Nonlinear Physical Systems

Spectral Analysis, Stability and Bifurcations

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Chapter 4

Dissipation Effect on Local and Global Fluid-Elastic Instabilities

4.1. Introduction

In many physical systems, damping has been observed to have a stabilizing or destabilizing effect. Various discrete mechanical systems display this unusual feature. The most famous is probably the one studied by Ziegler in 1952 [ZIE 52]. It consists of a double-pendulum subjected to a follower force, which can practically be achieved by dry friction [BIG 11, BIG 12]. It was found by Ziegler that the critical value of the load at which the straight equilibrium position becomes unstable decreases and changes in a discontinuous manner when an infinitesimal amount of viscous damping is added in the model. Many gyroscopic systems present this feature [RUI 93, KIR 10], as well as aeroelastic systems [LEE 97], and the experimental evidence of a destabilizing effect of a viscous fluid has been obtained in the context of rotor dynamics [ANT 96, GRU 96]. In the case of a general dynamical system, Bottema [BOT 55] has described the required conditions for the damping to induce instability. After Kirillov and Verhulst [KIR 10], the discontinuous behavior of the critical value of the parameter for instability is linked to the existence of singularities of maps between manifolds of dimension n to spaces of dimension $2n - 1$, as studied by Whitney [WHI 43] in a purely mathematical context, independently of the effect of dissipation on mechanical systems.

Destabilizing the effect of damping in continuous but finite systems has also been evidenced. The cantilevered beam subjected to a follower force has been studied by Sugiyama and Langthjem [SUG 07] in terms of energy transfers at the free end. In

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the case of a fluid-conveying pipe, the stabilizing or destabilizing nature of damping has been shown to depend on the fluid–solid mass ratio [LOT 86]. Similar conclusions have been drawn for the fluttering flag [GUO 00].

In infinite systems described by wave equations, the effect of damping on stable waves has also been the subject of a large number of papers. In the problem consisting of a flow over a compliant panel, Landahl [LAN 62] showed that viscosity was able to destabilize some neutral waves. The same phenomenon was observed in the fluid-conveying pipe problem [ROT 64]. Such destabilizing phenomena were first encountered in the plasma physics discipline, where the concept of wave of negative energy was introduced [VON 05, BRI 64, BEK 69]. This concept has been a considerable addition to the discussion of stability of these media and has been introduced in the research field of mechanics by Cairns [CAI 79], who showed that the classical expression of the wave energy introduced in plasma physics represents here the work done on the system to generate the wave from $t = -\infty$ to $t = 0$.

The objective of this chapter is to present a joint local/global stability analysis of a model problem of fluid-elastic instabilities: the fluid-conveying pipe. As emphasized in a recent review by Païdoussis [PAÏ 08], the fluid-conveying pipe may be seen as a model problem for many physical systems, where a slender structure is coupled to an axial flow, and has applications in many fields, such as paper and nuclear industries, aeronautics, musical acoustics and biomechanics. The fluid-conveying pipe shares many similarities with all the systems mentioned above: once a critical value of a parameter is reached (here the flow velocity), the finite length system exhibits an instability and damping can, in some cases, lower the critical velocity value. In the infinite length system, unstable waves (temporally or spatially amplified) may be identified, as well as the neutral waves that become unstable waves when a small amount of damping is added. However, in none of the above-cited works, a joint local and global analysis is performed to determine the role of the negative energy waves in the global instability.

This chapter is organized as follows: in section 4.2, the local and global analyses are briefly described. In section 4.3, the linear equations governing the dynamics of a fluid-conveying pipe are presented. Effect of damping on the local and global stability of the fluid-conveying pipe is analyzed in section 4.4. An application to energy harvesting is discussed in section 4.5 and, finally, section 4.6 concludes the chapter.

4.2. Local and global stability analyses

In this section, the local and global stability analyses of a general one-dimensional mechanical system are presented.

4.2.1. *Local analysis*

Consider an infinite length one-dimensional medium governed by a wave equation of the form

$$\frac{\partial^2}{\partial t^2} \mathcal{M}\left[y(x,t)\right] + \frac{\partial}{\partial t} \mathcal{C}\left[y(x,t)\right] + \mathcal{K}\left[y(x,t)\right] = 0 \quad \text{on } \Omega = [-\infty, +\infty], \quad [4.1]$$

where $\mathcal{M}$, $\mathcal{C}$ and $\mathcal{K}$ are the mass, damping and rigidity operators, respectively. After introducing, into this equation, a disturbance of the form of a plane harmonic wave $y = y_0 e^{i(kx-\omega t)}$, a dispersion relation that links frequency to wave number is obtained,

$$D(k, \omega) = 0. \tag{4.2}$$

The medium is stable if, for any sinusoidal wave of infinite extent in the x-direction and associated with a real wave number k, the corresponding frequencies given by equation [4.2] are such that the displacement remains finite in time. The *local* instability criterion is then

$$\text{Instability if } \exists k \ \in \mathbb{R} \setminus \text{Im}[\omega(k)] > 0. \tag{4.3}$$

This approach is called *temporal* because it consists of examining the *temporal* evolution of waves in time.

4.2.2. *Global analysis*

The global analysis considers the same local wave equation [4.1] in a finite domain $\Omega = [0, l]$, associated with a set of boundary conditions, denoted by $\mathcal{B}_i(y) = 0$ $i = 1..N$, where N is the maximal order of the spatial derivatives in operators $\mathcal{K}$ and $\mathcal{M}$. Considering ansatz solutions of the form $y = \phi(x)e^{-i\omega t}$, we obtain a Sturm–Liouville problem whose solutions are an infinite set of eigenfunctions $\phi_n(x)$ and eigenfrequencies ω_n. In this case, the instability condition is,

$$\text{Instability if } \exists \omega_n \setminus \text{Im}[\omega_n] > 0. \tag{4.4}$$

Numerical methods are generally used to solve this kind of problems. In the following, the Galerkin method is used to compute approximate solutions. The solution $y(x,t)$ is decomposed on a truncated function basis that satisfies the

boundary conditions

$$y(x,t) = \sum_{n=1}^{N} \psi_n(x)q(t).$$ [4.5]

After defining a scalar product

$$\langle f, g \rangle = \int_\Omega fg\,dx,$$ [4.6]

the approximated form of y defined in equation [4.5] is introduced in [4.1], which is then projected on a mode $\psi_m(x)$. We finally obtain a discrete set of coupled oscillator equations

$$M\ddot{\vec{q}} + C\dot{\vec{q}} + K\vec{q} = 0.$$ [4.7]

The coefficients of the matrices M, C and K result from the projection of the inertia, damping and rigidity forces of equation [4.1]. Finally, looking for harmonic solutions in the form of $\vec{q} = \vec{q}_0 e^{-i\omega t}$ leads to a second-order eigenvalue problem for the eigenfrequency. The criterion for global instability is then given by [4.4], and the corresponding eigenvector gives the eigenmode in the form of a combination of functions $\psi_j(x)$.

4.3. The fluid-conveying pipe: a model problem

The simplest equation describing the linear dynamics of this system consists of an Euler–Bernoulli equation for a beam of mass per unit length m, flexural rigidity EI, in which an internal fluid of mass per unit length M and of negligible viscosity is flowing at constant velocity U. The dimensional wave equation governing the vertical displacement $Y(X,T)$ is [PAÏ 98]

$$EI\frac{\partial^4 Y}{\partial X^4} + m\frac{\partial^2 Y}{\partial T^2} + M\frac{\partial^2 Y}{\partial T^2} + MU^2\frac{\partial^2 Y}{\partial X^2} + 2MU\frac{\partial^2 Y}{\partial X\partial T} = 0$$ [4.8]

The first two terms in this equation are the flexural rigidity and inertia terms of the linearized Euler–Bernoulli equation. The third term is an inertia term that comes from the presence of the fluid inside the pipe. The fourth term may be understood as a centrifugal term that arises as soon as the beam experiences a local curvature. Finally, the fifth term is generally referred to as a Coriolis force and may be interpreted by

considering a portion of the pipe moving at a constant velocity. Due to the presence of a moving mass inside, a force is exerted on this portion of the pipe.

When damping is to be considered, we may add one or both of these two additional forces to the wave equation

$$\mathcal{D}_f(Y) = C\frac{\partial Y}{\partial T}, \quad \mathcal{D}_s(Y) = E^*I\frac{\partial^5 Y}{\partial X^4 \partial T}. \tag{4.9}$$

The first case is referred to as viscous damping and is generally a consequence of the presence of a viscous fluid around the pipe, while the second is called structural damping and is the consequence of a viscoelastic behavior of the material that constitutes the pipe.

After introducing the characteristic length $\eta = \left(\frac{EI}{MU^2}\right)^{1/2}$ and time $\tau = \left(\frac{(m+M)\eta^4}{EI}\right)^{1/2}$ and rescaling all dimensional quantities using these two parameters, the wave equation takes a form that depends on only one independent parameter β

$$\frac{\partial^2 y}{\partial t^2} + \frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial x^2} + 2\sqrt{\beta}\frac{\partial^2 y}{\partial x \partial t} = 0, \tag{4.10}$$

where β is the mass ratio

$$\beta = \frac{M}{m+M} \in [0,1]. \tag{4.11}$$

Note that the case $\beta = 0$ is stricly equivalent of cantilevered beam with a follower force, referred to as the Beck's column [BEC 52]. In their dimensionless forms, the damping operators now are

$$d_f(y) = c\frac{\partial y}{\partial t}, \quad d_s(y) = \alpha\frac{\partial^5 y}{\partial x^4 \partial t}. \tag{4.12}$$

When considering a finite length system, the non-dimensional length has to be introduced

$$l = \frac{L}{\eta} = UL\sqrt{\frac{M}{EI}}, \tag{4.13}$$

and two boundary conditions have to be specified at each boundary. They are

$$y(x = x_0, t) = y'(x = x_0, t) = 0 , \quad y''(x = x_0, t) = y'''(x = x_0, t) = 0, \text{[4.14]}$$

for a clamped end and a free end, respectively, where the primes $(.)'$ denote the derivation with respect to x and x_0 takes the value 0 or l.

4.4. Effect of damping on the local and global stability of the fluid-conveying pipe

4.4.1. Local stability

The dispersion relation of the undamped fluid-conveying pipe is

$$D(k, \omega) = k^4 - \omega^2 + k^2 + 2\sqrt{\beta}k\omega = 0. \tag{4.15}$$

The frequency associated with a real wave number k then becomes

$$\omega_\pm = \sqrt{\beta}k \pm k\sqrt{\beta + k^2 - 1}. \tag{4.16}$$

For $\beta \in [0, 1[$ and $k \in [0, \sqrt{1 - \beta}]$, frequencies $\omega_\pm$ are complex conjugate and the positive imaginary part of one of them gives rise to a wave with an amplitude exponentially growing in time. For $k > \sqrt{1 - \beta}$, $\omega(k) \in \mathbb{R}$ and the waves are called neutral. In conclusion, the medium is locally unstable, $\forall \beta \in [0, 1[$. Conversely for $\beta \geq 1$, the medium is neutrally stable. However, it has to be noted that $\beta > 1$ has no physical meaning in the present context.

In various studies on the effect of damping on wave propagation, the key role of the wave energy has been evidenced. Although introduced in the context of shear layer waves between two non-miscible fluids [CAI 79], the definition is generic and can be readily used in any mechanical system. Consider a harmonic wave with ω and $k \in \mathbb{R}$ and $D(k, \omega) = 0$. The wave energy is defined as the work done on the system to establish this neutral wave from $t = -\infty$ to $t = 0$, and is expressed as

$$E = -\frac{\omega}{4}\frac{\partial D}{\partial \omega}y_0^2. \tag{4.17}$$

If E is negative, it means that energy has to be removed from the system to establish the wave. The latter is referred to as a *negative energy wave* (NEW) [VON 05].

Now consider that a small amount of viscous damping is added in the system so that the dispersion relation takes the form

$$D_1(k, \omega + \delta\omega) = D(k, \omega + \delta\omega) - ic(\omega + \delta\omega) = 0, \qquad [4.18]$$

where $\delta\omega \ll \omega$ is a small perturbation to the frequency introduced by the damping, which satisfies at order one

$$\delta\omega \left. \frac{\partial D}{\partial \omega} \right|_{(k,\omega)} = ic\omega. \qquad [4.19]$$

We readily deduce from this expression the perturbation on the growth rate $\delta\sigma = \text{Im}(\delta\omega)$

$$\delta\sigma = \frac{c\omega}{\partial D/\partial \omega}. \qquad [4.20]$$

This quantity has the opposite sign of the wave energy E. A NEW is hence destabilized by viscous damping. The same calculation performed with viscoelastic damping gives

$$\delta\sigma = \frac{\alpha k^4 \omega}{\partial D/\partial \omega}, \qquad [4.21]$$

which leads us to the same conclusion.

In the fluid-conveying pipe case, the wave energy is expressed as

$$E_\pm = \frac{1}{2} k^2 \sqrt{k^2 + \beta - 1} \left(\sqrt{k^2 + \beta - 1} \pm \sqrt{\beta} \right), \qquad [4.22]$$

and E_- has negative values in the range $k \in]\sqrt{1 - \beta}, 1[$. Hence, the range of temporally unstable waves becomes $[0, 1[$ when damping is added, whereas it was $k \in [0, \sqrt{1 - \beta}]$ in the conservative case. Damping enlarged the range of unstable wave numbers. Moreover, the system is now temporally unstable for any value of β, when it was for $\beta \in [0, 1[$ in the conservative case.

In Figure 4.1, the ranges of unstable waves in the damped and undamped cases are compared when parameter β, quantifying the Coriolis force, varies from 0 to 1. It can be concluded from the figure that the Coriolis force stabilizes waves, which are, in turn, destabilized by a small amount of damping. The same kind of behavior was observed by Thomson and Tait in discrete systems [THO 79].
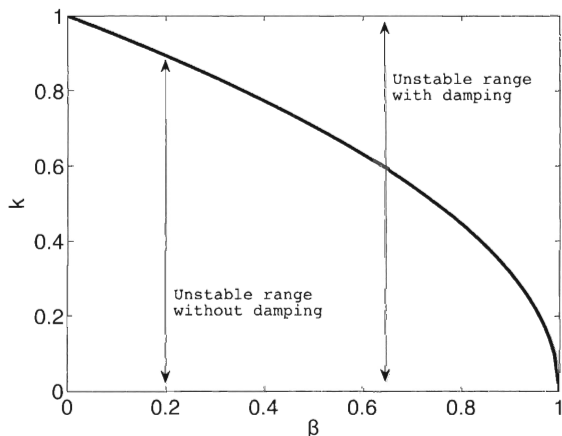
Figure 4.1. *Range of unstable wave numbers as a function of β. This illustrates the fact that the range of wave numbers $\sqrt{1 - \beta}$ is stabilized by the Coriolis force and this range is destabilized when damping is added in the medium*

4.4.2. *Global stability*

Boundary conditions and the finite length parameter l are now introduced. The dimensionless parameter l in equation [4.13] is proportional to both L and U, indicating that it can be seen as a dimensionless length or flow velocity. Although the limit $l = 0$ has no meaning when it is sought as a length, it can be achieved by letting the flow velocity vanish. In the global approach, it is more convenient to use L to rescale the lengths so that the dimensionless wave equation becomes

$$\frac{\partial^2 y}{\partial t^2} + \frac{\partial^4 y}{\partial x^4} + v^2 \frac{\partial^2 y}{\partial x^2} + 2\sqrt{\beta}v \frac{\partial^2 y}{\partial x \partial t} = 0, \tag{4.23}$$

where the length of the dimensionless problem is the unity and with,

$$v = l. \tag{4.24}$$

The Galerkin method presented in section 4.2.2 is used here to obtain the present results. The chosen test functions are the eigenmodes of the pipe without flow. The functions $\psi_n(x)$ are hence the eigenfunctions of equation [4.23] with the same set of boundary conditions and $v = 0$. These eigenfunctions are basically the eigenmodes of a beam and are known and well-documented analytic functions (see, for instance,

[BLE 90]). Equation [4.23] is then projected on each mode ψ_m, leading to N ordinary differential equations for the time variable, which is

$$\ddot{\vec{q}} + 2\sqrt{\beta}v\, C\, \dot{\vec{q}} + (K_d + v^2 K_f)\vec{q} = 0. \qquad\qquad [4.25]$$

The coefficients of the matrices C, K_d and K_f result from the projection of the Coriolis, flexural rigidity and centrifugal operators, respectively. Note that K_d is diagonal, as discussed above. The coefficients of these matrices can be found in [GRE 66]. In the clamped-clamped case, C is skew-symmetric and K_f is symmetric. In the clamped-free case, C and K have both symmetric and skew-symmetric parts. This has consequences on the bifurcation properties of these two systems. The bifurcation types have been studied in such discrete systems. The role of matrix symmetries in the bifurcation is out of the scope of this chapter and can be found in other mathematical studies [KIR 07, KIR 10].

4.4.2.1. *Evolution of eigenfrequencies and bifurcations*

The evolution of the eigenfrequencies when v is increased from 0 is shown in Figure 4.2 with four typical cases: a pipe clamped at both ends, a clamped-free pipe at $\beta = 0$, and a clamped-free pipe at $\beta = 0.2$ and $\beta = 0.95$. These graphs illustrate the typical behaviors of the eigenfrequencies when the flow velocity increase. Different bifurcations are evidenced. In the case of a clamped-clamped pipe, instability always arises through a saddle-node bifurcation. The instability is called static instability, or buckling. In the case of a clamped-free pipe, the bifurcation depends on the value of the mass ratio β. If $\beta = 0$, the dissipation matrix vanishes in equation [4.25] and the instability occurs via a Hopf bifurcation after the merging of two eigenfrequencies on the real axis. In the fluid-elastic community, this instability is referred to as flutter instability, as it results in self-sustained oscillations of the structure once the amplitude of the solution has been saturated by the nonlinear effects. When $\beta \neq 0$, the increase of the flow velocity has a stabilizing effect at first, due to a flow-induced damping through the matrix C: all the eigenfrequencies travel toward the negative imaginary part half plane. When further increasing the flow velocity, one eigenfrequency changes its trajectory and crosses the real axis, giving rise to a flutter instability.

4.4.2.2. *Effect of damping on marginal stability of the clamped-free pipe*

The global stability of the system is now characterized by plotting in the (β, l) plane the marginal stability curve (Figure 4.3(a)) for $c = 0$ (no damping). This curve corresponds to the line in the (β, l) plane where the maximum growth rate $\max_n[\mathrm{Im}(\omega_n)] = 0$. In the same figure, the local stability criterion $\beta = 1$ is plotted. It appears, then, that the long system limit for global instability is the local stability criterion. In Figure 4.3(b), different values of the dimensionless damping c from 0 to 1,000 are considered. The resulting marginal stability curve moves continuously from the undamped limit to a horizontal limit. For $\beta \in [0, 0.2]$, the damping appears to have a stabilizing effect while it has a destabilizing effect for $\beta > 0.2$. Hence, also

for the finite length clamped-free pipe, damping can have a destabilizing effect. While in absence of damping the instability criterion of the finite length tends to the local one when l is increased, no such limit can be observed in the damped case because the damped medium is locally unstable $\forall\beta$. However, the horizontal limit observed at high values of the damping cannot be predicted by a local criterion.
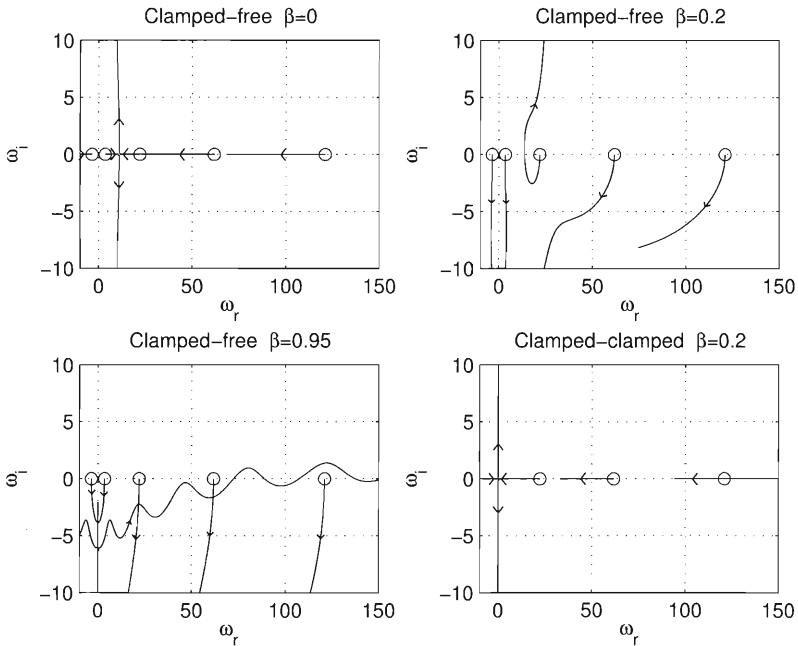
Figure 4.2. *Evolution in the complex plane of the eigenfrequencies of the clamped-clamped and clamped-free pipe when the velocity parameter v is increased from 0, for different values of the mass ratio β*

4.4.2.3. *Kulikovskii's criterion*

We observed in section 4.4.2.2 that the long system limit of the marginal stability curve in the undamped case is the local stability criterion. In the case of the Ginzburg–Landau equation, which is a simplified amplitude equation describing fluid-mechanics systems such as jets, vortices and shear layers, the long system limit for global instability criterion was found to be that of transition from local convective instability to absolute instability [TOB 98, COS 97]. Convective and absolute instabilities distinguish the long time behavior the temporally growing wave packet generated by an impulse forcing on an unstable medium. In the case of convective instability, the growing wave packet is advected by the stream, while in the case of absolute instability, the wave packet grows in place. For the fluid-conveying pipe, the transition between these two kinds of local instabilities occurs at $\beta = 8/9$ [KUL 88].

a)   b)
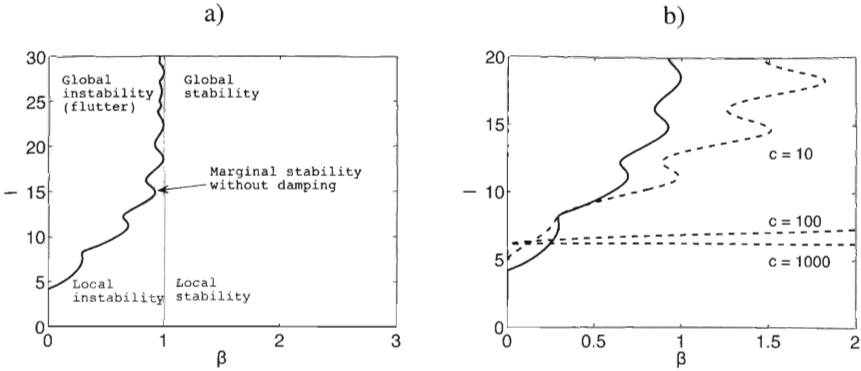
Figure 4.3. *a) Marginal global stability curve of the pipe conveying fluid in the parameter plane* (β, l) *(thick black) compared with the local stability criterion (thin grey); b) marginal global stability curves for increasing viscous damping*

Hence, the long system limit is predicted by a local criterion, which is not unique. A criterion introduced by Kulikovskii in 1966 in the context of electron beam instability and cited in the book by Lifshitz and Pitaevskii [LIF 81] permits us to determine which local criterion should prevail. Kulikovskii showed that as the length of the system tends to infinity, the eigenvalues tend to a continuous line in the complex frequency plane such that the most spatially amplified (or least evanescent) downstream propagating wave is exactly balanced by the least spatially evansescent (or most amplified) upstream propagating wave, which is formally written as:

$$\mathrm{Im}(k^+ - k^-) = 0, \tag{4.26}$$

where k^+ is the downstream wave number that has the minimum imaginary part, and k^- is the upstream wave number that has the maximum imaginary part. It can be shown in the fluid mechanics media mentioned above that in case of convective instability, no frequency of positive imaginary part satisfies this criterion. Conversely, it is possible to satisfy this criterion with unstable frequencies in the fluid-conveying pipe case when convectively unstable. Consequently, in the long system limit, the convectively unstable pipe is unstable while it is not the case for the Ginzburg–Landau equation. To illustrate this phenomenon, the Kulikovskii criterion is shown in Figure 4.4 in the complex plane of frequencies and compared to the locii of eigenfrequencies when the length of the system is increased to large values. The value of $\beta = 0.95$ is chosen so that the medium is convectively unstable. Two different sets of boundary conditions are presented. It seems that although the occurences of bifurcations are different in both systems, the eigenfrequency maps converge to the same limit when $l \to \infty$.
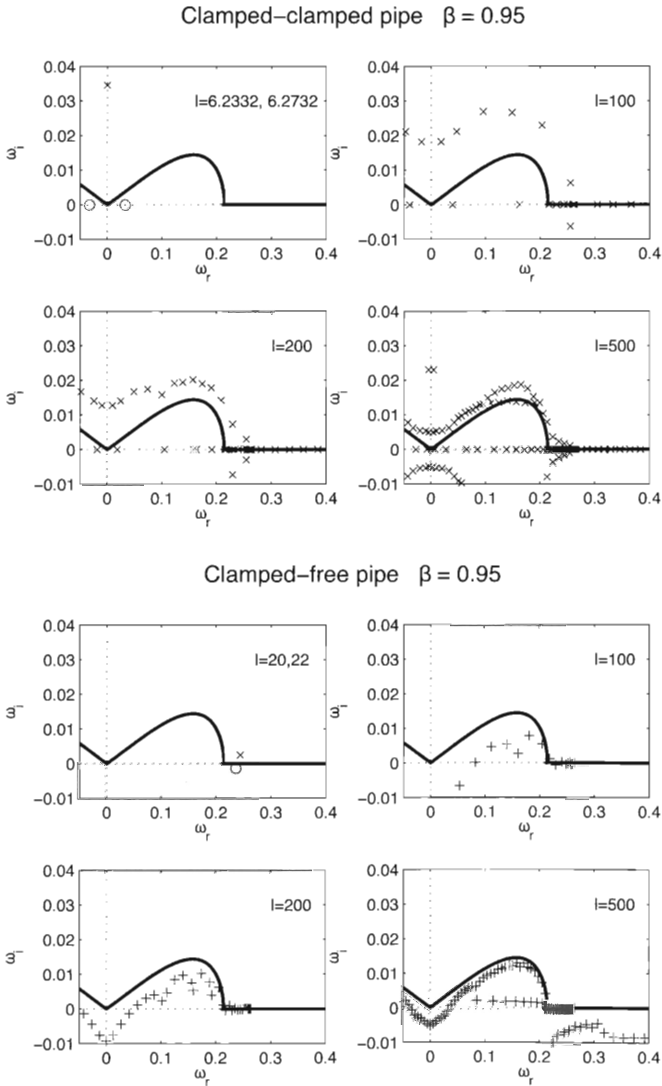
Figure 4.4. *Eigenfrequencies of the clamped-clamped and clamped-free pipe at $\beta = 0.95$ and for different values of the length l, compared to the Kulikovskii criterion. In the upper left plot of each case, the circles and crosses represent the locii of the eigenvalues just below and just above the instability threshold, respectively. These plots illustrate the fact that although the apparition of instability is different between these two sets of boundary conditions, all the eigenfrequencies approach a common limit when the length of the system is increased to infinity*

4.4.2.4. *Length scale criterion*

We discussed the long system limit in the previous section. We now address the opposite case of short systems. In Figure 4.3(a), for the values of the parameters comprised between the local and global stability curves, and below the dashed lines in Figure 4.3(b), the system is globally stable although locally unstable. In this situation, the confinement induced by considering a short system has, consequently, to prevent unstable waves from playing a role in the dynamics. This confinement effect can be quantified and can give an approximate criterion of stability. Let us state that an unstable wave can give rise to an unstable mode of the finite system only if its wavelength is smaller than the length of the system. The smallest unstable wavelengths are

$$\lambda = \frac{2\pi}{\sqrt{1-\beta}} \, , \quad \lambda_d = 2\pi, \qquad\qquad [4.27]$$

in the undamped and damped cases, respectively. Plotted against the marginal stability curves in Figure 4.5, these criteria show a good agreement. The marginal stability curve goes continuously from the length criterion without damping to that with damping.
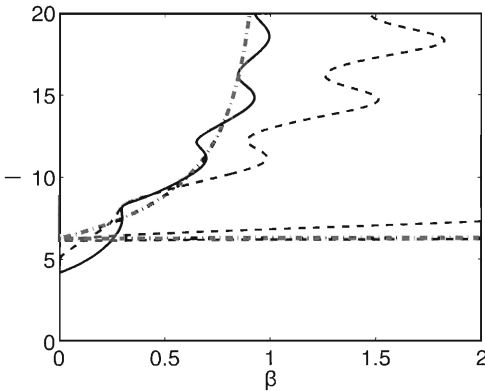
Figure 4.5. *Global stability curves of the pipe conveying fluid in the*
(β, l) plane for increasing values of the viscous damping (solid and dashed
lines), compared with the length criteria defined in equation [4.27]
(dashed-dotted lines)

4.5. Application to energy harvesting

The possibility to take advantage of the energy transfer between a flow and a structure through the flutter phenomenon has recently received significant attention.

Various harvesting systems have been considered theoretically or experimentally, such as the simple Kelvin–Voigt-type dampers [SIN 12], electromagnetic coupling [TAN 09] and piezoelectric coupling [DOA 11, AKC 12]. In this section, we summarize the study of a clamped-free plate equipped with a continuous distribution of piezoelectric patches [DOA 11]. The piezoelectric effect couples deformation in the plate to the electric field and electrical energy may then be harvested at the outlets of a piezoelectric patch. Although more sophisticated circuits can be considered [LEF 06], in the model presented here, the harvested energy is dissipated in a shunting resistance. The dimensionless model presented here takes the form of two coupled wave equations for the vertical displacement of the plate y and the electrical charge displacement per unit length q:

$$\frac{1}{V^2}\left(1+\alpha^2\right)\frac{\partial^4 y}{\partial x^4} + \frac{\partial^2 y}{\partial t^2} - \frac{\alpha}{V}\frac{\partial^2 q}{\partial x^2} = -[p], \qquad [4.28]$$

$$\gamma\frac{\partial q}{\partial t} + q - \frac{\alpha}{V}\frac{\partial^2 w}{\partial x^2} = 0. \qquad [4.29]$$

In [4.28], the first two terms are the rigidity and inertia terms of the linear Euler–Bernoulli beam model, the third term quantifies the momentum exerted on the plate consecutively to the electric field in the piezoelectric patches and the right-hand term is the pressure jump between both sides of the plate. The later can be written as a linear function of the small amplitude displacement y and its effective expression may take different forms depending on the approximations and geometries of the system. For instance, if the typical length of deformation is large compared to the width of the plate, the pressure jump has the same three fluid terms as in the fluid-conveying pipe equation [4.8] [LEM 05]. In the opposing case of an infinite span plate, we obtain the same three terms, scaled by $1/k$, where k is the wave number [CRI 91]. In equation [4.29], the first two terms stand for Ohm's law, while the third term expresses the inverse piezoelectric coupling: a deformation in the piezoelectric material leads to an electrical charge displacement. The parameters of the problem are V, the non-dimensional flow velocity, α, the piezoelectric coupling coefficient, and γ is the ratio between the fluid–solid and electrical characteristic timescales.

The uncoupled medium ($\alpha = 0$) bears temporally unstable waves, as well as neutral waves, some of them being negative energy waves. An analysis similar to that performed in section 4.4.1 shows that the temporal growth rate of neutral waves is perturbed by the following amount when piezoelectric coupling is added:

$$\delta\sigma \simeq \frac{\omega\alpha^2\gamma k^4}{V^2(1+\omega^2\gamma^2)\dfrac{\partial D}{\partial \omega}}. \qquad [4.30]$$

This expression shows that the perturbation of the growth rate has the opposite sign of the wave energy and scales as α^2. Piezoelectric coupling hence destabilizes negative energy waves. Moreover, $\delta\sigma$ presents a maximum when $\gamma = 1/\omega$, i.e. when the time scale of the RC circuit is the same as that of the wave.

In the linear analysis, the conversion efficiency of the system can be defined for an unstable wave as the ratio between the power dissipated in the resistances during one period and the mean of the energy present in the system over this same period.

It is found in [DOA 11] that the maximum efficiency is always maximized for a negative energy wave destabilized by piezoelectric coupling, when $\gamma \sim 1/\omega$. In other words, energy harvesting may destabilize negative energy waves, and conversion efficiency is maximized with these waves.

The finite length system may also be analyzed and similar results to those in the case of the fluid-conveying pipe are found. The system can, indeed, be destabilized or stabilized by piezoelectric coupling. An analysis of the efficiency of the conversion shows that the maximum efficiency exists in the domain of parameters where destabilization by piezoelectric coupling occurs. This happens for long systems, originating from the same confinement effect discussed in the previous section.

4.6. Conclusion

The counterintuitive effect of damping on the stability of the fluid-conveying pipe has been analyzed both in terms of waves propagating in the infinite medium (local approach) and in terms of modes in the finite length system (global approach). The fluid-conveying pipe is a model problem for many fluid-elastic systems where a compliant structure interacts with a flow, such as flags, plates, shells, walls and wings.

It was shown that the Coriolis force has a stabilizing effect on the infinite medium and damping paralyzes this effect. The destabilizing effect of damping was shown to be due to the creation of negative energy waves by the Coriolis term. The finite length system can be stabilized or destabilized by damping. A length scale analysis has shown that destabilization by damping may be due to the destabilization of negative energy waves. In [DOA 02, DOA 10], additional rigidity forces are considered on the pipe, such as elastic foundation or tension. A more rich picture of stability properties is then evidenced.

Extension to energy harvesting systems has been briefly discussed. It was shown that energy harvesting has a similar destabilizing effect on such fluid-elastic systems and that negative energy waves maximize the energy conversion efficiency.

4.7. Bibliography

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