave that maximizes r is one of these neutral waves destabilized by the piezoelectric coupling, as shown in Fig.2a, where the wavenumber that maximizes the efficiency is plotted as function of the non-dimensional velocity V^* for different values of the coupling coefficient $\alpha = \chi/\sqrt{cB}$.

Energy conversion through bending modes (global analysis)

When a finite length system is taken into account, a similar analysis can be conducted on the eigenmodes of the system. Now the plate is not always unstable. It is well known that a flutter instability appears at a finite value of the flow velocity [3, 4]. In the finite length problem, it is found that depending on the mass ratio $M^* = \rho L/\mu$, the piezoelectric coupling can increase or decrease the critical velocity for instability. Thus piezoelectric coupling can have a stabilizing or destabilizing effect. On Fig.2b, color levels the conversion efficiency at marginal stability of the marginally unstable mode are represented in the (M^*, β) plane. Here, β is a measure of the ratio between fluid-solid and electrical timescales. On this figure is also represented with a black line the region where piezoelectric coupling has a destabilizing effect. Similarly to the wave analysis, efficiency is maximum where the system is destabilized by damping. We predict that this is due to the fact that *Negative Energy Waves* are implied in the unstable mode.

Non-linear results

The effect of piezoelectric coupling on the saturated mode as well as the harvested power in the saturated regime are addressed numerically and experimentally. As an illustration, Fig.3 presents some experimental results obtained with a mylar plate equipped with two piezoelectric elements occupying the whole length of the plate. The unstable deformation during the oscillation, involving an appropriate image processing from a picture sequence obtained by a high speed camera is given on Fig.3a. The tension at the outlets of the piezoelectric elements in series, with a 330kΩ resistance in parallel is plotted as function of time in Fig.3b. Finally, the power harvested is plotted as function of the value of the resistance and compared with a model in Fig.3c.

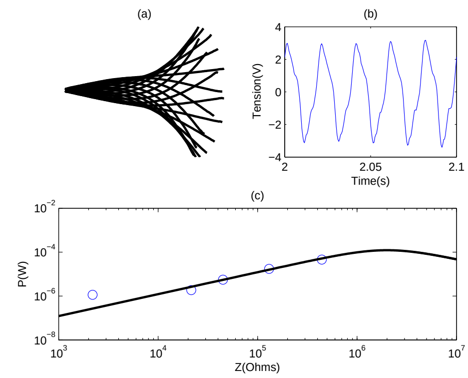

Figure 3: (a) Experimental deformation during one cycle of oscillation. (b) Tension at the outlets of the piezoelectric element. (c) Harvested power as function of the shunting resistance.

Conclusions

We have derived a fully coupled fluid-solid-electrical wave equation which allows to address the optimization of energy conversion from a flowing fluid to electrical circuits trough piezoelectric coupling. We have found that this coupling has a destabilizing effect both on waves propagating in the infinite medium (local stability) and modes of the finite system (global stability). Moreover, the maximal efficiency of the energy conversion is observed in situations where piezoelectric coupling has a destabilizing effect. Nonlinear saturation of the dynamics of the system is now considered, to address the efficiency of energy conversion of saturated regimes.

References

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