Filtrage Particulaire appliqué à la localisation

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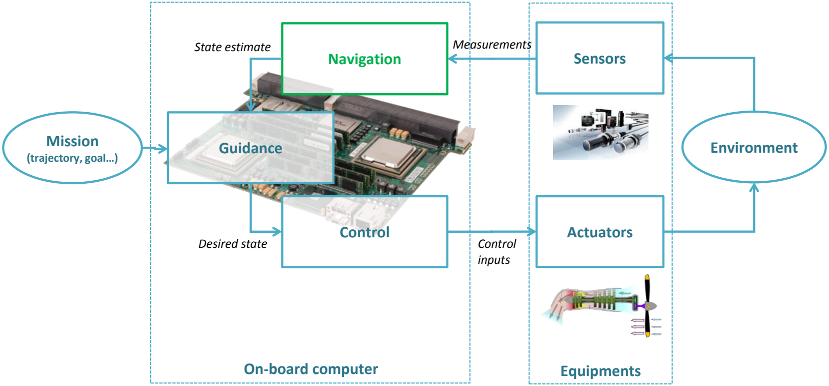
Motivation

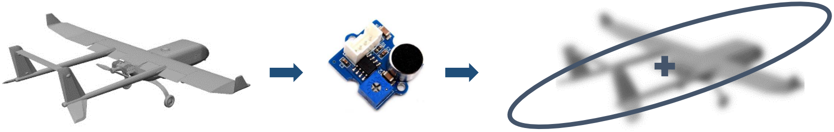
Autonomous systems need to perform accurate state estimation (e.g., self localization, objective assessment...)

... with limited embedded calculation capabilities.

Guidance, Navigation and Control (GNC)



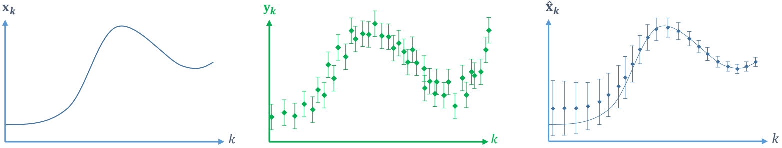
State estimation for navigation



$\mathbf{x}_k =$ position, velocity...

$\mathbf{y}_k =$ measurements

$\hat{\mathbf{x}}_k =$ estimated position, velocity...



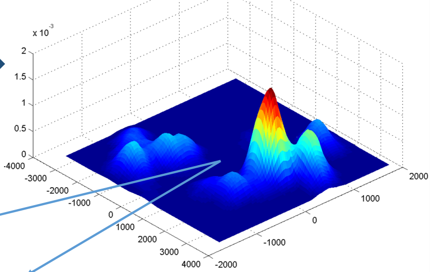
State estimation consists in retrieving the vehicle's state from noisy and incomplete measurements and uncertain evolution model.

Different ways to model the state density

$x_k =$ position, velocity...

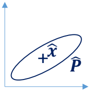
$y_k =$ measurement

Density: $p(x_k|y_1, \dots, y_k)$

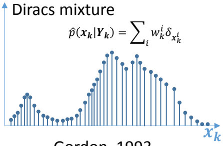


Kalman Filters

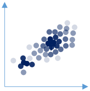
Particle Filters



Kalman, 1960



Gordon, 1993



What we know

Theoretical state evolution (dynamical model):

$$\begin{aligned} \dot{\mathbf{x}} &= F(\mathbf{x}, \mathbf{u}) \\ \mathbf{x}_k &= f(\mathbf{x}_{k-1}, \mathbf{u}_k) \end{aligned} \tag{1}$$

Theoretical observation equation (sensor model):

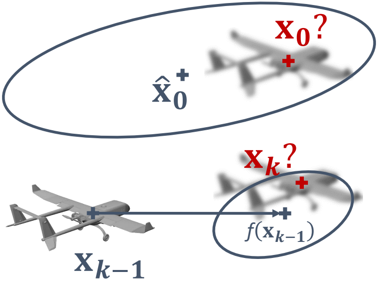
$$\mathbf{y}_k = h(\mathbf{x}_k) \tag{2}$$

However, these equations are not totally representative of the actual system, e.g.:

- unexpected wind, friction, unmodeled dynamics...
- sensor noise, unmodeled disturbances...

What we **don't** know (uncertainties)

- Initial state uncertainty

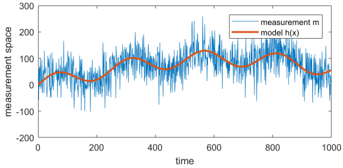


- Process noise (dynamics)

$$\mathbf{x}_k = f(\mathbf{x}_{k-1}, \mathbf{u}_k) + \mathbf{w}_k$$

- Measurement noise (and potentially some bias)

$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{v}_k$$



A way to model uncertainties: probability distribution functions

- State distribution:

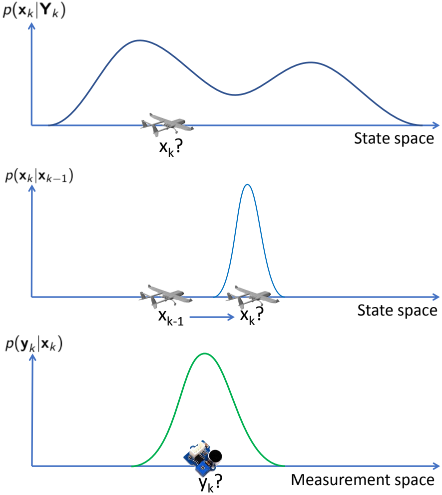
$$p(\mathbf{x}_k|\mathbf{y}_1,\ldots,\mathbf{y}_k) \triangleq p(\mathbf{x}_k|\mathbf{Y}_k)$$

- Process noise distribution

$$p(\mathbf{x}_k|\mathbf{x}_{k-1})$$

- Measurement distribution

$$p(\mathbf{y}_k|\mathbf{x}_k)$$



Optimal filter equations (Bayesian filtering)

State density propagation (dynamics, Chapman-Kolmogorov equation):

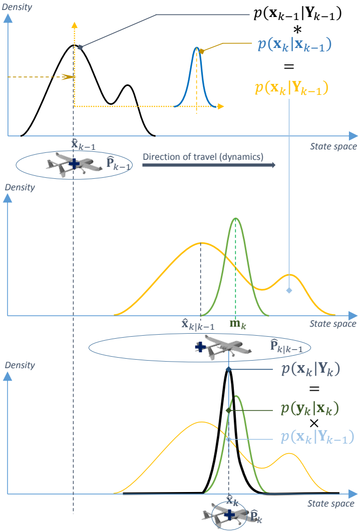
$$p(\mathbf{x}_k|\mathbf{Y}_{k-1}) = \int p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{Y}_{k-1})\mathrm{d}\mathbf{x}_{k-1} \tag{3}$$

State density correction/update (measurements, Bayes rule):

$$p(\mathbf{x}_k|\mathbf{Y}_k) = \frac{p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{Y}_{k-1})}{\int p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{Y}_{k-1})\mathrm{d}\mathbf{x}_k} \tag{4}$$

(Measurements $\mathbf{y}_i$ and $\mathbf{y}_j$ ($\forall\ i \neq j$) are assumed to be statistically independent)

Optimal filter equations (Bayesian filtering)



(a) Prediction

Convolution of the **prior conditional density** with the state transition density. The transition density accounts for the deterministic dynamics f_k and its uncertainty (process noise $\mathbf{w}_k$).

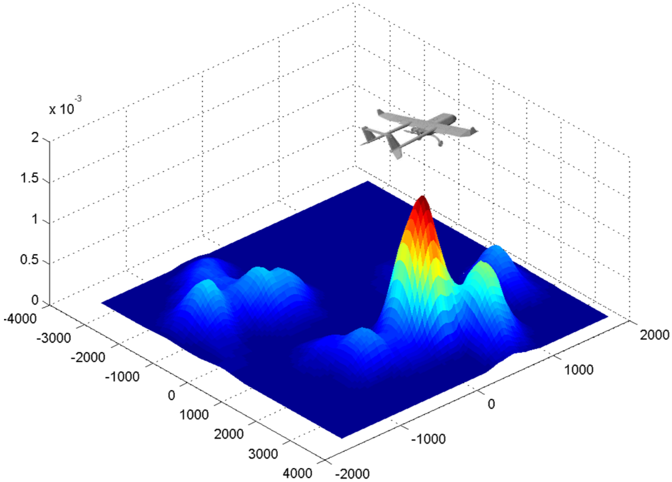
(b) Predicted density, new measurement

Step (a) results in the **predicted conditional density**, whose support is usually larger than the prior density.
A measurement $\mathbf{y}_k$ is now available. It will introduce information in the estimation.

(c) Correction

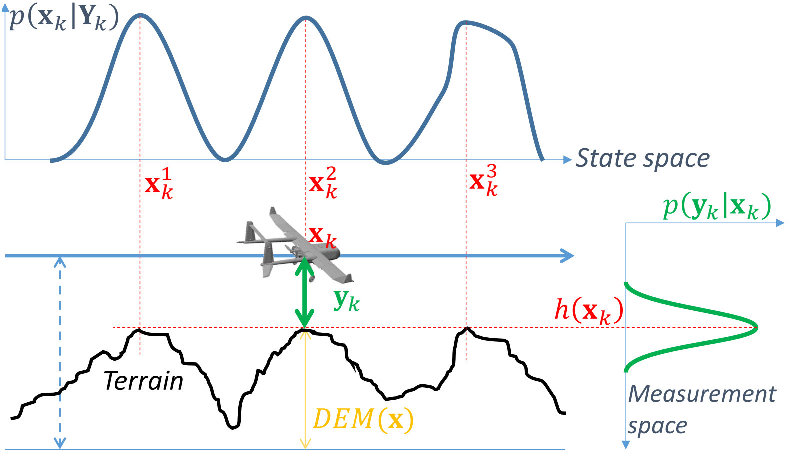
The predicted density is multiplied with the measurement density, leading to the **posterior conditional density**.
Its support is usually smaller than the predicted density. It yields a refined estimate $\hat{\mathbf{x}}_k$ and covariance $\hat{\mathbf{P}}_k$.
Then, one can iterate back to step (a) for further time-steps.

Multimodal state density (example: Terrain Aided Navigation)



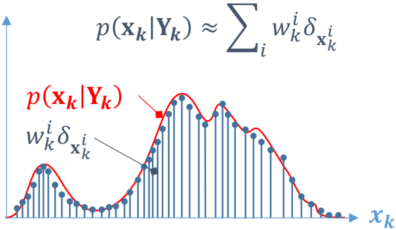
State density $p(\mathbf{x}_k|\mathbf{Y}_k)$

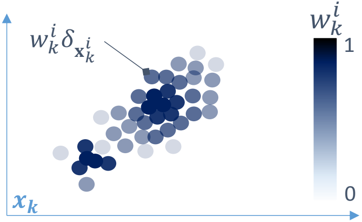
Multimodal state density (example: Terrain Aided Navigation)



Non injective observation equation (e.g. terrain elevation) may yield several admissible states (peaks in posterior state density).

State density approximation





$$p(\mathbf{x}_k|\mathbf{Y}_k) \approx \sum_{i=1}^{N} w_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i) \tag{5}$$

$\mathbf{x}_k$: actual state
$\mathbf{x}_k^i$: particle i state
w_k^i : particle i weight $(\sum_i w_k^i = 1)$

Particle Filter (theory): prediction and correction

State density propagation (dynamics, Chapman-Kolmogorov equation):

$$\begin{array}{ccc} p(\mathbf{x}_k|\mathbf{Y}_{k-1}) & = \int & p(\mathbf{x}_k|\mathbf{x}_{k-1})p(\mathbf{x}_{k-1}|\mathbf{Y}_{k-1})\mathrm{d}\mathbf{x}_{k-1} \\ \sum_{i=1}^N w_{k-1}^i \delta(\mathbf{x}_k - \mathbf{x}_k^i) & \leftarrow & \sum_{i=1}^N w_{k-1}^i \delta(\mathbf{x}_k - (f(\mathbf{x}_{k-1}^i, \mathbf{u}_k) + \mathbf{v}_k^i)) \end{array} \tag{6}$$

State density correction/update (measurements, Bayes rule):

$$\begin{array}{ccc} p(\mathbf{x}_k|\mathbf{Y}_k) & \propto & p(\mathbf{y}_k|\mathbf{x}_k)p(\mathbf{x}_k|\mathbf{Y}_{k-1}) \\ \sum_{i=1}^N w_k^i \delta(\mathbf{x}_k - \mathbf{x}_k^i) & \leftarrow & \sum_{i=1}^N w_{k-1}^i p(\mathbf{y}_k|\mathbf{x}_k^i)\delta(\mathbf{x}_k - \mathbf{x}_k^i) \end{array} \tag{7}$$

Particle Filter (in practice): prediction and correction

State density propagation (dynamics, Chapman-Kolmogorov equation):

$$\mathbf{x}_k^i = f(\mathbf{x}_{k-1}^i, \mathbf{u}_k) + \mathbf{w}_k^i \tag{8}$$

$\mathbf{w}_k^i \ \forall i \in [1, N]$: random sample of process noise

State density correction/update (measurements, Bayes rule):

$$w_k^i \propto w_{k-1}^i p(\mathbf{y}_k | \mathbf{x}_k^i) \tag{9}$$

Particle Filter: state estimation

Maximum *a posteriori* (hard to compute in the general case):

$$\widehat{\mathbf{x}}_k = \underset{\mathbf{x}_k}{\operatorname{argmax}}\, p(\mathbf{x}_k|\mathbf{Y}_k) \tag{10}$$

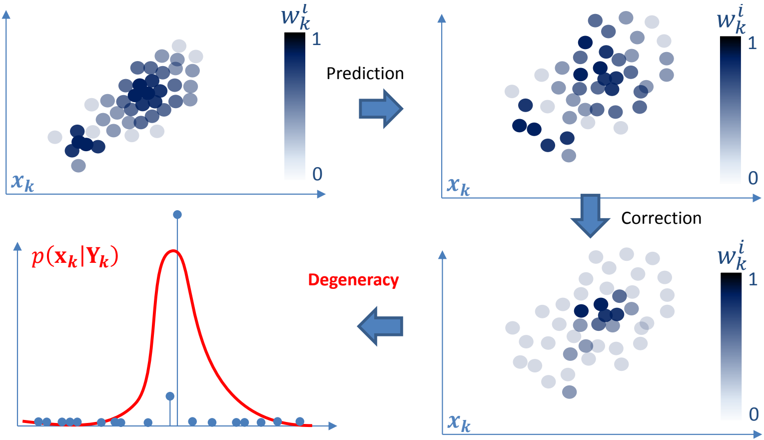
Least square approximation:

$$\widehat{\mathbf{x}}_k = \sum_{i=1}^{N} w_k^i \mathbf{x}_k^i \tag{11}$$

Empirical covariance:

$$\widehat{\mathbf{P}}_k = \sum_{i=1}^{N} w_k^i (\mathbf{x}_k^i - \widehat{\mathbf{x}}_k)(\mathbf{x}_k^i - \widehat{\mathbf{x}}_k)^T \tag{12}$$

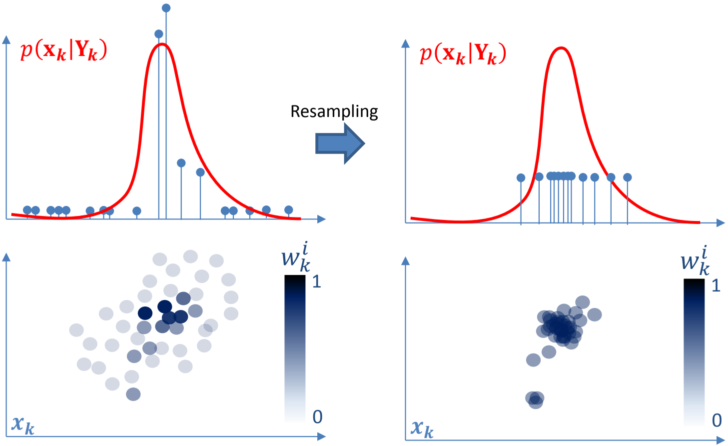
Prediction, correction, and degeneracy phenomenon



After a number of [prediction-correction] cycles, a majority of particle weights will tend to 0 while a small number (or only one) will tend to 1:

That is the weights degeneracy phenomenon.

Resampling



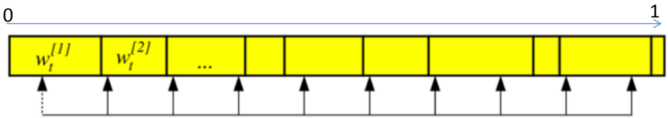
The resampling step aims to duplicate strong-weighted particles to keep an appropriate description of the state density **when degeneracy is about to occur**. Low-weighted particles are destroyed to keep N unchanged.

Multinomial Resampling

The most commonly used resampling technique is Multinomial Resampling:

Input: particle weights $\{w^i\}_{i\in[1,N]}$
Output: number of new instances per particles $\{n^i\}_{i\in[1,N]}$

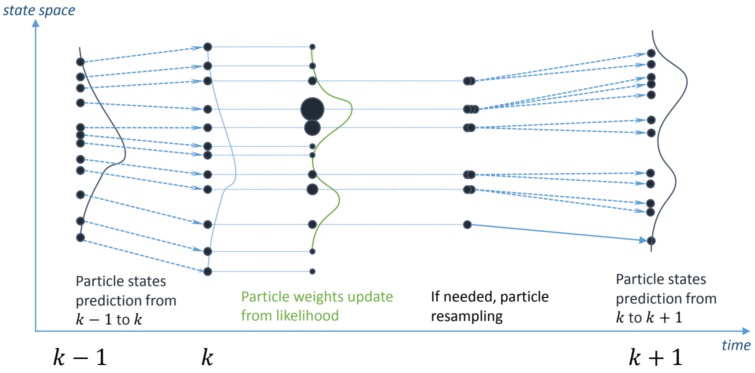
Initialise the duplication counters to $n^i = 0\ \forall i \in [1, N]$
for $i = 1$ to N **do**
  Draw $u^i \sim \mathcal{U}_{[0,1]}$
  Find $j \in [1, N]$ such that $u^i \in \left]\sum_{l=1}^{j-1} w^l, \sum_{l=1}^{j} w^l\right]$
  Count $n^j = n^j + 1$
end for
Return $n^i\ \forall i \in [1, N]$



for $i = 1$ to N **do**
Draw $u^i \sim \mathcal{U}_{[0,1]}$

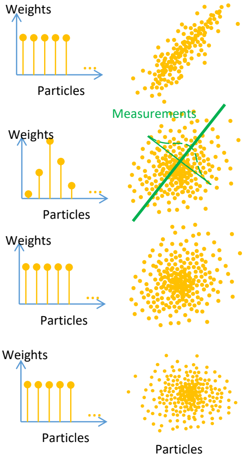
New particle distribution

Prediction, correction, and resampling



SIR-PF scheme

Particle Filter



Steps

Resampling

Low-weighted particles are replaced with copies of high-weighted particles

Correction

$$\begin{cases} p(\mathbf{x}_k|\mathbf{Y}_k) = \sum_{i=1}^{N} \omega_k^i \delta_{\mathbf{x}_k^i} \\ \omega_k^i \propto p\left(\mathbf{y}_k|\mathbf{x}_k^i\right)\omega_{k-1}^i \end{cases}$$

Prediction

$$\begin{cases} p(\mathbf{x}_k|\mathbf{Y}_{k-1}) = \sum_{i=1}^{N} \omega_{k-1}^i \delta_{\mathbf{x}_k^i} \\ \mathbf{x}_k^i = f\left(\mathbf{x}_{k-1}^i, \mathbf{u}_k\right) + \mathbf{w}_k^i \end{cases}$$

Initialization

$$\begin{cases} p(\mathbf{x}_0) = \sum_{i=1}^{N} \omega_0^i \delta_{\mathbf{x}_0^i} \\ \left\{\mathbf{x}_0^i\right\}_{i=\{1,\ldots,N\}} \sim p(\mathbf{x}_0) \end{cases}$$

PF formulation example with Gaussian uncertainties

Initialization: draw:

$$\left\{ \mathbf{x}_0^i \sim \mathcal{N}(\widehat{\mathbf{x}_0}, \widehat{\mathbf{P}}_0) \right\}_{i \in [1,N]} \tag{13}$$

Prediction step:

$$\mathbf{x}_k^i = f(\mathbf{x}_{k-1}^i, \mathbf{u}_k) + \mathbf{w}_k^i \tag{14}$$

where $\mathbf{w}_k^i \sim \mathcal{N}(0, \mathbf{Q}_k) \quad \forall i \in [1, N]$

Correction step:

Weights update (where $\mathbf{y}_k - h(\mathbf{x}_k^i)$ is the *innovation* term):

$$\tilde{w}_k^i = w_{k-1}^i \exp\left(-\frac{1}{2} (\mathbf{y}_k - h(\mathbf{x}_k^i))^T \mathbf{R}_k^{-1} ((\mathbf{y}_k - h(\mathbf{x}_k^i)) \right) \tag{15}$$

Weights normalization: $w_k^i = \frac{\tilde{w}_k^i}{\sum_i \tilde{w}_k^i}$

Estimation:

$$\widehat{\mathbf{x}_k} = \sum_{i=1}^{N} w_k^i \mathbf{x}_k^i \qquad \widehat{\mathbf{P}}_k = \sum_{i=1}^{N} w_k^i (\mathbf{x}_k^i - \widehat{\mathbf{x}}_k)(\mathbf{x}_k^i - \widehat{\mathbf{x}}_k)^T \tag{16}$$

Resampling (cf slide 19) if:

$$N_{eff} = \frac{1}{\sum_i (w_k^i)^2} < \theta_{eff} N \tag{17}$$

where $\theta_{eff} \in (0, 1)$, (usually 0.5)

Adaptive particle filters

KLD-sampling

At each iteration of the particle filter, determine the number of samples such that, with probability $1-\delta$, the error between the true posterior and the sample-based approximation is less than ε.

$$N = \frac{k-1}{2\varepsilon}\left[1 - \frac{2}{9(k-1)} + \sqrt{\frac{2}{9(k-1)}}z_{1-\delta}\right]^3 \qquad (18)$$

where $z_{1-\delta}$ is the upper $1-\delta$ quantile of the standard normal $\mathcal{N}(0,1)$ distribution and k is the numbers of bins.

Typical parameter values: $\varepsilon = 0.07$ and $1-\delta = 0.99$.

KLD: Kullback-Leibler Divergence.

KLD-sampling algorithm

Initialisation

NX=0; N=0; k=0 ;

Paramètres de réglage

ε = 0.07;

NMIN= 500 ;

Bins = [0,...,0]; les cases de la grille

While *N<NX ou N< NMIN*

N=N+1;

Tirage aléatoire de l'indice p de la particule

$\{X_{k-1}^p, w_{k-1}^p\}$

Prédiction

$X_k^p = f(X_{k-1}^p) + W_k^p$

Correction

$w_k^p \propto w_{k-1}^p p(y_k | X_k^p)$

Recherche du numéro de la case j occupée de la grille par la particule X_k^p

j = find(X_k^p *)*

Si bins(j) < 1

k = k+1; % incrémentation du nombre de case occupée par les particules

bins(j) = bins (j)+1; % la case est occupée

Si k>= 1

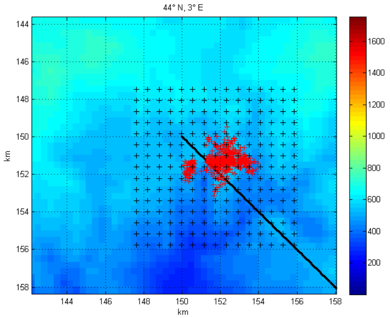
$$NX = \frac{k-1}{2\varepsilon}\left[1 - \frac{2}{9(k-1)} + \sqrt{\frac{2}{9(k-1)}}\, z_{1-\delta}\right]^3$$

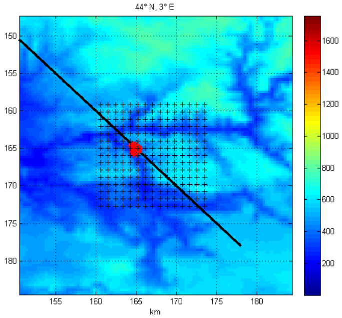
end

end

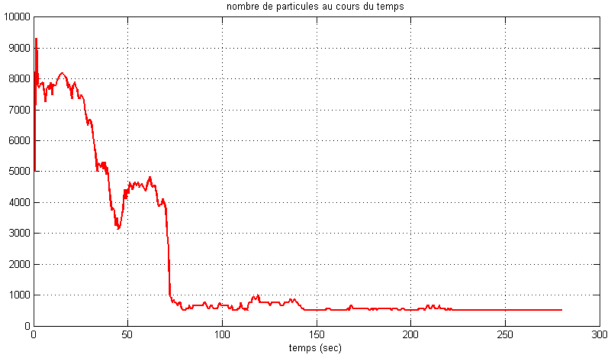
end

KLD-sampling

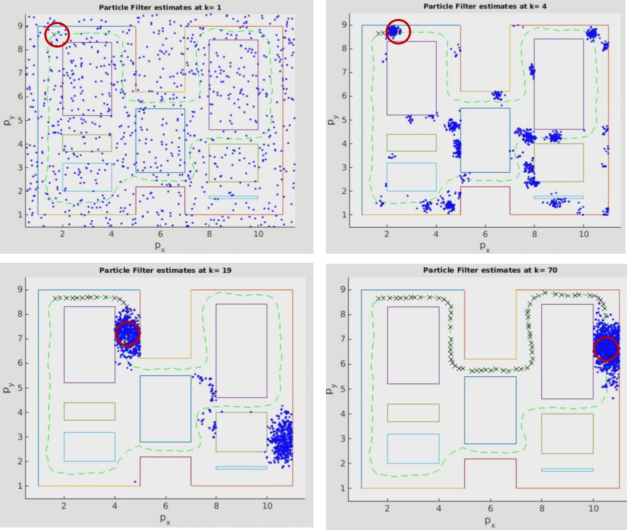




KLD-sampling

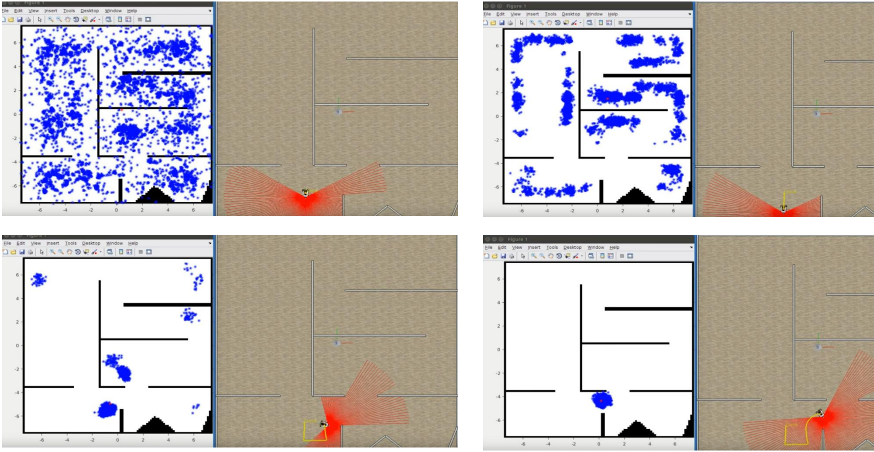


Ex1: Indoor map navigation (odometry only)



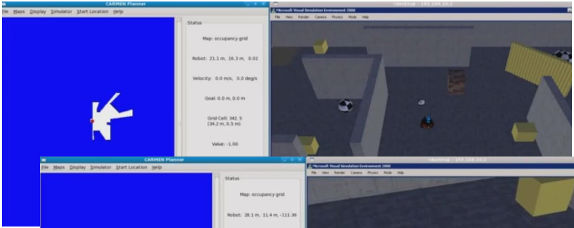
Position estimation using Particle Filter
https://www.youtube.com/watch?v=qsNGoHi7o2U

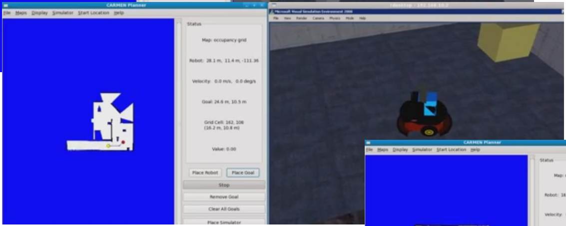
Ex2: Indoor map navigation with LIDAR data

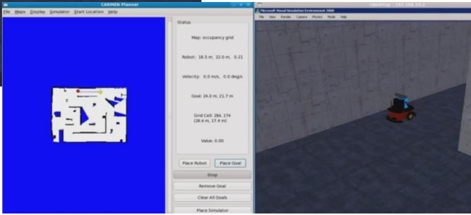


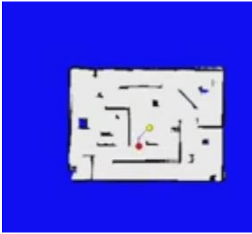
EMOR - Project 3: particle filter (with LIDAR data)
https://www.youtube.com/watch?v=Xlt1aoz5fUw

Ex3: Particle Filter SLAM



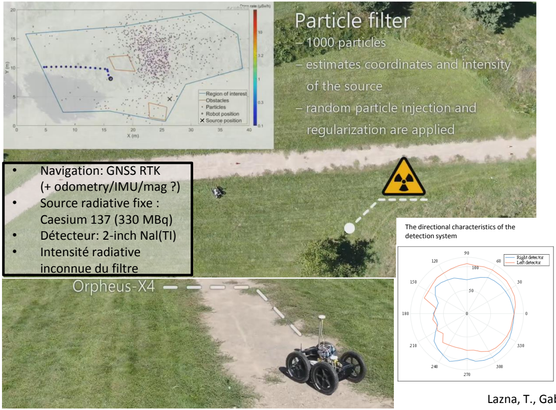


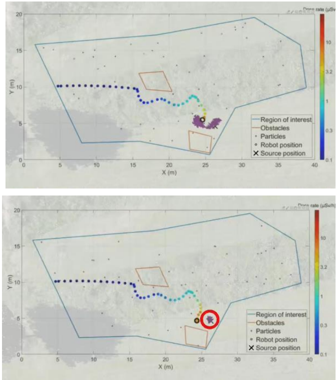




Particle Filter SLAM with Occupancy Grid Map
https://www.youtube.com/watch?v=CWpwrpZD0J4

Ex4: Autonomous Radiation Source Localization Using a Particle Filter

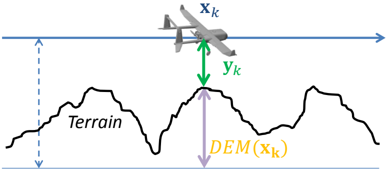


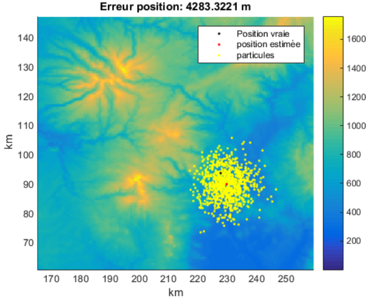


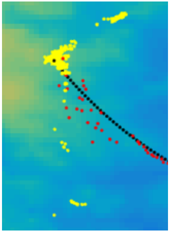
Autonomous Radiation Source Localization Using a Particle Filter
https://www.youtube.com/watch?v=Ox4J_Jov2XE

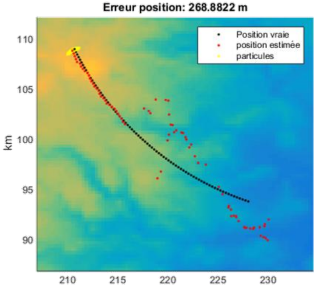
Lazna, T., Gabrlik, P., Jilek, T., & Zalud, L. (2018). Cooperation between an unmanned aerial vehicle and an unmanned ground vehicle in highly accurate localization of gamma radiation hotspots. *International Journal of Advanced Robotic Systems*, *15*(1), 1729881417750787.
https://journals.sagepub.com/doi/pdf/10.1177/1729881417750787

Ex5: Terrain aided navigation









SIR-PF advantages and limitations

Advantages

- Tackles non-linear dynamics and measurement models
- Tackles non-differentiable models (no linearization)
- Tackles non-Gaussian uncertainties and multimodal state densities (ambiguities)
- Can start with huge initial uncertainty
- Relatively easy to implement

Limitations of the original version of the algorithm:

- Computationally demanding
- May converge to erroneous local state density maximum
- Can be hard to tune (number of particles, choice of noise modeling, resampling threshold)

More advanced Particle Filters

Original algorithm:

- Sequential Importance Resampling Particle Filter (SIR-PF) [2]

More advanced algorithms:

- Rao-Blackwellized Particle Filter (RBPF) [7]
- (Weighted) Ensemble Kalman Filter (WEnKF/EnKF) [3]
- Regularized Particle Filter (RPF) [4]
- Kalman Particle Kernel Filter (KPKF) [6]
- Box Particle Filter (BPF) [8] and Box Regularized Particle Filter (BRPF) [9]
- Adaptive Approximate Bayesian Computational Particle Filter (A2BC-PF) [10]
- ...

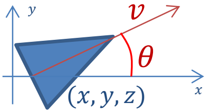
TP

1. Prendre en main et décrire sous forme de schéma la structure du code *FiltrageParticulaire_terrainNavigation.m* et repérer les différents paramètres (réglage du filtre, simulation...).
2. Compléter le code avec les équations du filtre PF (prédiction, correction, ré-échantillonnage slide 22), le modèle dynamique et le modèle de mesure (slide 35) à l'aide de la fonction *hobs*($\mathbf{x}_k$, *params*) et commenter les résultats;
3. Faire varier le bruit de dynamique du filtre (matrice $\mathbf{Q}_f$) sur les différentes variables d'état et expliquer le comportement des particules.
4. Faire varier le bruit de mesure du filtre (matrice $\mathbf{R}_f$) entre 10^2 et 100^2 et expliquer les résultats.
5. Faire varier le nombre de particules N et expliquer les résultats.
6. Faire varier le seuil de ré-échantillonnage *threshold_resampling*. Pour les valeurs de 0, 0.5 et 1, tracer des histogrammes des poids (*hist(wp)*) et identifier le phénomène de dégénérescence et l'impact du ré-échantillonnage.
7. Simuler un trou de mesures entre $t = 50$ s et $t = 75$ s en utilisant la variable *is_measurementValid* et expliquer les résultats.
8. Modifier la fréquence des mesures (passer à 0.1 Hz) en utilisant la variable *is_measurementValid* et expliquer les résultats.
9. Proposer une autre façon de ré-échantillonner les poids, en remplacement de la fonction select.p (qui s'utilise comme: $indp = select(wp)$ avec wp l'ensemble des poids et $indp$ la liste des indices des nouvelles particules par rapport aux anciennes), la coder puis commenter les résultats. Vous pourrez vous aider de [12] (référence slide 36, pdf fourni avec les codes).

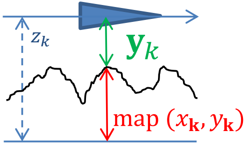
/!\ Les rapports de TP sont individuels.

State: $\mathbf{x}_k = [x_k, y_k, z_k, \theta_k]^T$, Control: $\mathbf{u}_k = [v_k, \omega_k]^T$

Dynamics:



$$\mathbf{x}_{k+1} = f(\mathbf{x}_k, \mathbf{u}_k) = \begin{bmatrix} x_k + v_k \Delta t \cos \theta_k \\ y_k + v_k \Delta t \sin \theta_k \\ z_k \\ \theta_k + \Delta t \omega_k \end{bmatrix}$$



Measurements (e.g., radar altimeter or laser telemeter):

$$\mathbf{y}_k = h(\mathbf{x}_k) + \mathbf{v}_k = z_k - \text{lectureCarte}(x_k, y_k) + \mathbf{v}_k$$

where $\mathbf{v}_k \sim \mathcal{N}(0, \mathbf{R})$ and $\mathbf{R} = 20^2\ \text{m}^2$.

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