Postdoctoral project

Numerical study of hybrid resonances in cold plasma

Context

Time-harmonic wave propagation in cold magnetized plasmas is described by the Maxwell equations with a frequency-dependent spatially-varying tensor of dielectric permittivity. Interaction of the exterior magnetic field with media may lead to plasma heating. Mathematically, plasma heating is related to the presence of the singularities in the electric field (which is no longer L^2, unlike in the case of electromagnetic wave propagation in dielectrics). Rigorous mathematical theory of hybrid resonances in plasmas was initiated by Bruno Després and his co-workers, see [1, 2, 3, 4]. They have shown that the corresponding singularities can be studied by examining the limiting absorption solution to a certain boundary-value problem. In the simplest setting, this problem is described by a second-order PDE with a sign-changing coefficient in the principal part of the operator. Unlike in the case of plasmonic applications, this coefficient is typically smooth, and the corresponding problem is well-posed in a non-standard functional framework. This naturally leads to questions of the choice of the appropriate variational formulation, as well as of the design of the numerical methods to solve such problems.

While some of these questions had been answered in recent years, the mathematical and numerical analysis of the non-simplified two-dimensional Maxwell equations, as well as full three-dimensional Maxwell system is still in its infancy stage (see e.g. [2] for a related study).

Postdoctoral project

This **2-year** postdoctoral research subject is a continuation of the PhD thesis of Étienne Peillon [5], see also [6]. It is dedicated to the design and analysis of alternative variational formulations, and their extension to the case of the Maxwell equations.

We aim at tackling the following questions:

- design of an alternative mixed formulation for the solution of the scalar 2D problem $\operatorname{div}(\alpha\nabla u) = 0$ with α changing its sign across a regular interface, in a well-adapted functional framework. The key idea is a re-interpretation of this problem as a special transmission problem, with well-chosen trace operators, see [7].
- implementation of this formulation, its mathematical and numerical analysis.

- extension to the realistic case of 2D Maxwell equations, where the tensor of dielectric permittivity is no longer diagonal.

Keywords: plasma resonance, singular solutions, Maxwell equations

Desired skills: strong programming skills, numerical analysis and computational mathematics, analysis of PDEs

Supervision: The postdoctoral research subject will be supervised by Patrick Ciarlet and Maryna Kachanovska. The succesful candidate will be based at ENSTA in Palaiseau.

References

[1] Bruno Després, Lise-Marie Imbert-Gérard, and Ricardo Weder. Hybrid resonance of Maxwell's equations in slab geometry. *J. Math. Pures Appl. (9)*, 101(5):623–659, 2014.

[2] M. Campos Pinto and B. Després. Constructive formulations of resonant Maxwell's equations. *SIAM J. Math. Anal.*, 49(5):3637–3670, 2017.

[3] Anouk Nicolopoulos, Martin Campos Pinto, Bruno Després, and Patrick Ciarlet, Jr. Degenerate elliptic equations for resonant wave problems. *IMA J. Appl. Math.*, 85(1):132–159, 2020.

[4] Bruno Després, Lise-Marie Imbert-Gérard, and Olivier Lafitte. Solutions to the cold plasma model at resonances. *J. Éc. polytech. Math.*, 4:177–222, 2017.

[5] Étienne Peillon. *Modélisation et analyse des équations de Maxwell, avec changement de signe.* PhD thesis, 2024.

[6] Patrick Ciarlet, Maryna Kachanovska, and Étienne Peillon. Study of a degenerate non-elliptic equation to model plasma heating. *ESAIM Math. Model. Numer. Anal.*, 58(5):1785–1821, 2024.

[7] Patrick Ciarlet, Maryna Kachanovska, and Étienne Peillon. Limiting absorption principle for a degenerate equation with a smooth sign-changing coefficient. In preparation.