

Exercice 7.6: Changement de variables

Soit, pour $a > 0$,

$$J(a) = \int_{\frac{1}{a}}^a \frac{\ln x}{1+x^2} dx = \int_{\frac{1}{a}}^a \frac{\ln x}{1+\frac{1}{x^2}} \frac{dx}{x^2}.$$

Posons

$$\begin{aligned} u = \frac{1}{x} &\longrightarrow \text{bornes: } u : \frac{1}{\frac{1}{a}} = a \rightarrow \frac{1}{a}, \\ du &= -\frac{1}{x^2} dx, \\ \frac{\ln x}{1+\frac{1}{x^2}} \frac{dx}{x^2} &= \frac{\ln \frac{1}{u}}{1+u^2} (-du) = \frac{\ln u}{1+u^2} du \end{aligned}$$

Donc:

$$\begin{aligned} J(a) &= \int_a^{\frac{1}{a}} \frac{\ln u}{1+u^2} du \\ &= -\int_{\frac{1}{a}}^a \frac{\ln u}{1+u^2} du \\ &= -J(a) \end{aligned}$$

Donc:

$$J(a) = 0$$