Exercice 8.5**: Changement de variables**

Soit

$$I = \int_0^{36} f(x)\ \mathrm{d}x = 36.$$

(1) $J = \int_0^{12} f(3x)\ \mathrm{d}x$.

Posons (changement de variables)

$$\begin{aligned} u = 3 \cdot x \longrightarrow\ & \text{bornes: } u : 3 \cdot 0 = 0 \to 3 \cdot 12 = 36, \\ & du = 3dx, \\ & f(3x)\mathrm{d}x = f(u)\frac{\mathrm{d}u}{3}. \end{aligned}$$

Donc:

$$\begin{aligned} J &= \int_0^{36} f(u)\ \frac{\mathrm{d}u}{3} \\ &= \frac{1}{3} \cdot \int_0^{36} f(u)\ \mathrm{d}u \\ &= \tfrac{1}{3} \cdot I \end{aligned}$$

Donc:

$$J = 8$$

(2) $K = \int_0^6 xf(x^2)\ \mathrm{d}x$.

Posons (changement de variables)

$$\begin{aligned} u = x^2 \longrightarrow\ & \text{bornes: } u : 0^2 = 0 \to 6^2 = 36, \\ & du = 2x\mathrm{d}x, \\ & x \cdot f(x^2)\mathrm{d}x = f(u)\frac{\mathrm{d}u}{2}. \end{aligned}$$

Donc:

$$\begin{aligned} K &= \int_0^{36} \frac{1}{2} \cdot f(u)\ \mathrm{d}u \\ &= \frac{1}{2} \cdot \int_0^{36} f(u)\ \mathrm{d}u \\ &= \tfrac{1}{2} \cdot I \end{aligned}$$

Donc:

$$K = 12$$