Mathematical justification and analysis of the hyperbolic metamaterial models

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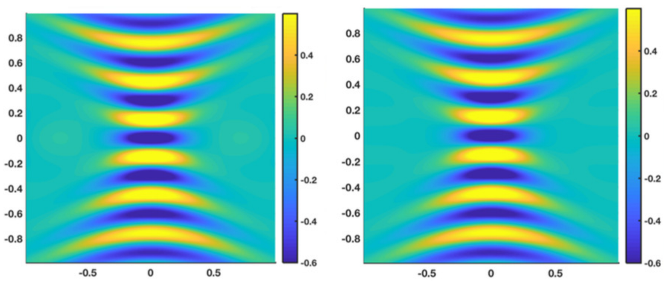
Context: Hyperbolic metamaterials are artificially engineered anisotropic materials which exhibit some unusual properties, such as negative refraction and backward wave propagation. Applications include: enhanced particle absorption, emission, and collection, e.g. for sensors and antennas; devices with tunable optical properties; rogue wave generation, ... The respective phenomena are described by Maxwell's equations with currents, coupled with ODEs for currents. A particularity of hyperbolic metamaterials lies in the fact that in the frequency domain the respective problem is hyperbolic, at least for a range of frequencies. This non-standard behaviour poses exciting mathematical questions, both from the analysis (well-posedness, regularity of the solution) and numerical modelling point of views.



Limiting amplitude vs. limiting absorption

PhD thesis proposal: We propose to investigate one of the simplest models of this type, namely, the 3D Maxwell's equations describing a cold plasma in a strong magnetic field. The PhD thesis will be split into two parts.

The first part will be dedicated to the investigation of the 3D model in free space, namely, to the extension of the results already obtained for its 2D counterpart. This extension is not straightforward, due to the vector nature of the Maxwell's equations. In particular, we aim at obtaining the well-posedness result (and thus, the respective radiation condition), results about the regularity of the solution, as well as limiting amplitude and limiting absorption principles.

The subject of the second part could be altered depending on the interests of the candidate. More precisely, the more theory-inclined candidate will address the above questions for the exterior boundary-value problems (at least in 2D). The main difficulty here lies in the fact that such problems can be ill-posed, depending on the geometry of the scatterer. The more numerical analysis-inclined candidate will address the problem of the numerical approximation of such models, in particular, construction of efficient numerical methods for their resolution. The main difficulty lies in a lower (compared to the elliptic problems) regularity of the solutions of the respective models. Techniques exploiting the knowledge about how the singularities propagate will be investigated.

Requirements: We are looking for a candidate with a strong background in numerical analysis for PDEs. Skills in programming and analysis of PDEs are desired (but not necessary).

Keywords: Maxwell equations, analysis of PDEs, numerical analysis, numerical implementation

Supervision: Patrick Ciarlet and Maryna Kachanovska. The succesful candidate will be based at ENSTA in Palaiseau.