On the Optimality of the Median Cut Spectral Bisection Graph Partitioning Method

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Abstract. Recursive Spectral Bisection (RSB) is a heuristic technique for finding a minimum cut graph bisection. To use this method the second eigenvector of the Laplacian of the graph is computed and from it a bisection is obtained. The most common method is to use the *median* of the components of the second eigenvector to induce a bisection. We prove here that this median cut method is optimal in the sense that the partition vector induced by it is the closest partition vector, in any l_s norm, for $s \geq 1$, to the second eigenvector. Moreover, we prove that the same result also holds for any m-partition, that is a partition into m and $(n-m)$ vertices, when using the mth largest or smallest components of the second eigenvector. ©*1997 Society for Industrial and Applied Mathematics.*

Keywords. graph partitioning, recursive spectral bisection, graph Laplacian, Fiedler vector, parallel computing.

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