#### Partz

# An introduction to mobile robotics seen through the eyes of distributed computing

Eric Goubault Bernardo Hummes

June 15, 2022

# Out/ine

- · Robotics and distributed computing models
- · Equivalence between models
- · Distributed consensus and gathering
- · (Some) curent results
- · Robot tasks

· Exploration as a task

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# LOOK-COMPUTE-MOVE

• Cheoretical framework for studying different robotic scenarios in an Unified View

LCM cycle perception of the environment decision based on the previous observation COMPUTE execution of the decision



# LOOK-COMPUTE-MOVE

# Agents

- Movement
  - random or deterministic
- ·Identity labeled or anonymous
- Knowledge mop and other agents
- ·COmmunication
  - time and distance

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* (Cmong many other classifiations
```



# LOOK-COMPUTE-MOVE

# Agents

#### ·Movement

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·Identity labeled or anonymous

• Knowledge mop and other agents

·Communication

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#### Environment

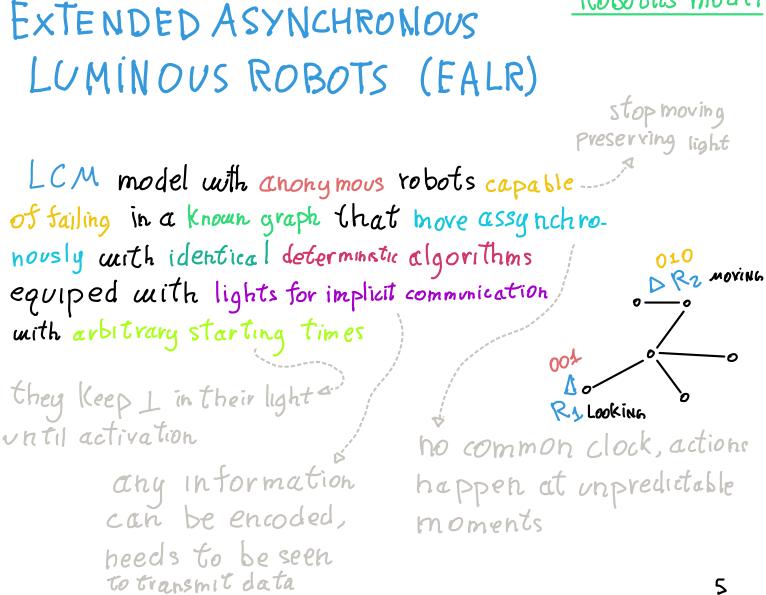
- World
- graph or euclidean space • Labeling local or global / sense of direction
- Time

synchonous or asynchonous

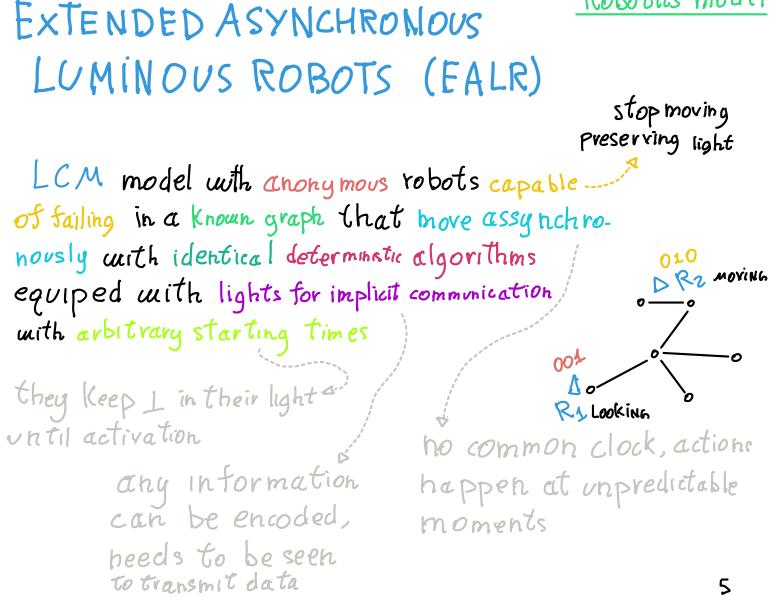
• Storage

tokens and messages











EXTENDED ASYNCHROMOUS LUMINOUS ROBOTS (EALR) stop moving Preserving light LCM model with anonymous robots capable .... of failing in a known graph that move assynchronously with identical determinatic algorithms DR2 MOVING equiped with lights for implicit communication with arbitrary starting times 001 they keep 1 in their light Ry Looking until activation no common clock, actions any intermation happen at unpredictable can be encoded, moments peeds to be seen to transmit data



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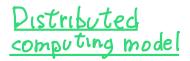
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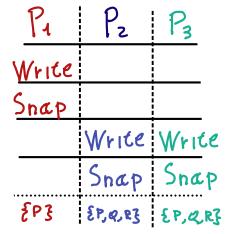
# WAIT FREE SHARED MEMORY (WFSM)

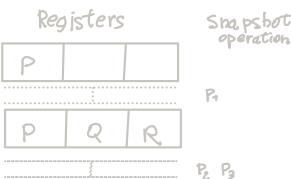
Distributed computing model where undistinguishable programs that can fail assgnchronovsly execute atomic operations of SNAPSHOT, compute and write in a shared memory space with arbitrary starting times

they cannot wait for others to finish their operations

> a program unites to a single register but reads all of them







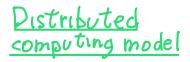
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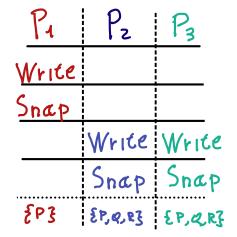
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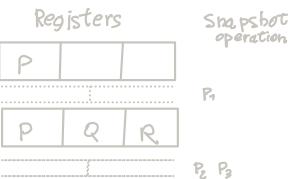
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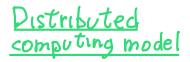
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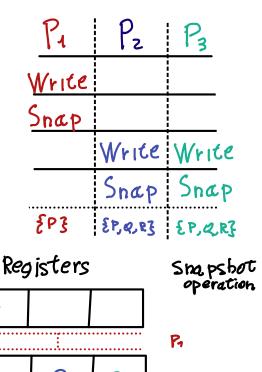
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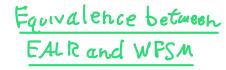


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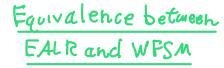


### EQUIVALENCE BETWEEN EALR AND WFSM

<u>Theorem</u>:

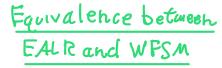
A robot task T on a graph is solvable in the EALR model with  $N \gg 2$  robots tolerating f failures i f and only if T is solvable in the WFSM model with  $N \gg 2$  processes tolerating up to f fairures.

\* From Alcantara et al, The topology of look-compote-move vobot unit-fre c corithms with hard termination.



### SIMULATING EALR IN WESM

global view of the graph global view of all registers with the positions and lights, and their contents, Lis writen for mactive robots L lights are kept invisible SNAPSHOT LOOK COMPUTE COMPUTE  $MOVE(Vi, (i) - \mathbf{V})$ WRITE((Vi,Ci)) robot i moves to The process writes ((Vi, (i)) to vertex Vi and sets the memory at its register the color Ci



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-0

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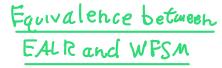
SNAPSHOT

COMPUTE

WRITE ((Vi, Ci))

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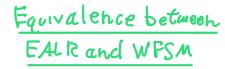
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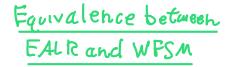
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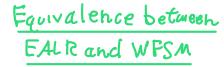
# SIMULATING WFSM in EALR

global view of the graph with the positions and lights, global view of all registers inactive processes have 1 and their contents in their lights SNAPSHOT LOOK COMPUTE COMPUTE MOVE(-, x) -WRITE (X) robot i moves to whatever process writes X value to adjacent vertex and sets the memory at its register its color to X



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# BİNARY AND APPROXIMATE CONSENSUS PROBLEM

We wish for N72 nameless processes, with private inputs among a set of possible ones, To decide on a value, respecting the following properties: ·Termination: every correct process decides avalue. · Validity: the decided value must be proposed by at least one process. · Agreement all decided values are the same. 1 1 .... Dinary consensus 1  $\rightarrow$  0 2 1 - NK-Cot N. ALA. 0

$$\frac{0}{(K=2)}$$

### VERTEX AND APPROXIMATE GATHERING PROBLEM

We wish for N72 undistinguishable robots executing identical algorithms to decide on a vertex to move to, respecting the following properties: • Termination : every robot decides a vertex in a 1-gathering bounded number of LCM cycles. · Validity: the decided vertex cannot be fixed in advance. · Agreement: all decided vertices are the same. edge gathering gathering

# IMPOSSIBILITY OF GATHERING WITH TERMINATION

#### Theorem

The gathering problem with termination is unsolvable by any algorithm in the ALR model even if robots have pomertal capabilities, namely, they are non-oblivious and able to detect multiplicities, share the same labeling of G and have an unbounded number of lights.

Distributed consensus

and gathering

#### Distributed consensus and gathering

IMPOSSIBILITY OF GATHERING WITH TERMINATION

Idea of the proof

Algorithm A in EALR That respects termination and validity while solving gathering.

Algorithm B in WFSM that solves binary consensus con be built using simulated A.

Binary consensus is Knowingly impossible.

Gathering with termination is impossible

Algorithm A cannot exist.

Kmain reason is the Validity property

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Xmain reason is the Validity property IMPOSSIBILITY OF EDGE-GATHERING ON CYCLIC GRAPHS

#### Theorem

Let G be a connected cyclic graph. There is no algorithm that solves edge gathering on G for N>3 robots tolerating two failures in the STRONG version of the EALR model.

Distributed consensus

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Distributed consensus IMPOSSIBILITY OF and gathering EDG7E-GATHERING ON CYCLIC GRAPHS at most k different I dea of the proof values are decided Algorithm A in EALR That respects termination and Validity while solving edge-gathering on a cyclic graph Algorithm B in WFSM that solves 2-set agreement for 3 processes and 2 failures using simulated A. Algorithm C in WESM that is similar to B, but with NZ3 processes using B vie BGz simulation Edge gather Edge gathering on cyclic graphs is 2-Set agreement is Algorithm A impossible to solve cannot exist impossible

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#### POSSIBILITY OF EDGE-GATHERING ON TREES

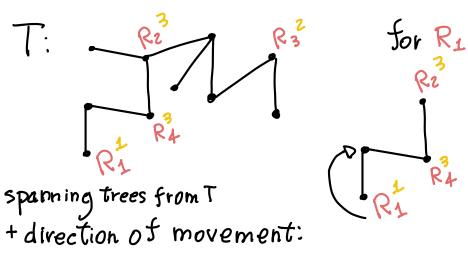
#### Theorem

The edge gathering problem is solvable in the EALR model for NZZ robots on any tree T in diam(TJ-1 vounds using diam(T)-1 distinct light colors an toleroting UP TO N-1 crash failures.

Distributed consensus POSSIBILITY OF EDGEand gathering GATHERING ON TREES extreme case for Edge - gathering R Each vobot vi executes the same algorithm for diam(T) - I rounds: The robots count Their rounds and 1. A spanning tree is constructed of the vertices occupied by robots in the maximal round + ri Show in their lights Yi will either be at a leaf of the tree or not
 IIF ri is in a leaf, it moves deeper into the tree robots in the maximal round 2.2 Otherwise, ri stays still serve as leaders, orienting the others • As Their positions are constrained by the mitial solution and They will always get closer, convergence happens up to an edge 19

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Distributed consensus and gathering





for R3 R2 R3



\*: unless they are past round diam(T)-1

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(SOME) RESULTS

Problem	# robots	#lights	graph	solvability
gathering Cdge-gathering Cdge-gathering Cdge-gathering 1-gathering 1-gathering 1-gathering	N7/2 N7/3 N7/3 N7/2 N7/2 N7/2 N7/3	unbounded O bounded unbounded	tree connected + diam 73 with a cycle	Impossible Possible Impossible Clique Impossible graph is Possible & tree Possible A Impossible
triangle vertex in a tree				

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SOLUTIONS are down wards Indusive, all subsolutions are also solutions

Robotic missions can be expressed and analyzed using using structures from combinatorial topology. A robot task < I, O, A> will be solvable if it is possible to find the subdivision and simplicial mep bellow, such that  $\delta(S_{ubd}(\mathcal{I})) \subseteq \mathcal{O}$ 7 4 Input complex all sets of -> () Output complex " vertices where Ill, catrier, map all sets of vertices robots can start each initial configu. where robots can end ration is carried to their Subd(I) to setisty the objective acceptable solutions decision Protocol complex simplicial map final state achie. decided vertex OJ  $\mathbf{v}$  () ved after cycles each robot at the end of execution ofexecution 24



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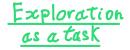


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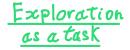
#### MAP EXPLORATION

Terminating exploration: each verte x is visited by at least one robot, eventually all robots stop.
 Variations: Soraging, pursuit/chase

• Exclusive perpetual exploration: each vobot visits each vertex infinetely many times, no two robots troverse the same edge or vertex at the same time.

Lo variations: surveillance, patrolling

Note: not representable in the formalization of vobot tasks above!



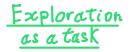
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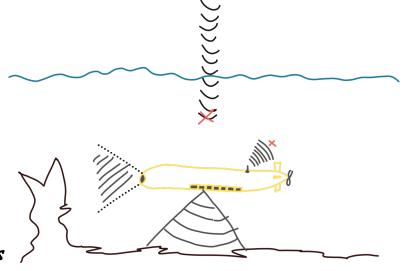


### REALISTIC SCENARIOS

Together with different types of tasks, it is interesting To explore their possible scenarios, prioritizing realistic ones.

There are several limitations to consider.

- · Asynchronous operations
- · Limited visibility
- · Limited communication
- · Faulty movements
- Heterogeneous environments
  Imprecise localization



#### CONCLUSION

• Cheoretical Framemorks are helpfulin dealing with different problems with the same point of view.

• Equivalencies between models are useful for "borrowing" tools, such as The topological approach in robot tasks

• There are multiple formalizations of robotic activities to chose according to context and level of obstraction, It is useful to explore those closer to the applications.

# FUTURE WORK



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· Formalization of exploration tasks and how to extend the current oppoach for them, soluting to underwater exploration.

• In the same context, study variations of the communications model for robots with limited visibility

• Study The usage of grid graphs for the representation of the reachable area of a robot in an homogeneous environment

• Explore the connection with temporal and epistemic modal logics, already studied distributed computing.

## Further reading

• The topology of look - compute - move algorithms with hard termination, by Alcantara, Castañeda, Flores-Peñazola and Rajsbaum.

- Brief announce ment: variants of a pproximate agreement On graphs and simplicial complexes, by Ledent.
- Distributed computing by mobile entities: current research in moving and computing, by Flocchini, Prenupe and Santoro.
- · Distributed computing through combinatorial topology, by Herlihy, Kozlov and Rajsbaum.

