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ERRATUM

WELL-POSEDNESS OF THE DRUDE–BORN–FEDOROV MODEL FOR CHIRAL MEDIA

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There is an error in Sec. 4.1.2 devoted to the invertibility of the space operator $a_\beta = I + \beta\,\mathbf{curl}$ from $W = H_0(\operatorname{div} 0, \Omega)$ to W with domain $W \cap H_0^1(\Omega)^3$. More precisely, on pp. 468–469, the problem lies in the equivalence between (4.4) and (4.5). Indeed, to prove that a solution to (4.5) is a solution to (4.4) (as intended in the original text), one can try a *mixed* approach, introducing a Lagrange multiplier, denoted by p, accounting for the divergence constraint satisfied by the fields in W. This yields

$$\boldsymbol{w} + \beta\,\mathbf{curl}\,\boldsymbol{w} = \boldsymbol{\nabla} p \text{ in } \Omega.$$

However, one fails to show that p is zero. Another way is to set problem (4.4) in X_N, thus *relaxing* the constraint $\boldsymbol{u} \cdot \boldsymbol{n}_{|\partial\Omega} = 0$. It fails as well, as one then finds

$$\boldsymbol{w} \cdot \boldsymbol{n} = \boldsymbol{u} \cdot \boldsymbol{n} \text{ on } \partial\Omega.$$

To summarize, the first equation of (4.6) is not satisfied if one uses a Lagrange multiplier, whereas the last equation of (4.6) is not verified if one relaxes the constraint on the normal component of $\boldsymbol{u}$ on the boundary. As a consequence, the invertibility of a_β is not established.

One can even go further by showing that this operator is *not invertible*. We know that $\ker(a_\beta) = \{\mathbf{0}\}$ (cf. p. 468) and that the range of a_β, $R(a_\beta)$, is closed (cf.

p. 469). In addition, one can show that $R(a_\beta)$ is a strict subset of W (see Ref. 1), even with domain X_N.

To recover an existence result for the time-dependent problem of interest ($\boldsymbol{E}$ and $\boldsymbol{H}$ governed by Eqs. (3.1)), a solution is to relax the constraint on the divergence. For that, consider the operator $a_\beta^r = I + \beta\,\mathbf{curl}$ from $H(\mathrm{div}, \Omega)$ to itself, with domain $H_0^1(\Omega)^3$. Then, one looks for *invariant subspaces* $S(\subset R(a_\beta^r))$ such that $(a_\beta^r)^{-1}S \subset S$. One can trivially check that $S_{\min}$, defined by

$$S_{\min} = \boldsymbol{\nabla} H_0^2(\Omega),$$

is such an invariant subspace. Also, one can prove that the largest invariant subspace $S_{\max}$ is a closed subspace of $R(a_\beta^r)$ (see Ref. 1). An open question is whether or not it coincides with $S_{\min}$.

Theorems which deal with the existence of a solution to the time-dependent Maxwell system must be modified by assuming that the initial data also satisfy

$$\boldsymbol{E}_0 \in (a_\beta^r)^{-1}(S_{\max}), \quad \boldsymbol{H}_0 \in \boldsymbol{H}_{\boldsymbol{J}}(0) + (a_\beta^r)^{-1}(S_{\max}).$$

All subsequent results remain valid, to the possible exception of the non-observability of the Maxwell system in chiral media (see Sec. 7.3) which is tied to the pending question on the determination of the largest invariant subspace $S_{\max}$.

Reference

1. S. Nicaise, Private communication.