ENSTA Paris
2020-2021

Numerical methods for dynamical systems

Homework n° 5

Goal(s)

- ★ Implementation of shooting methods to solve two point boundary value problems
- ★ Implementaion of finite difference method to solve two point boundary value problems

Exercise 1 – Shooting method

The goal of this exercice is to implement simple shooting method to solve two-point boundary value problem for ODE
We recal that IVP-ODE is defined by

$$\dot{\mathbf{y}} = f(t, \mathbf{y}) \quad \text{with} \quad \mathbf{y}(0) = \mathbf{y}_0 \ . \tag{1}$$

Application of forward Euler's methods for this IVP-ODE with a step-size h defined an iterative function

$$\mathbf{y}_{n+1} = \mathbf{y}_n + hf(t_n, \mathbf{y}_n) \quad \text{et} \quad t_{n+1} = t_n + h \ .$$

Dans un premier temps, la résolution des BVP pourra utiliser cette méthode mais rien n'interdit d'utiliser les méthodes plus sophistiquées vues en cours.
We will consider this simple BVP problem

$$\ddot{y} = \frac{3}{2} y^2$$

avec $y(0) = 4$ et $y(1) = 1$

Note that two solutions exist for this problem (see `https://en.wikipedia.org/wiki/Shooting_method`)
- $\dot{y}(0) = -8$
- $\dot{y}(0) \approx -35.858548$

Question 1

Implement the simple shooting method with a bisection approach

Question 2

Change the numerical method to solve this problem to use variable step-size approach still using bisection approach. Compare the speed to solve this problem.

Question 3

Change the root finding method to use Newton approach with finite difference method to approximate the Jacobian. Compare the speed to solve this problem with fixed step-size and variable step-size integrator.

Exercise 2 – Finite difference method

Question 1

Apply finite difference approach to solve the BVP-ODE problem. Compare the speed to solve this problem with various discretization step-size h.

Note you should use Python function in numpy to solve the system

TO SUBMIT

- A small report should be sent summarize the answers to the questions.

— This report should be associated to the source code.

Send the archive containing the report and the source codes in a mail which title is

[numerical methods for dynamical systems] FIRSTNAME LASTNAME

to `alexandre.chapoutot@ensta-paris.fr`
before the next lecture, Friday October 23, 2020.