Revisting Box-RRT algorithm with DynIBEX

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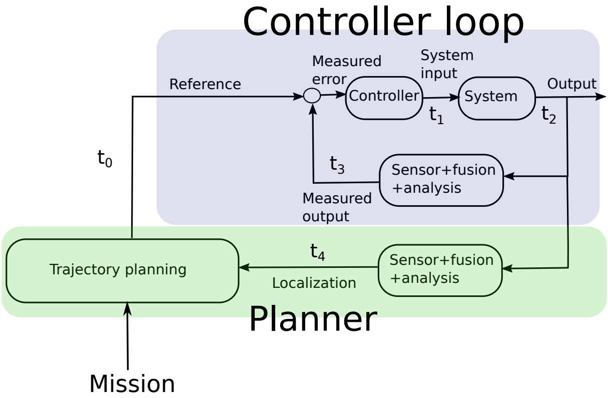
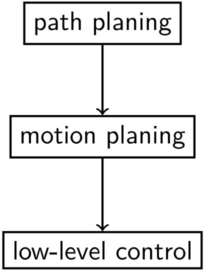
Autonomous vehicle



Goal of the project try to understand main pieces of the system to validate their behavior and the behavior of the overall system.

A hierarchical control



- **Path planing** generates a set of way points (does not take into account the dynamics of the vehicle) from a map (totally or partially) known, take into account obstacles (static)
- **Motion planing** generates a set of trajectories feasible for the dynamics considered and take into account obstacles (static and dynamic)
- **Low-level controller** tries to follow the (discretized) trajectory w.r.t. the dynamic of the vehicle

This talk

We focus on **trajectory planing algorithms** taking into account

- A model of the vehicle
- A model of a map with obstacles (static)
- Bounded uncertain information on the position/orientation, etc.

This work is based on

Reliable robust path planning. Romain Pepy, Michel Kieffer, Eric Walter. Journal Applied Mathematical Computing. 2009.

More precisely, we implement the **BoxRRT algorithm** with **DynIBEX**.

DynIBEX in few words

A library combining of **CSP solver** (IBEX[1]) with **validated numerical integration methods** à la Runge-Kutta.

What can we simulate?

Our main tool for set-based simulation of dynamical systems

- Ordinary differential equations (ODE)
- Algebraic-differential equations (DAE) of index-1

$$S \equiv \begin{cases} \dot{\mathbf{y}} = F(t, \mathbf{y}, \mathbf{x}, \mathbf{p}, \mathbf{u}) \\ 0 = G(t, \mathbf{y}, \mathbf{x}, \mathbf{p}, \mathbf{u}) \\ 0 = H(t, \mathbf{y}, \mathbf{p}, \mathbf{u}) \\ \mathbf{y}(0) \in \mathcal{Y}_0, \mathbf{x}(0) \in \mathcal{X}_0, \mathbf{p} \in \mathcal{P}, \mathbf{u} \in \mathcal{U}, t \in [0, t_{\text{end}}] \end{cases} .$$

How checking temporal properties on S?

[1] Gilles Chabert (EMN) et al. `http://www.ibex-lib.org`

QCSDP

Let S be a differential system and $t_{\text{end}} \in \mathcal{R}_+$ the time limit. A QCSDP is a CSP defined by

- a set of variables $\mathcal{V}$ including *at least* t, a vector $\mathbf{y}_0$, $\mathbf{p}$, $\mathbf{u}$
  We represent these variables by the vector $\mathbf{v}$;
- an initial domain $\mathcal{D}$ containing *at least* $[0, t_{\text{end}}]$, $\mathcal{Y}_0$, $\mathcal{U}$, and $\mathcal{P}$;
- a set of constraints $\mathcal{C} = \{c_1, \ldots, c_e\}$ composed of predicates over sets, that is, constraints of the form

$$c_i \equiv Q\mathbf{v} \in \mathcal{D}_i . f_i(\mathbf{v}) \diamond \mathcal{A}, \qquad \forall 1 \leqslant i \leqslant e$$

  with $Q \in \{\exists, \forall\}$, $f_i : \wp(\mathcal{R}^{|\mathcal{V}|}) \to \wp(\mathcal{R}^q)$ stands for non-linear arithmetic expressions defined over variables $\mathbf{v}$ and solution of differential system S, $\mathbf{y}(t; \mathbf{y}_0, \mathbf{p}, \mathbf{u}) \equiv \mathbf{y}(\mathbf{v})$, $\diamond \in \{\subseteq, \cap_\emptyset\}$ and $\mathcal{A} \subseteq \mathcal{R}^q$ where $q > 0$.

Note: we follow the same approach that Goldsztejn et al.[2]

[2] Including ODE Based Constraints in the Standard CP Framework, CP10

Box-QCSDP as abstraction of QCSDP

Box-QCSDP

Let S be a differential system and $t_{\text{end}} \in \mathcal{R}_+$ the time limit A Box-QCSDP is defined by

- a set of variables $\mathcal{V}$ including *at least* t, a vector $\mathbf{y}_0$, $\mathbf{p}$, $\mathbf{u}$
  We represent these variables by the vector $\mathbf{v}$;
- an initial box $[\mathbf{d}]$ containing *at least* $[0, t_{\text{end}}]$, $[\mathbf{y}_0]$, $[\mathbf{u}]$, and $[\mathbf{p}]$;
- a set of interval constraints $\mathcal{C} = \{c_1, \ldots, c_e\}$ composed of predicates over sets, that is, constraints of the form

  $$c_i \equiv Q\mathbf{v} \in [\mathbf{d}_i].[f_i](\mathbf{v}) \diamond \alpha(\mathcal{A}), \qquad \forall 1 \leqslant i \leqslant e$$

  with $Q \in \{\exists, \forall\}$, $[f_i] : \mathcal{IR}^{|\mathcal{V}|} \to \mathcal{IR}^q$ stands for non-linear arithmetic expressions defined over variables $\mathbf{v}$ and interval enclosure solution $[\mathbf{y}](t; \mathbf{y}_0, \mathbf{p}, \mathbf{u}) \equiv [\mathbf{y}](\mathbf{v})$, $\diamond \in \{\subseteq, \cap_{\emptyset}\}$ and $\alpha \in \{\text{Hull}, \text{Int}\}$

Note: using boxes is not so straightforward to preserve soundness
TODO: A more formal definition of the abstraction has to be defined!

DynIBEX: a Box-QCSDP solver with restrictions

Solving arbitrary quantified constraints is hard!

We focus on particular problems of robotics involving quantifiers

- Robust controller synthesis: $\exists \mathbf{u}$, $\forall \mathbf{p}$, $\forall \mathbf{y}_0$ + temporal constraints
- Parameter synthesis: $\exists \mathbf{p}$, $\forall \mathbf{u}$, $\forall \mathbf{y}_0$ + temporal constraints
- etc.

We also defined a set of temporal constraints useful to analyze/design robotic application.

Verbal property	QCSDP translation
Stay in $\mathcal{A}$	$\forall t \in [0, t_{\text{end}}]$, $[\mathbf{y}](t, \mathbf{v}') \subseteq \text{Int}(\mathcal{A})$
In $\mathcal{A}$ at τ	$\exists t \in [0, t_{\text{end}}]$, $[\mathbf{y}](t, \mathbf{v}') \subseteq \text{Int}(\mathcal{A})$
Has crossed $\mathcal{A}$	$\exists t \in [0, t_{\text{end}}]$, $[\mathbf{y}](t, \mathbf{v}') \cap \text{Hull}(\mathcal{A}) \neq \emptyset$
Go out $\mathcal{A}$	$\exists t \in [0, t_{\text{end}}]$, $[\mathbf{y}](t, \mathbf{v}') \cap \text{Hull}(\mathcal{A}) = \emptyset$
Has reached $\mathcal{A}$	$[\mathbf{y}](t_{\text{end}}, \mathbf{v}') \cap \text{Hull}(\mathcal{A}) \neq \emptyset$
Finished in $\mathcal{A}$	$[\mathbf{y}](t_{\text{end}}, \mathbf{v}') \subseteq \text{Int}(\mathcal{A})$

Box-RRT Algorithm

Goal

- "quickly" find a trajectory going from an initial configuration $\mathbf{s}_i$ to a final configuration $\mathbf{s}_f$
- while avoiding obstacles $\mathbf{s}_o$
- and taking into account bounded uncertainties.

Main ingredients

- used a model of the vehicle
- based on RRT (Rapid Explore Tree) Algorithm
- combined with interval analysis tools (*e.g.*, guaranteed numerical integration)

Cinematic of a Robot in 2D

We consider an unicycle model of a robot

$$\begin{aligned}
\dot{x} &= v\cos(\theta) \\
\dot{y} &= v\sin(\theta) \\
\dot{\theta} &= v\tan\left(\delta + [-\varepsilon, \varepsilon]\right)
\end{aligned}$$

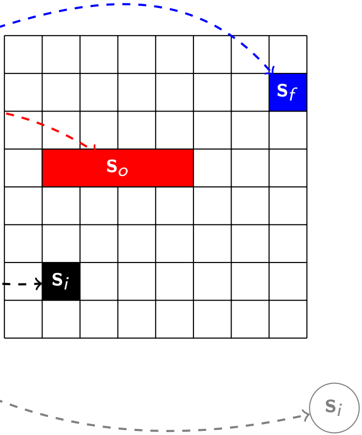
with constraints

- $v \in [-10, 10]$ and $\omega \in [-\pi/6, \pi/6]$
- $\varepsilon = 1e^{-3}$

How does Box-RRT work?

- A discretized map
- A final state
- Some obstacles
- An initial state
- A tree of possible trajectories

Initialization



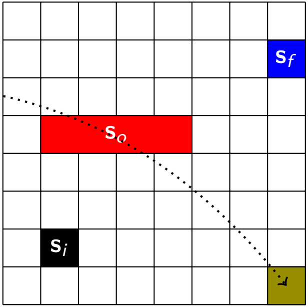
How does Box-RRT work?

Iteration 1

The building of the tree follows 3 main steps in a loop

1. Pick s_{random}
2. Find s_{nearest}
3. Compute s_{new}

until $\mathbf{s}_f$ is reached



Choose randomly a free state of the map

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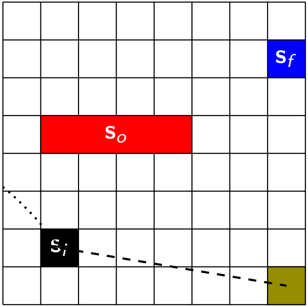

Find the nearest node in the tree following a Hausdorff distance

How does Box-RRT work?

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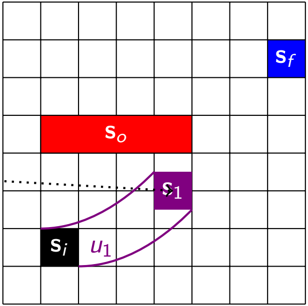
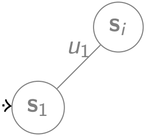


Predict the next state from a random control u

if no collision detected add $(\mathbf{s}_{\text{nearest}}, u, \mathbf{s}_{\text{new}})$ in the tree



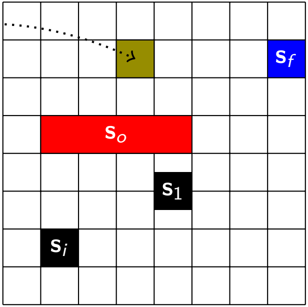
How does Box-RRT work?

Iteration 2

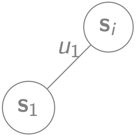
The building of the tree follows 3 main steps in a loop

1. Pick s_{random}
2. Find s_{nearest}
3. Compute s_{new}

until $\mathbf{s}_f$ is reached



Note that the choice of $\mathbf{s}_{\text{random}}$ can be biased to increase probability to be closer to $\mathbf{s}_f$



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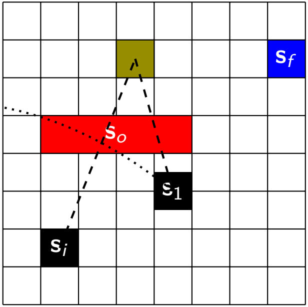
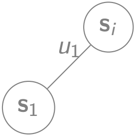


Computing distances has a linear complexity w.r.t. the number of nodes in the tree



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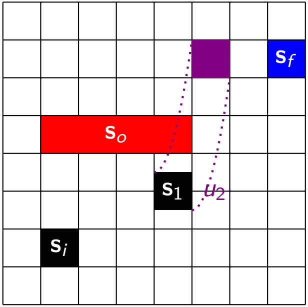
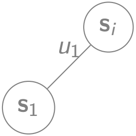


The absence of collision is detected when the **tube** does not intersect an obstacle



Box-RRT in DynIBEX

Three temporal constraints have been used:

- `stay_in` state-space;
- `has_crossed` (in negative form) obstacles;
- `one_in` target;

and one contractor on tube has been used

- `get_tight(t)` to get final state.

So this algorithm can be quickly implemented in DynIBEX.

Conclusion

- Defined a framework to analyze robotic application: Box-QCSDP
- Presented a small example of autonomous vehicle
- Shown one algorithm in the control hierarchy: Box-RRT

Future work

- Define properties we wan/can prove: Viability Kernel, etc.
- Make Box-RRT deterministic ?
- Combine Box-RRT with low-level controller (PID)

Under development

- DynIBEX and contractor and predicate on tubes (J. Alexandre dit Sandretto)
- Extension to n-dimensional case of viability computation (O. Mullier)
- Model this system in an appropriate language, *e.g.*, Zelus (F. Pessaux)
- Combining OpenSMT2 and DynIBEX $\Rightarrow$ SMT modulo ODE (R. Morier)