Continuous Galerkin methods for solving electromagnetic eigenvalue problems

Annalisa Buffa, Patrick Ciarlet, Erell Jamelot

Patrick.Ciarlet@ensta.fr

POEMS, UMR 2706 CNRS-ENSTA-INRIA, Paris, France

Time-harmonic Maxwell equations

- *In a bounded domain Ω.*
  Find $(\mathcal{E}, \mathcal{B}, \omega)$ such that

$$
\begin{cases}
\imath\omega\mathcal{E} - c^2 \mathbf{curl}\,\mathcal{B} = 0 \text{ in } \Omega\ ; \\
\imath\omega\mathcal{B} + \mathbf{curl}\,\mathcal{E} = 0 \text{ in } \Omega\ ; \\
\operatorname{div}\mathcal{E} = 0 \text{ in } \Omega\ ; \\
\operatorname{div}\mathcal{B} = 0 \text{ in } \Omega\ ; \\
\mathcal{E} \times \mathbf{n} = 0 \text{ on } \partial\Omega\ ; \\
\mathcal{B} \cdot \mathbf{n} = 0 \text{ on } \partial\Omega.
\end{cases}
$$

$\Big($ $\partial\Omega$ is the boundary, $\mathbf{n}$ is the unit outward normal to $\partial\Omega$. $\Big)$

- Goal: compute the EM eigenmodes in a resonator cavity, bounded by a perfect conductor, either polyhedral (3D) or polygonal (2D).

Time-harmonic Maxwell equations (2)

One of the two fields can be eliminated...

- Equivalent system: Find $(\mathcal{E}, \omega)$ such that

$$(PE) \left\{ \begin{array}{l} c^2 \mathbf{curl}\,\mathbf{curl}\,\mathcal{E} = \omega^2 \mathcal{E} \text{ in } \Omega\ ; \\ \operatorname{div} \mathcal{E} = 0 \text{ in } \Omega\ ; \\ \mathcal{E} \times \mathbf{n} = 0 \text{ on } \partial\Omega. \end{array} \right.$$

- Which *functional space* to measure the electric field?

  First choice:

$$\mathcal{H}_0(\mathbf{curl}\,, \Omega) := \{\mathcal{F} \in L^2(\Omega)^3 \,|\, \mathbf{curl}\,\mathcal{F} \in L^2(\Omega)^3,\ \mathcal{F} \times \mathbf{n}_{|\partial\Omega} = 0\}\,.$$

  cf. [Kikuchi'87/'89], [Demkowicz et al'9x], [Boffi et al'9x/'0x]...

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- Which *functional space* to measure the electric field?

  Second choice:

$$\mathcal{X} := \{\mathcal{F} \in \mathcal{H}_0(\mathbf{curl}\,, \Omega) \,|\, \operatorname{div} \mathcal{F} \in L^2(\Omega)\} \,.$$

  Ok in a convex domain Ω.

  cf. [Assous-Degond-Heintzé-Raviart-Segré'93].

  OK in a 2D or 2D1/2 non-convex domain Ω (Singular Complement Method).

  cf. [Assous-Jr et al'98/'00/'03], [Bonnet-Hazard-Lohrengel'99/'02]...

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- Which *functional space* to measure the electric field?

  Third choice:

$$\mathcal{X}_\gamma := \{\mathcal{F} \in \mathcal{H}_0(\mathbf{curl}\,, \Omega) \,|\, \operatorname{div} \mathcal{F} \in L^2_\gamma(\Omega)\}\,.$$

$$\Big(L^2_\gamma(\Omega) := \{v \in L^2_{\mathrm{loc}}(\Omega) \,|\, w_\gamma\, v \in L^2(\Omega)\}\,,\ ||v||_{0,\gamma} := ||w_\gamma\, v||_0.$$

  The weight w_γ is a function of the distance r to the reentrant edges:

$$w_\gamma(r) = (r/r_{\max})^\gamma,$$

  with a suitable $\gamma \in]\gamma_{min}, 1[$, $0 < \gamma_{min} < \frac{1}{2}$, cf. [Costabel-Dauge'02]. $\Big)$

  Scalar product: $(u, v)_{\mathcal{X}_\gamma} := (\mathbf{curl}\, u, \mathbf{curl}\, v)_0 + (\operatorname{div} u, \operatorname{div} v)_{0,\gamma}$.

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  Our choice from now on...

Variational Formulations

- Set $\lambda = \omega^2/c^2$ and $\mathcal{K}_\gamma := \{\mathcal{F} \in \mathcal{X}_\gamma \,|\, \mathrm{div}\,\mathcal{F} = 0\}$.
  An equivalent variational formulation of (PE) is
  Find $(\mathcal{E}, \lambda) \in \mathcal{K}_\gamma \times \mathbb{R}^+$ such that

$$(\mathbf{curl}\,\mathcal{E}, \mathbf{curl}\,\mathcal{F})_0 = \lambda(\mathcal{E}, \mathcal{F})_0, \ \forall \mathcal{F} \in \mathcal{K}_\gamma.$$

- How can one take into account the divergence-free constraint?

  Costabel and Dauge's choice [Costabel-Dauge'02]: parameterized eigenproblem
  Find $(\mathcal{E}_s, \lambda_s) \in \mathcal{X}_\gamma \times \mathbb{R}^+$ such that

$$(\mathbf{curl}\,\mathcal{E}_s, \mathbf{curl}\,\mathcal{F})_0 + s\,(\mathrm{div}\,\mathcal{E}_s, \mathrm{div}\,\mathcal{F})_{0,\gamma} = \lambda_s(\mathcal{E}_s, \mathcal{F})_0 \ \forall \mathcal{F} \in \mathcal{X}_\gamma,$$

  ($s > 0$ is a parameter.)

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- How can one take into account the divergence-free constraint?

  Our choice [Jr'05], cf. MAFELAP'03: mixed eigenproblem
  Find $(\mathcal{E}, p, \lambda) \in \mathcal{X}_\gamma \times L^2_\gamma(\Omega) \times \mathbb{R}^+$ such that

$$\begin{cases} (\mathcal{E}, \mathcal{F})_{\mathcal{X}_\gamma} + (p, \mathrm{div}\,\mathcal{F})_{0,\gamma} = \lambda(\mathcal{E}, \mathcal{F})_0 \ \forall \mathcal{F} \in \mathcal{X}_\gamma \\ (q, \mathrm{div}\,\mathcal{E})_{0,\gamma} = 0, \ \forall q \in L^2_\gamma(\Omega). \end{cases}$$

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Abstract theory

- A few spaces, forms, etc.
  - V and Q two Hilbert spaces ;
  - a a bilinear, continuous, symmetric, positive, semidefinite form on $V \times V$;
  - b a bilinear, continuous form on $V \times Q$;
  - f an element of V'.
  - L a third Hilbert space: $V \subset L$, V dense in L, and $L' \equiv L$ (the ’pivot’ space).

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- Introduce the mixed problem

$$(MP) \begin{cases} a(u,v) + b(v,p) = \langle f, v \rangle, \ \forall v \in V \\ b(u,q) = 0, \ \forall q \in Q. \end{cases}$$

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  Consider the operator T : $V' \to V$, with $u = \mathsf{T} f$.

  $\Big($ Its restriction from L to V is still denoted by T. $\Big)$

- The eigenproblem to be solved reads

  Find $(u, \lambda) \in V \times \mathbb{R}$ such that

$$\lambda \mathsf{T} u = u.$$

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Abstract theory (2)

- Discretization...
  - $V_h \subset V$;
  - $Q_h \subset Q$;
  - The *discrete kernel* $\mathbb{K}_h := \{v_h \in V_h \ : \ b(v_h, q_h) = 0, \ \forall q_h \in Q_h\}$;
- The discretized eigenproblem reads
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- For T *compact and self-adjoint*, uniform convergence of T_h to T in $\mathcal{L}(L, V)$ implies convergence of eigenvectors and eigenvalues...
  $\Big($ The convergence rate is governed by $r_0(h) := |||\mathsf{T} - \mathsf{T}_h|||$. $\Big)$

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- Four requirements [Boffi-Brezzi-Gastaldi'97]:
  - T compact and self-adjoint ;

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    $Q_0 := \{q \in Q \ : \ \exists v \in V$ s.t. (v, q) solution to (MP) for some $f \in L\}$.

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    $V_0 := \{v \in V \ : \ \exists q \in Q$ s.t. (v, q) solution to (MP) for some $f \in L\}$.

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Abstract theory (3)

[Boffi-Brezzi-Gastaldi'97] continued...

- The Weak Approximability of Q_0:
  $\exists r_1 \; : \; \mathbb{R}^+ \rightarrow \mathbb{R}^+$, such that $\lim_{h\rightarrow 0^+} r_1(h) = 0$ and

$$\sup_{v_h \in \mathbb{K}_h} \frac{b(v_h, q_0)}{\|v_h\|_V} \leq r_1(h) \|q_0\|_{Q_0}, \; \forall q_0 \in Q_0.$$

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- The Strong Approximability of V_0:
  $\exists r_2 \ : \ \mathbb{R}^+ \to \mathbb{R}^+$, such that $\lim_{h\to 0^+} r_2(h) = 0$ and

$$\forall v_0 \in V_0, \ \exists v^I \in \mathbb{K}_h \text{ s.t. } \|v_0 - v^I\|_V \leq r_2(h) \|v_0\|_{V_0}.$$

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- Theorem: provided the *four requirements* hold, one has

$$r_0(h) \leq C\,(r_1(h) + r_2(h)).$$

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Discretization and convergence results

In our case...

$$V = \mathcal{X}_\gamma\,;\, Q = L^2_\gamma(\Omega)\,;\, L = L^2(\Omega)^3\,;$$

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Error estimates can be *improved* with the use of graded meshes.

Numerical experiments

On a 'practical' example, taken from Monique Dauge's benchmark.

- 2D, L-shaped, domain, straight sides, corners in (0,0), (1,0), (1,1), (-1,1), (-1,-1), (0,-1).
- First five eigenvalues (with repetition), up to four digits:
  - $\lambda_1 = 1.476$, eigenmode has the strong unbounded singularity ;
  - $\lambda_2 = 3.534$;
  - $\lambda_3 = 9.870$;
  - $\lambda_4 = 9.870$;
  - $\lambda_5 = 11.39$.
- The weight is implemented with $\gamma = 0.95$.
- Experiments:
  - on *uniform meshes* ;
  - on *graded meshes* ;
  - without any weight on the divergence of the electric field.

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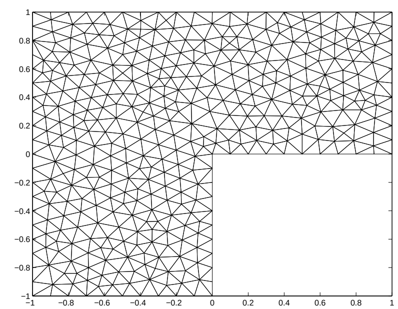
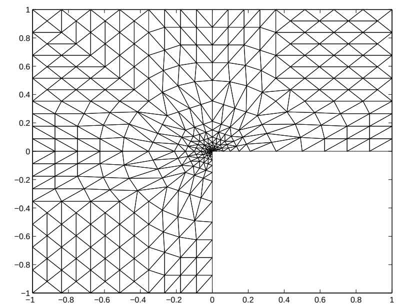
Uniform meshes



- Three meshes with
  - 738, 2952 and 11808 triangles ;
  - 410, 1557 and 6065 vertices ;
- Results:

$mesh$	$\lambda_{1,h}$	$\lambda_{2,h}$	$\lambda_{3,h}$	$\lambda_{4,h}$	$\lambda_{5,h}$
$uniform1$	2.162	3.536	9.871	9.871	11.39
$uniform2$	2.092	3.535	9.870	9.870	11.39
$uniform3$	1.963	3.534	9.870	9.870	11.39

Graded meshes



- Three meshes (courtesy of Beate Jung) with
  - 648, 2664 and 10728 triangles ;
  - 362, 1410 and 5522 vertices ;
- Results:

$mesh$	$\lambda_{1,h}$	$\lambda_{2,h}$	$\lambda_{3,h}$	$\lambda_{4,h}$	$\lambda_{5,h}$
$graded1$	1.742	3.534	9.872	9.872	11.39
$graded2$	1.484	3.534	9.764	9.870	11.39
$graded3$	1.478	3.534	9.801	9.870	11.39

No weight

- No weight ($\gamma = 0$):
  the electric field is measured with the usual L^2-norm for its divergence.
- No Singular Complement.
- Same three graded meshes...

- Results:

$mesh$	$\lambda_{1,h}$	$\lambda_{2,h}$	$\lambda_{3,h}$	$\lambda_{4,h}$	$\lambda_{5,h}$
$graded1$	3.553	6.073	9.872	9.872	11.40
$graded2$	3.535	6.068	9.870	9.870	11.39
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- Only the 'smooth' eigenmodes are captured numerically, as expected!
- One solves the mixed eigenproblem in $\mathcal{X} \cap H^1(\Omega)^3 \times L^2(\Omega)$.
  $\Big($ New eigenmodes appear... $\Big)$

Conclusion/Perspectives

- One can compute numerically EM eigenmodes:
  - eigenproblem expressed as a *mixed* Variational Formulation ,
  - discretized with *continuous* Galerkin methods.
- The mixed form is much simpler to solve numerically than the *parameterized* form.

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  - Extruded L-shaped domain ;
  - Fichera corner ;
  - More realistic geometries...
- Other uses of the mixed VF and continuous Galerkin methods:
  - Time-dependent Maxwell equations ([Jamelot'05], [Jr-Jamelot'06]);
  - Vlasov-Maxwell system of equations.

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