Renversements du champ magnétique : un modèle commun pour le champ magnétique terrestre et la dynamo VKS ?

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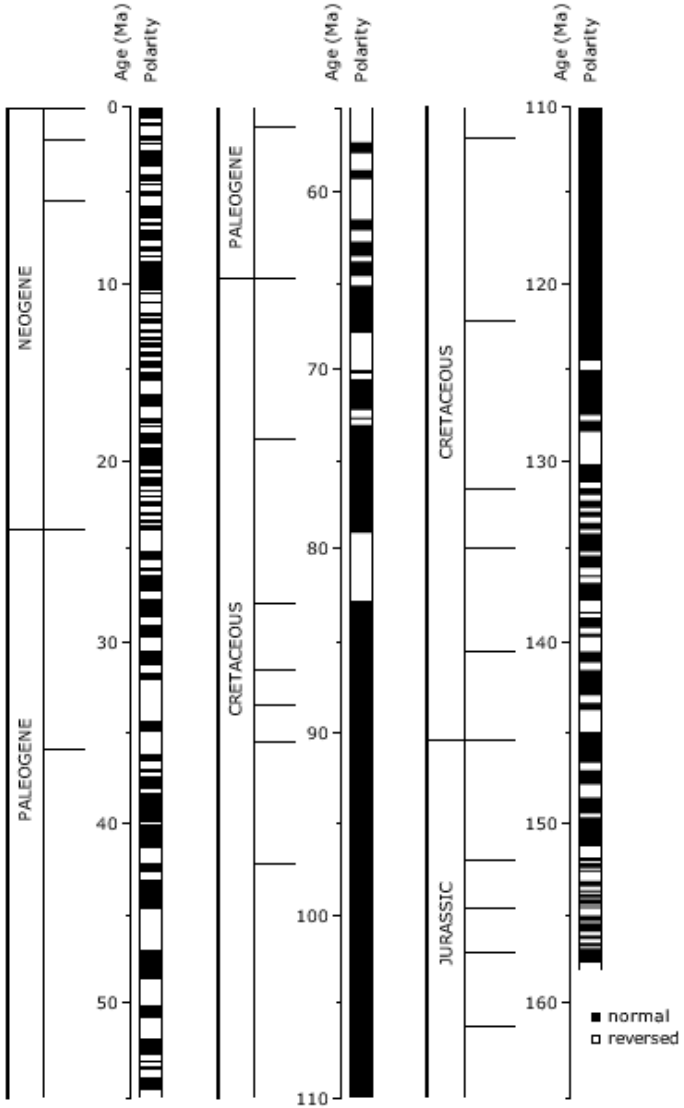
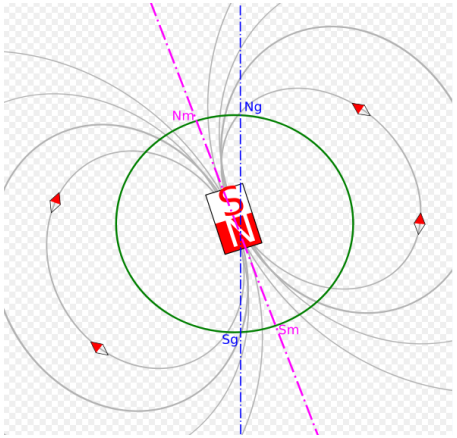
Troisième École Thématique Rennaise sur les Systèmes Complexes, 18 octobre 2011

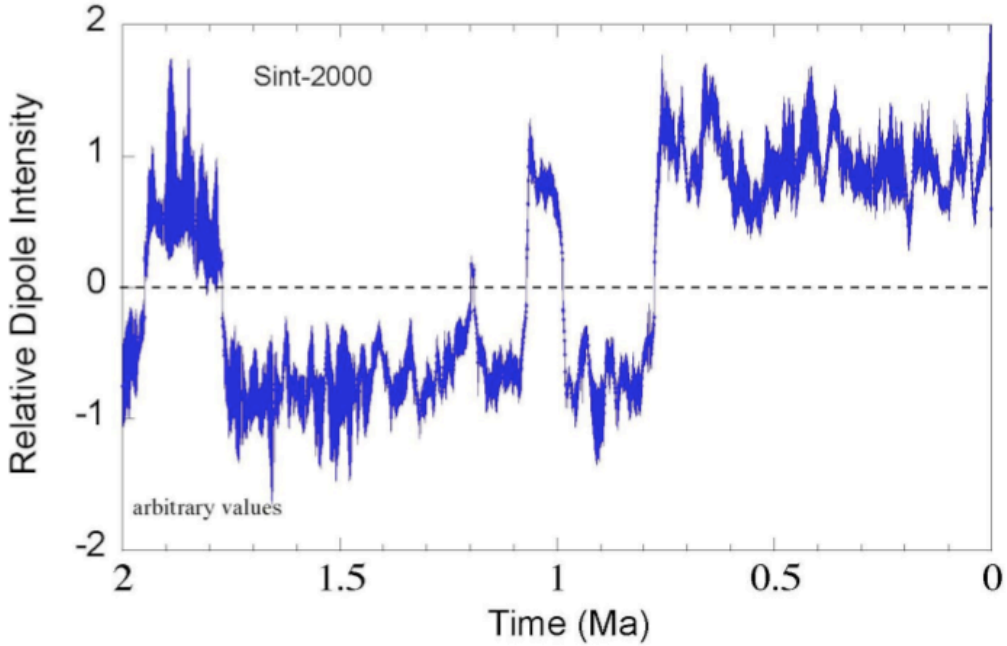
Renversements du champ magnétique de la terre



Lowrie (1997), "Fundamentals of Geophysics"



(Brunhes, J. Phys Theor. App.1906)



J.-P. Valet et al., Nature (2005)

Equations de la MHD et nombres sans dimension

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = -\nabla \left(\frac{\mathrm{p}}{\rho} + \frac{\mathbf{B}^2}{2\mu_0 \rho} \right) + \nu \nabla^2 \mathbf{v} + \frac{1}{\mu_0 \rho} (\mathbf{B} \cdot \nabla)\mathbf{B}.$$

- densité: ρ
- viscosité cinématique: ν
- vitesse: V
- taille du domaine: L
- perméabilité magnétique: μ_0
- conductivité électrique: σ

$Re = VL / \nu$

$R_m = \mu_0 \sigma VL$

$P_m = \mu_0 \sigma \nu$

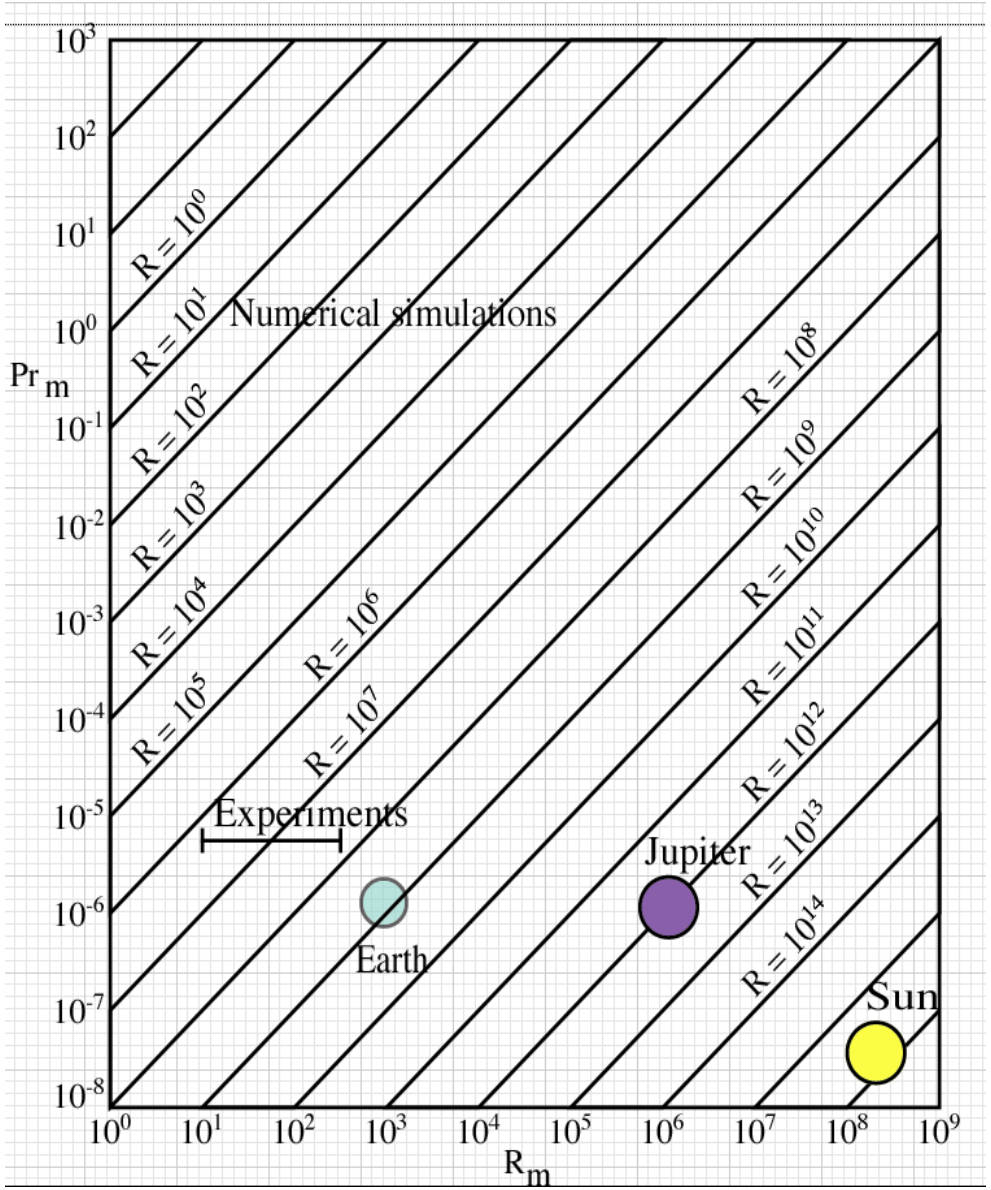
Effet dynamo

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{V} \times \mathbf{B}) + \frac{1}{\mu_0 \sigma} \nabla^2 \mathbf{B}$$

$$\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}.\nabla)\mathbf{v} = -\nabla \left(\frac{\mathrm{p}}{\rho} + \frac{\mathbf{B}^2}{2\mu_0 \rho} \right) + \nu \nabla^2 \mathbf{v} + \frac{1}{\mu_0 \rho} (\mathbf{B} \cdot \nabla)\mathbf{B}.$$

- Dynamo si la solution B=0 est instable ($R_m > R_{mc}$)
- Saturation de l'énergie magnétique par la force de Lorentz
- Si (V, B) est solution, alors(V, -B) autre solution
- Renversements : transitions de B(r) à –B(r)

Expériences, simulations numériques et champs cosmiques



$Rm = \mu_0 \sigma L V$

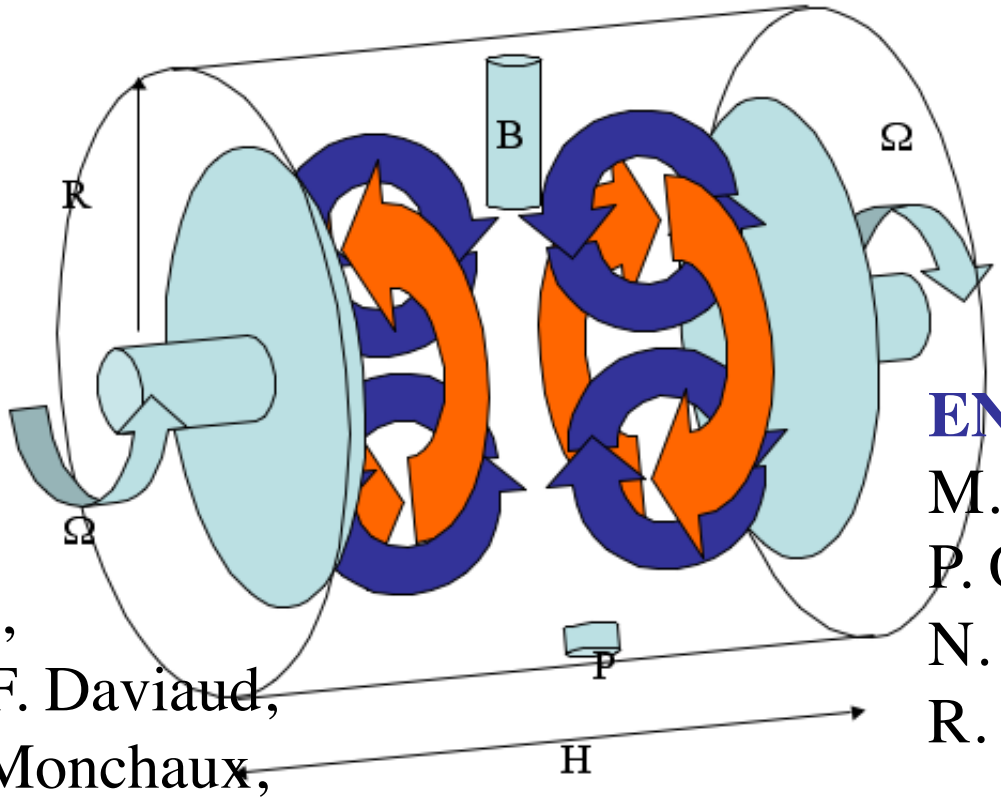
$Pm = \mu_0 \sigma \nu$

Puissance $P \propto \rho L^2 V^3$ nécessaire à engendrer un écoulement turbulent

$\Rightarrow Rm \propto \mu_0 \sigma \left(PL/\rho\right)^{1/3}$

Sodium liquide,
100 kW pour Rm = 50
avec L = 1 m

Ecoulement de von Karman: 2 disques en rotation



CEA-Saclay
S. Aumaître,
J. Boisson,
A. Chiffaudel,
B. Dubrulle, F. Daviaud,
L. Marié, R. Monchaux,
F. Ravelet

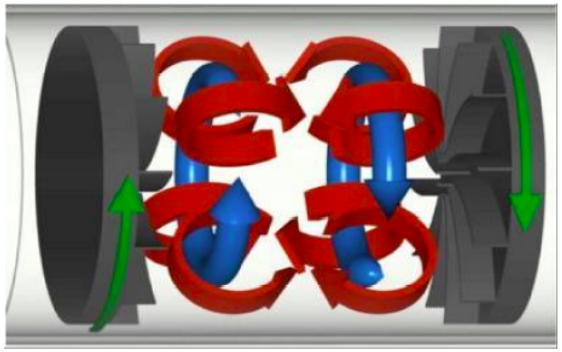
ENS-Lyon
M. Bourgoin,
P. Odier, J.-F. Pinton,
N. Plihon, G. Verhille,
R. Volk

ENS-Paris
M. Berhanu, S. Fauve, B. Gallet,
C. Gissinger, N. Mordant, F. Pétrélis

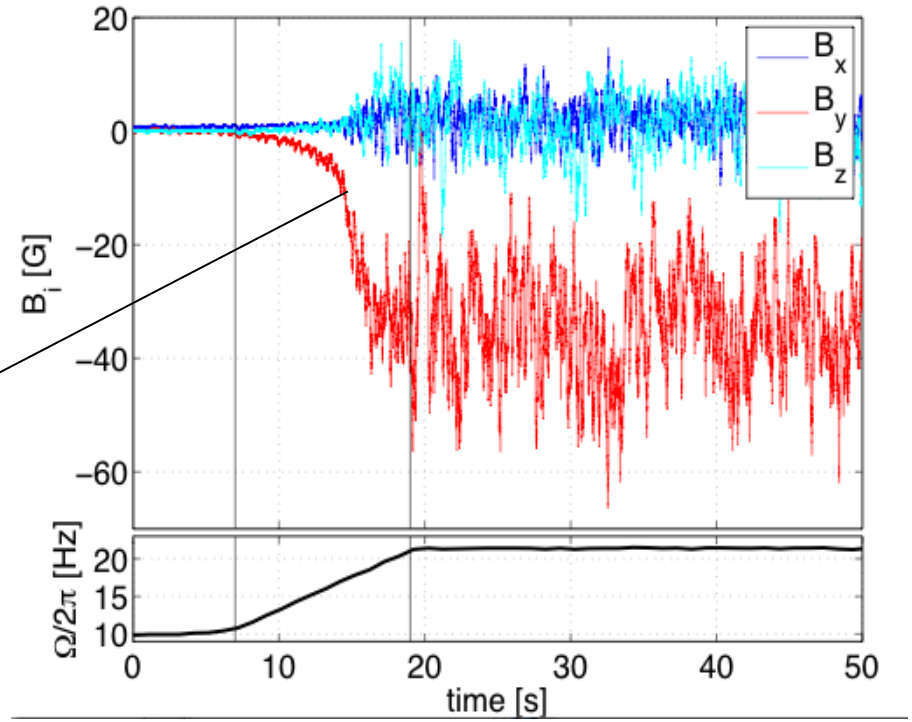
Champ magnétique stationnaire engendré par un écoulement symétrique

Monchaux et al., PRL 98, 044502 (2007)

<V>(r)



B(t)



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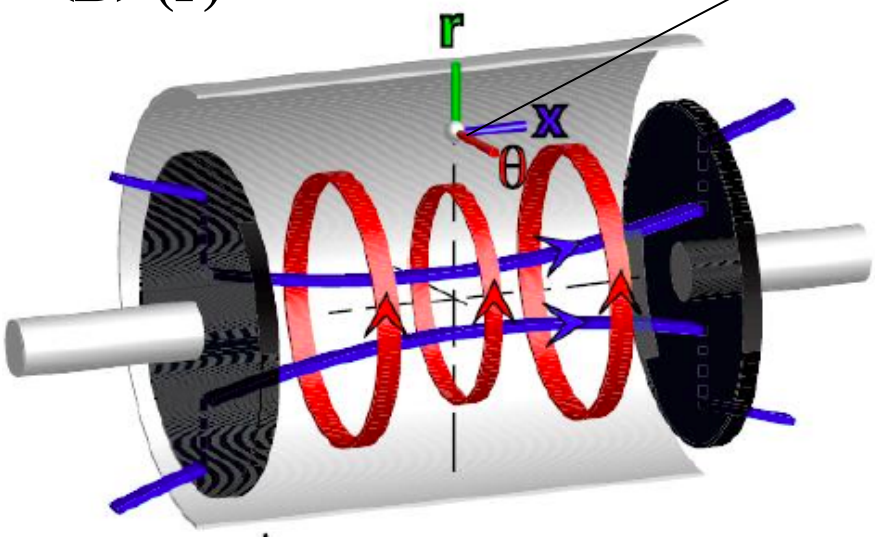
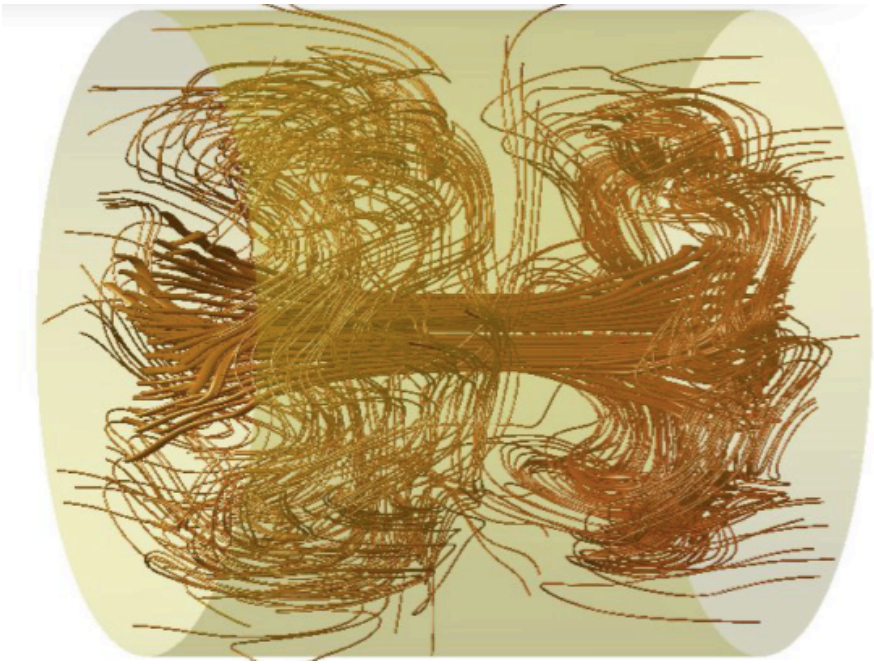


Un problème avec le théorème de Cowling ?

Géomètrie du champ magnétique engendré

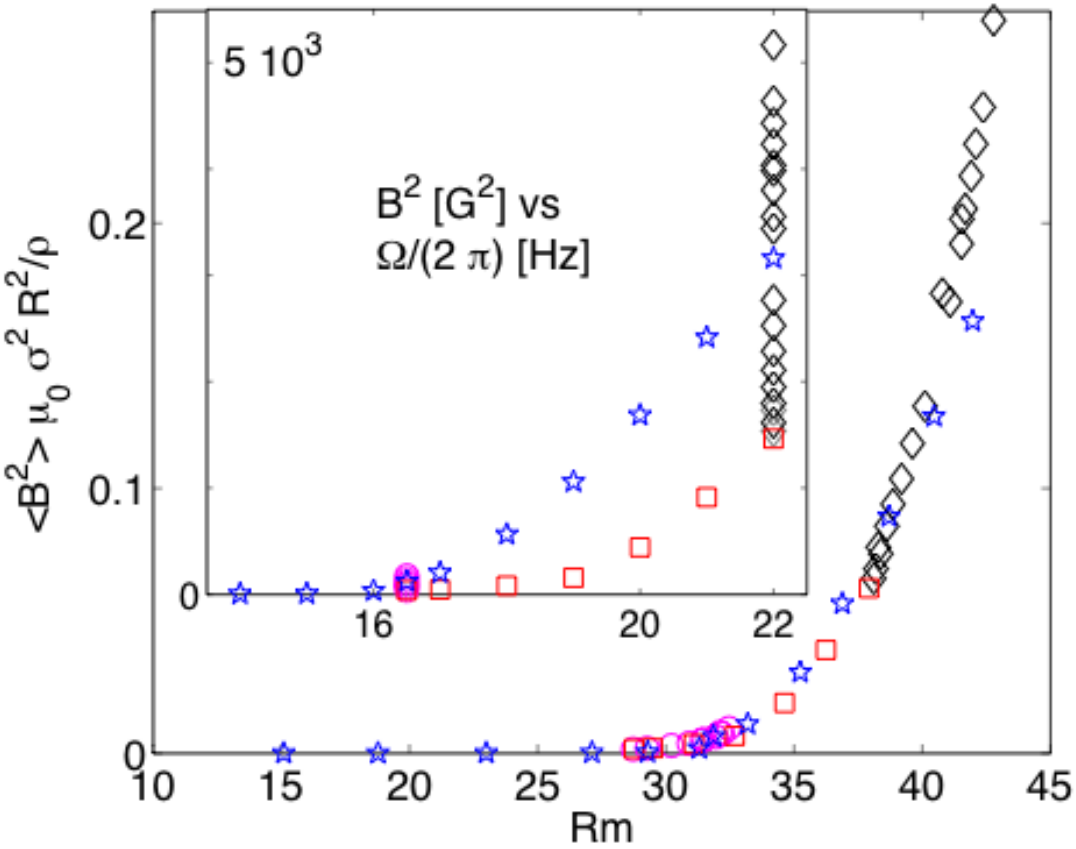


Ecoulement moyen seul:
Dipole équatorial; le champ magnétique doit briser l'axisymétrie (Cowling, 1934)

Ecoulement avec fluctuations non axisymmétriques; dipole axial (Gissinger, EPL 2009)

La dynamo VKS n'est pas engendrée par l'écoulement moyen

Densité d'énergie magnétique



Re faible

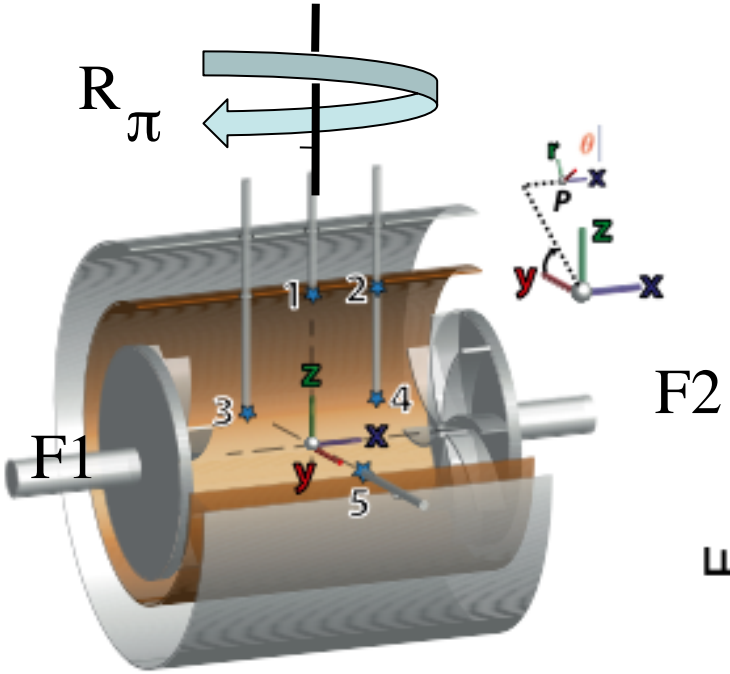
$$B^2 \propto \frac{\rho\nu}{\sigma L^2}\,(R_m - R_{mc})$$

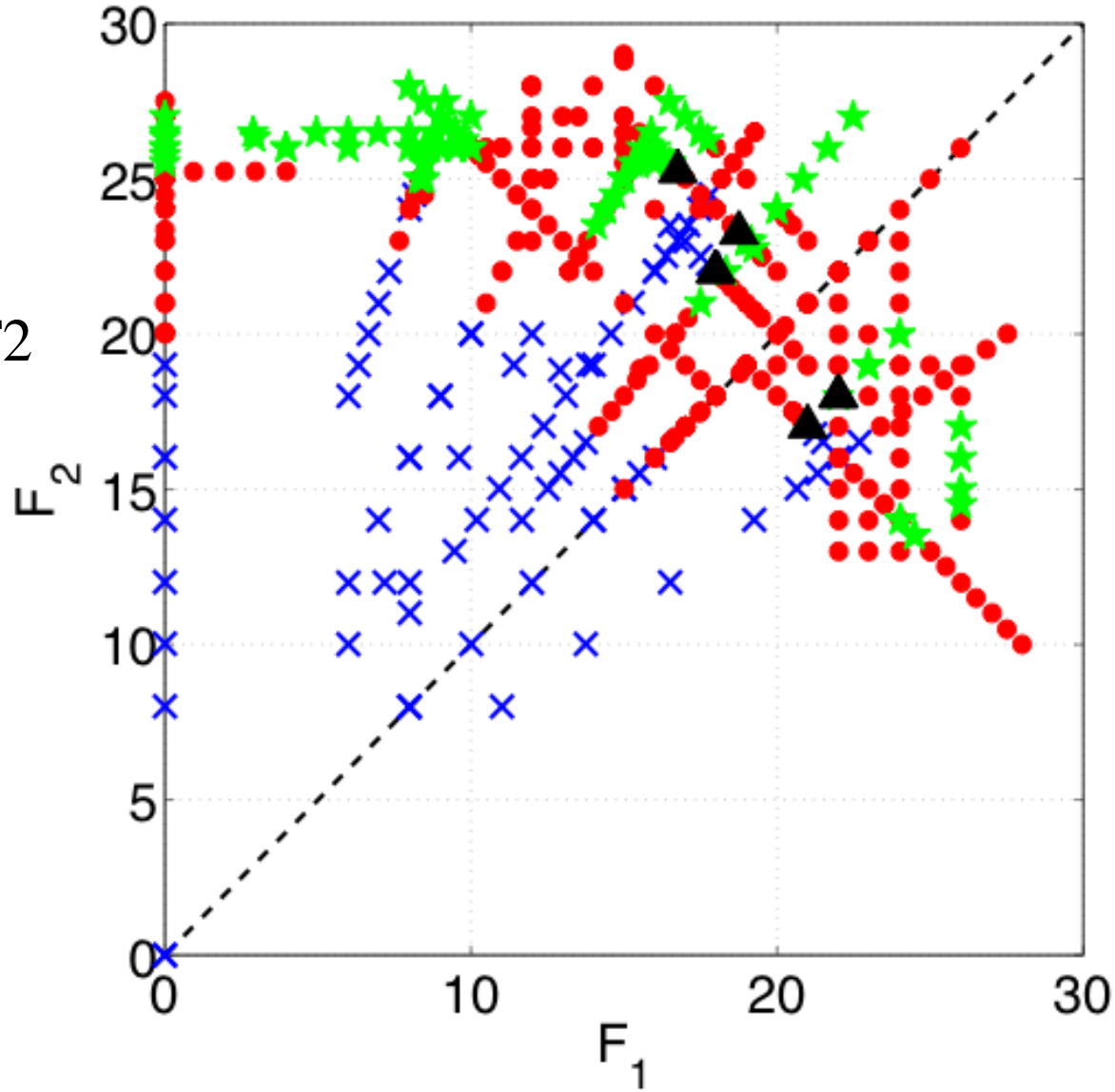
Re grand

$$B^2 \propto \frac{\rho}{\mu_0(\sigma L)^2}\,(R_m - R_m^c)$$

F. Pétrélis and S. Fauve, Eur. Phys. J. B 22, 273 (2001)

Régimes dynamiques engendrés par des écoulements non symétriques

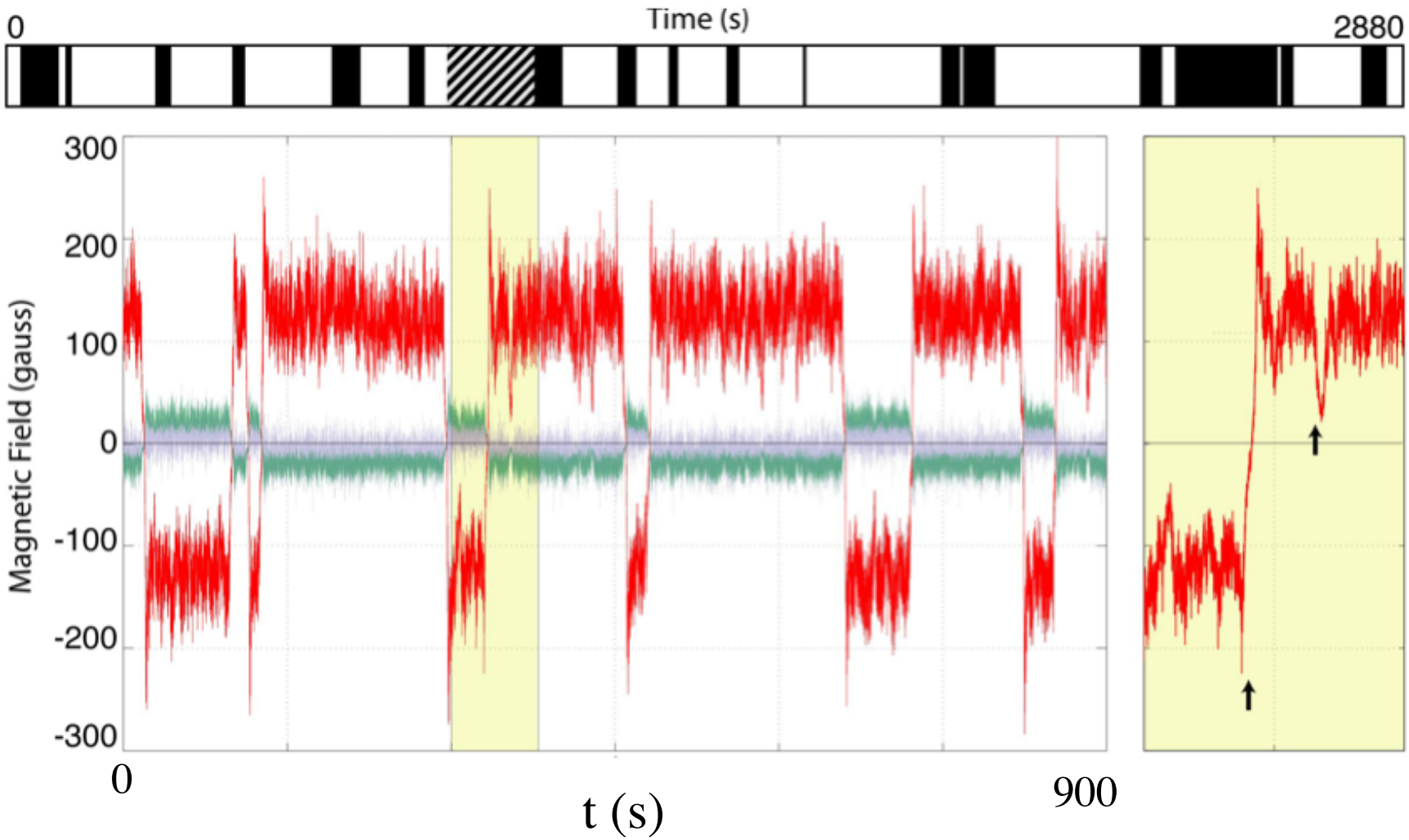




- Pas de dynamo
- Dynamos stationnaires
- Régimes dynamiques

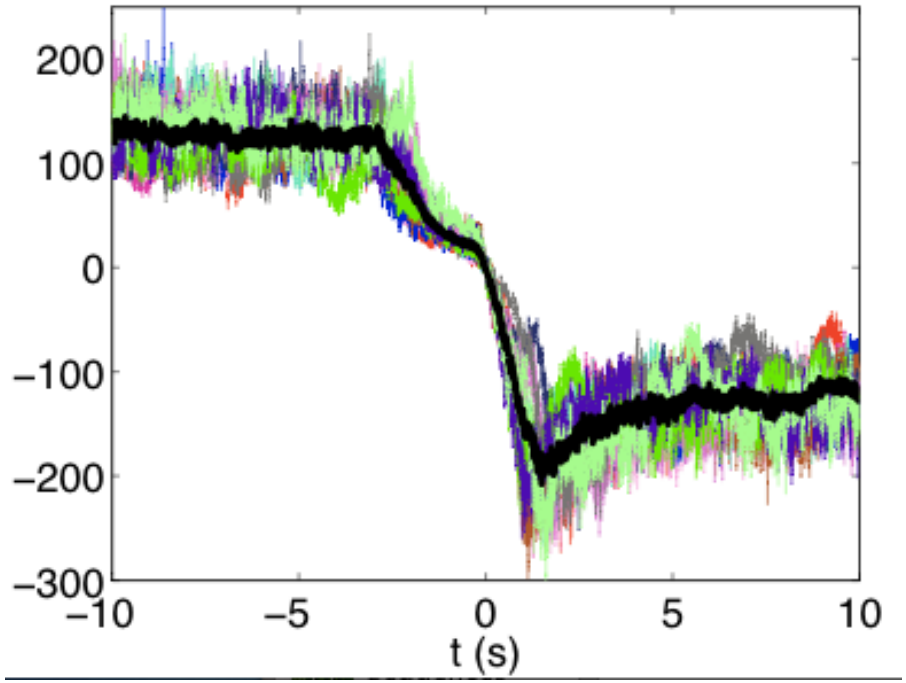
Renversements du champ magnétique

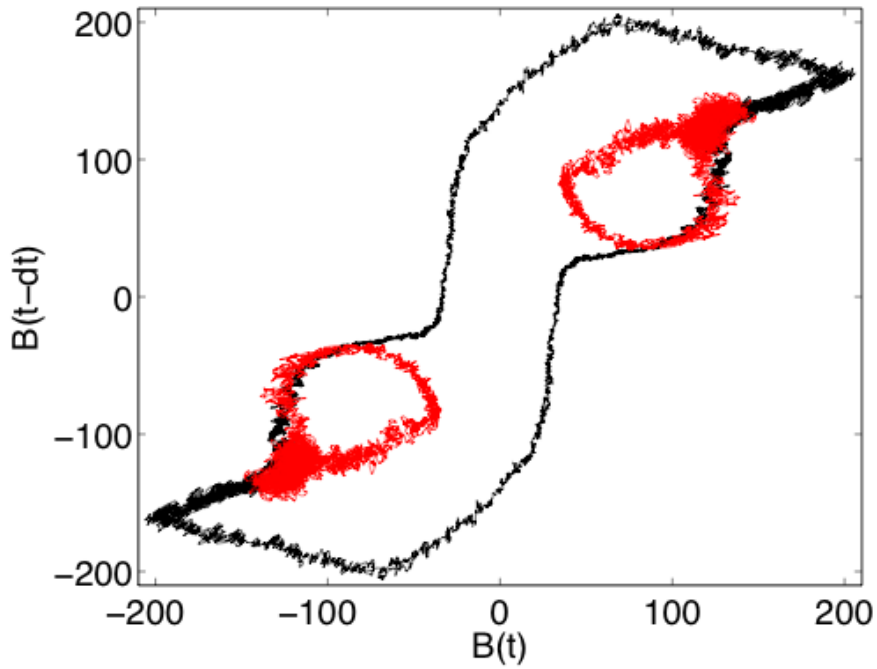
Berhanu et al., EPL 2007



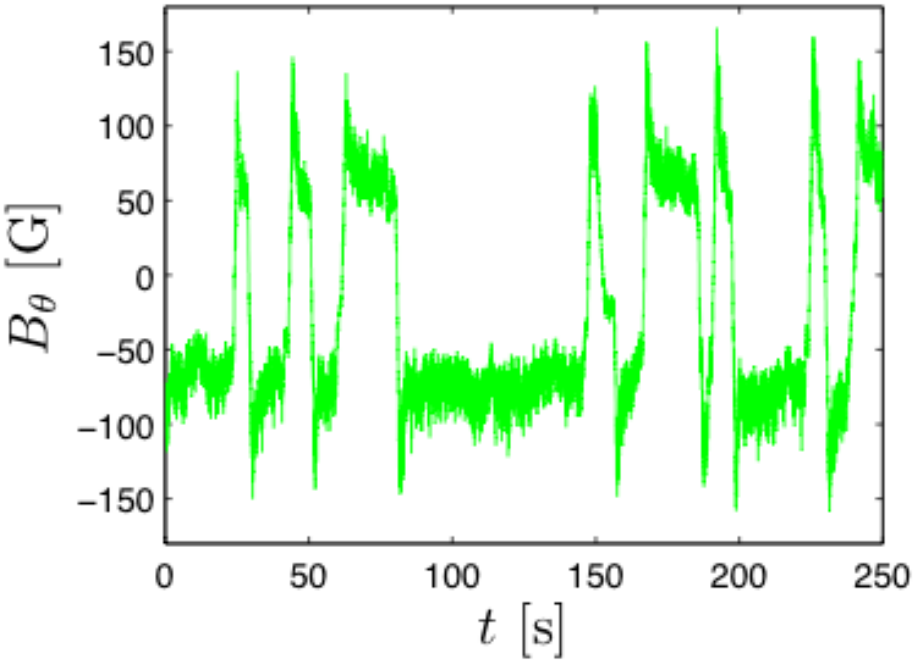
Trajectoires robustes des renversements malgré les fluctuations turbulentes

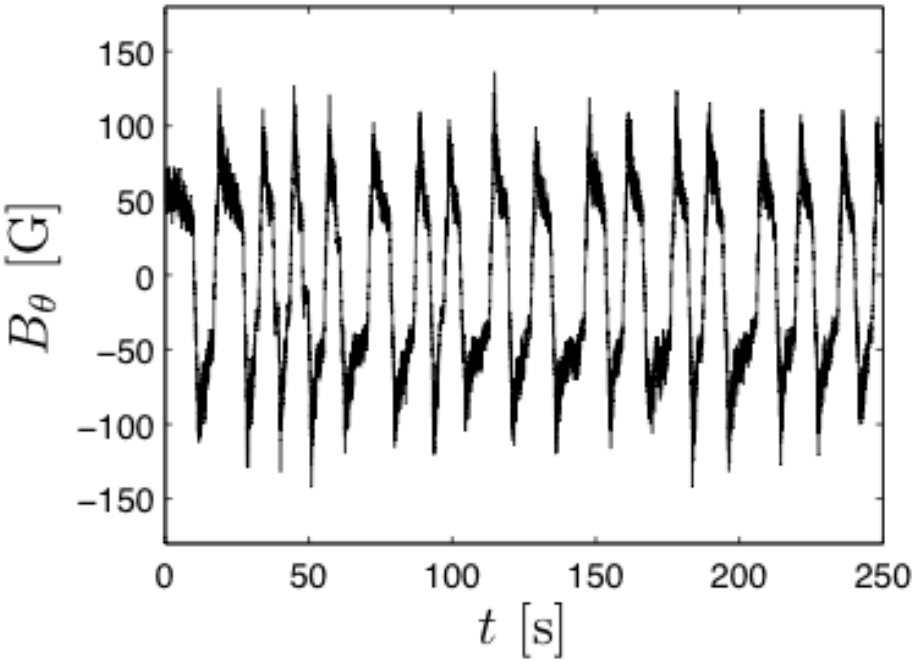
Superposition de 12 renversements (asymétrie, rebond)





Une transition continue entre renversements aléatoires et oscillation périodique

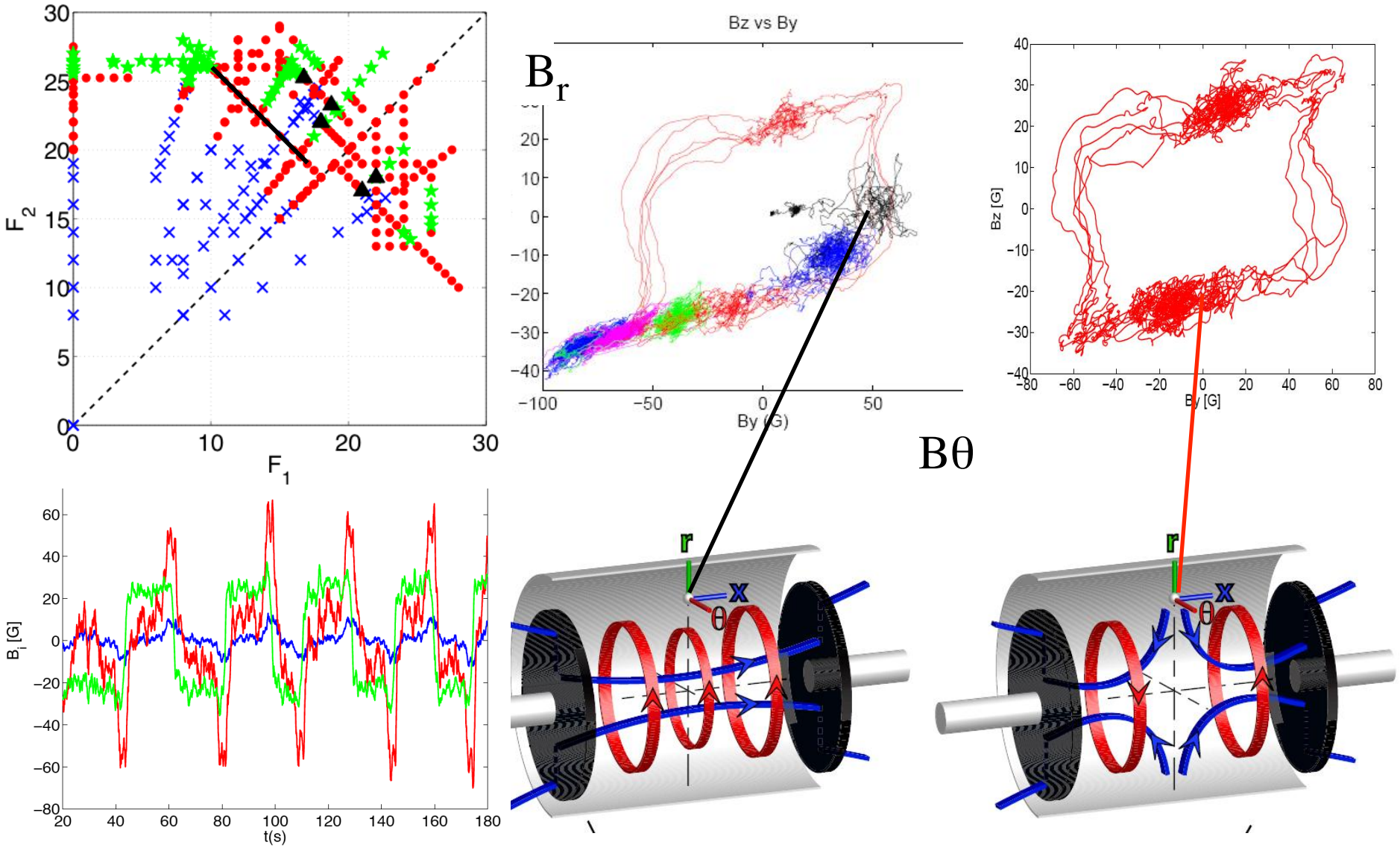




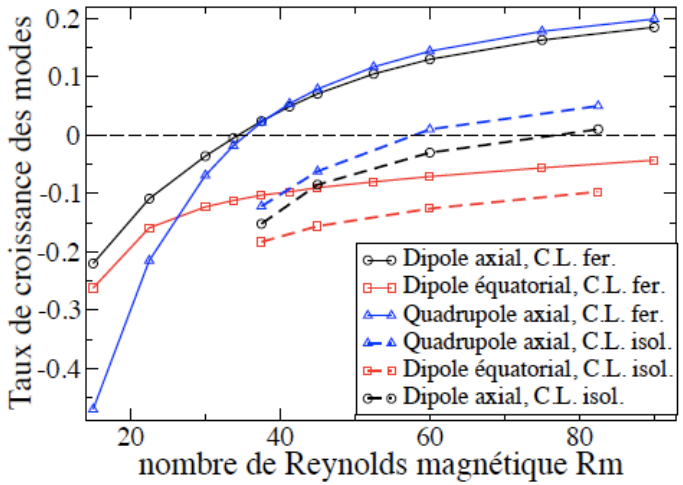
Augmentation de température telle que Rm augmente (Re diminue) de 5%

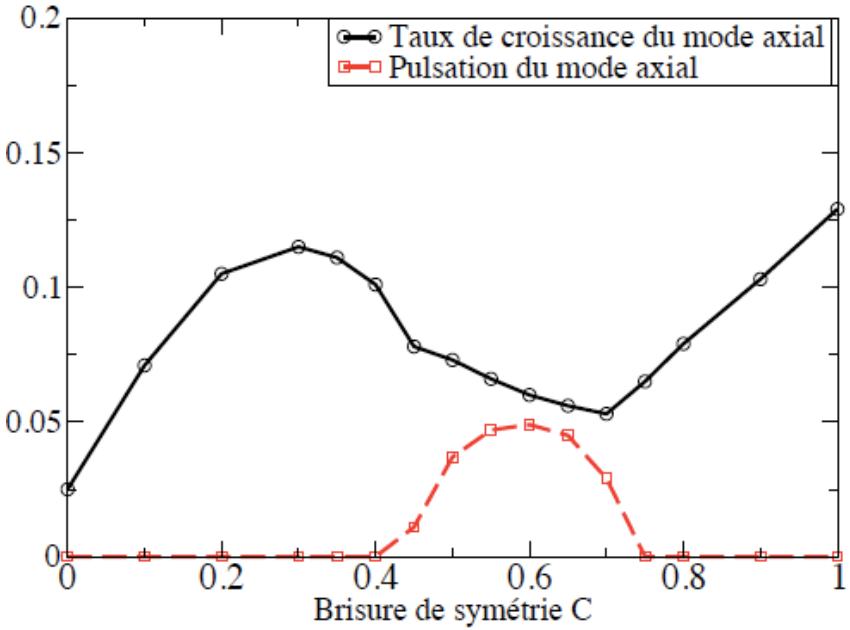
Il est donc difficile d'imaginer que les fluctuations turbulentes soient le mécanisme dominant des renversements

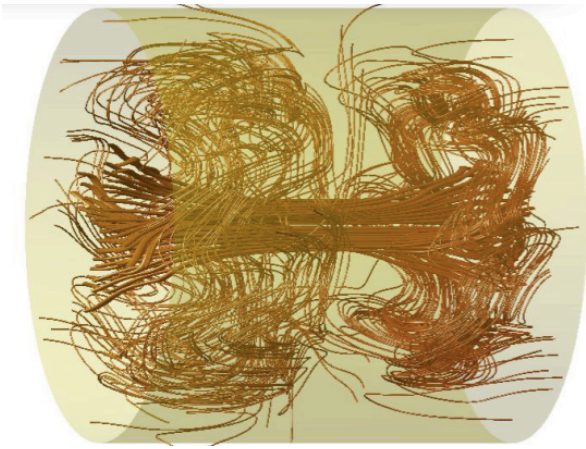
De la dynamo stationnaire au champ périodique



Simulations: oscillation dipole-quadrupole en brisant la symétrie du forçage (C. Gissinger, EPL 2009)





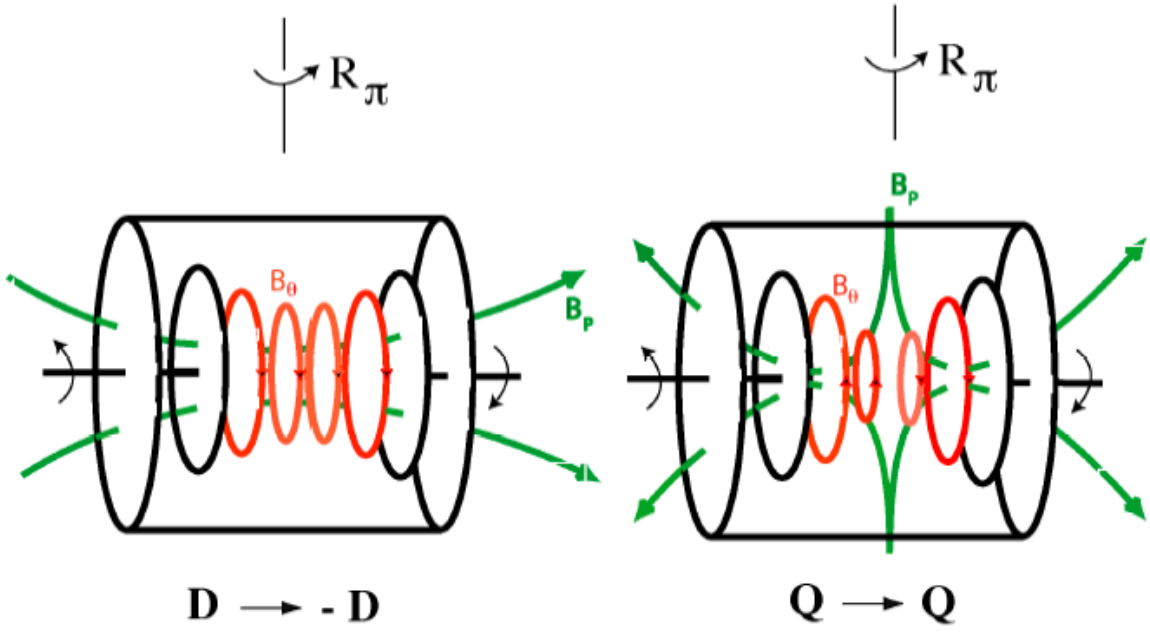


Dipole

Quadrupole

Competition entre modes dipolaires et quadrupolaires

F. Pétrélis, S. Fauve, J. Phys. Cond. Matt. 20, 494203 (2008)



$$\mathbf{B}(\mathbf{r}, t) = d(t)\,\mathbf{D}(\mathbf{r}) + q(t)\,\mathbf{Q}(\mathbf{r}) + \ldots$$

$$d_t = \alpha\, d + \beta\, q - a_1\, d^3 - a_2\, d^2 q - a_3\, d q^2 - a_4\, q^3$$

$$q_t = \gamma\, d + \delta\, q - b_1\, d^3 - b_2\, d^2 q - b_3\, d q^2 - b_4\, q^3$$

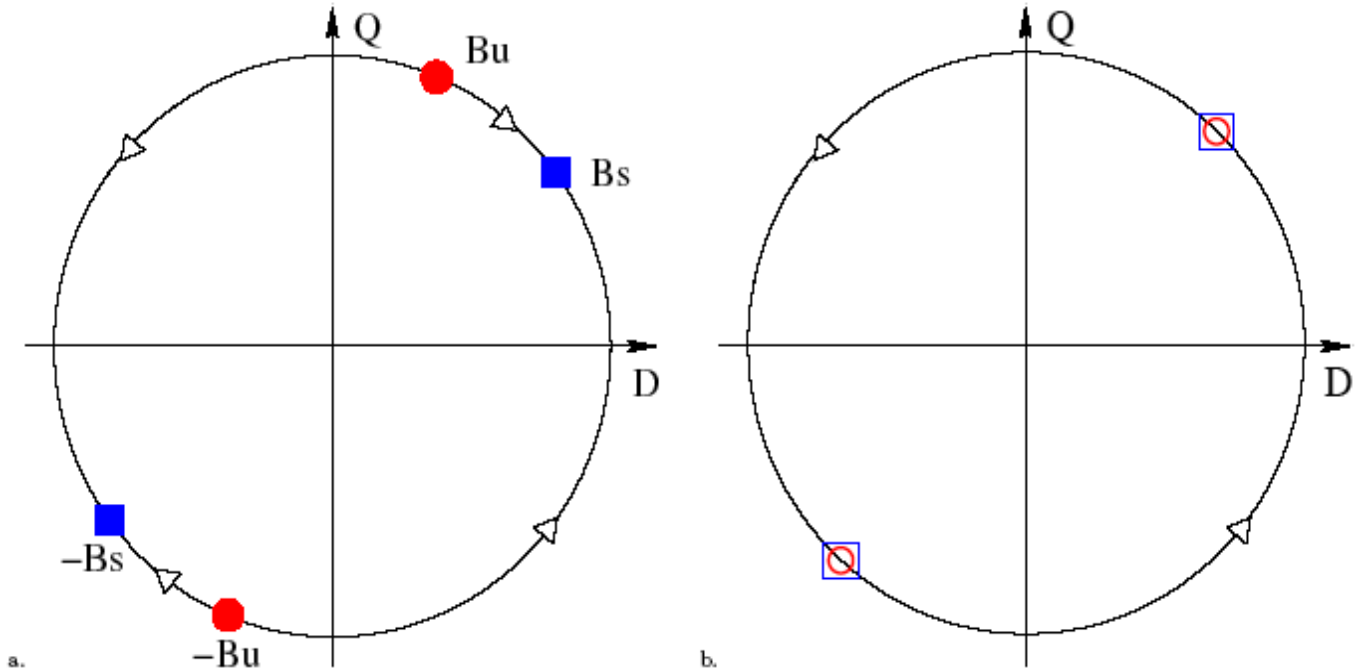
La brisure de symétrie couple modes dipolaire et quadrupolaire

Un cycle limite engendré par une bifurcation nœud-col

$$A = d + iq = R \exp i(\theta + \theta_0)$$

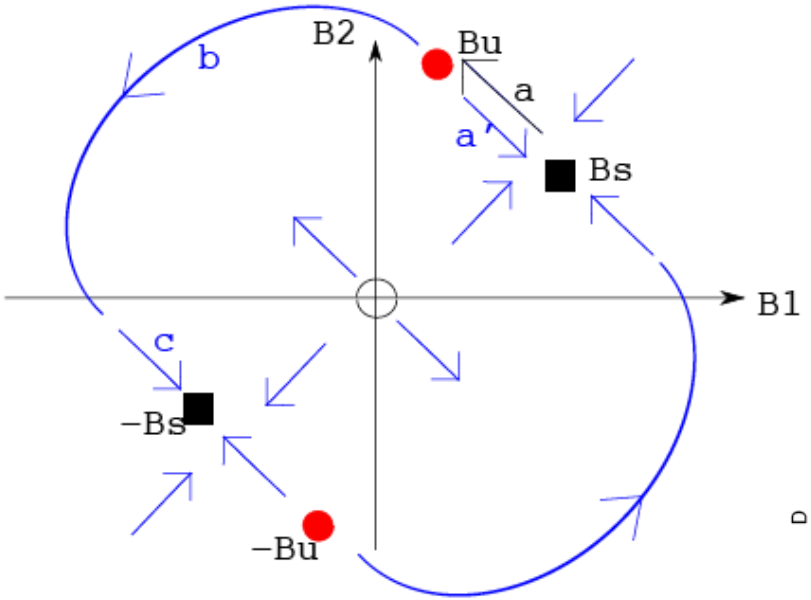
$$A_t = (\mu + i\,\nu)\,A + \rho\,A^* - \alpha\,A^3 - \beta\,A^2 A^* - \gamma\,A\,A^{*2} - \delta\,A^{*3}$$

$$\theta_t = \nu - \rho \sin 2\theta + \text{termes en } \cos 4\theta, \sin 4\theta$$



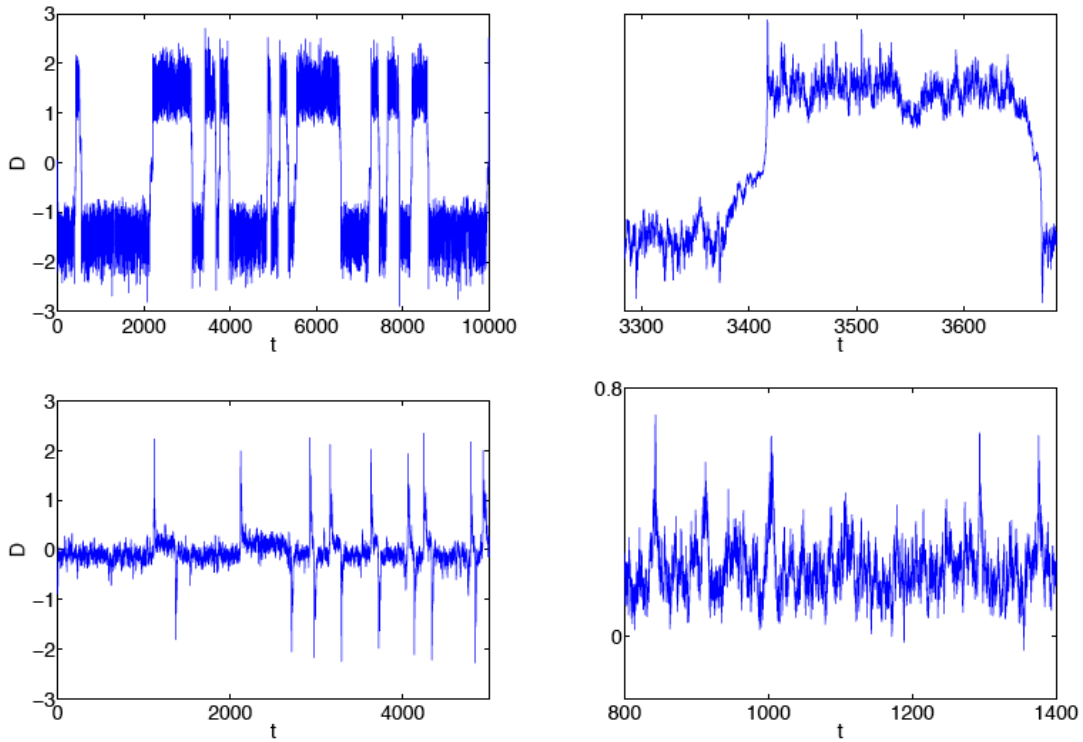
Augmenter l'asymétrie du forçage (ν) engendre un cycle limite

Mécanisme générique pour un système à 2 modes possédant la symétrie B-> -B



Connection de B à -B

Renversements induits par fluctuations avant la bifurcation nœud-col

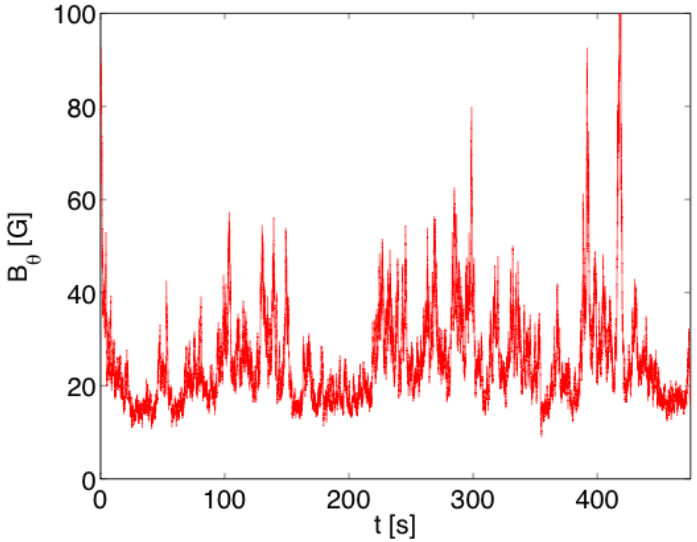


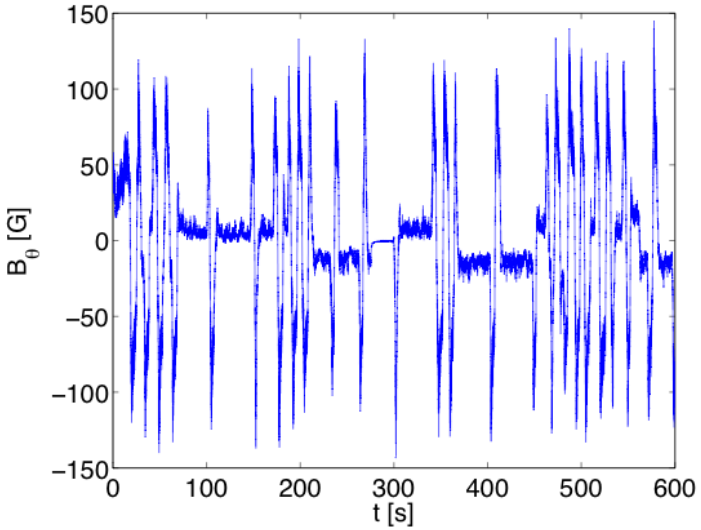
Predictions:

- forme des renversements
- échelles de temps
- transition continue renv-osc

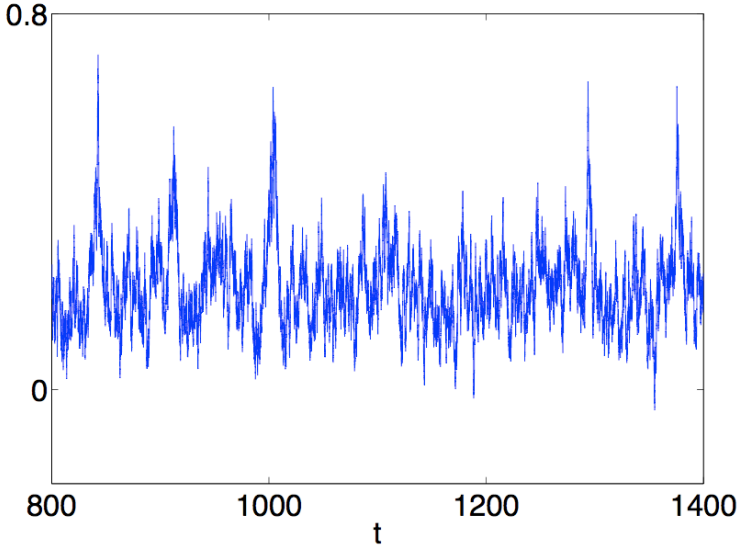
Bouffées intermittentes de champ magnétique

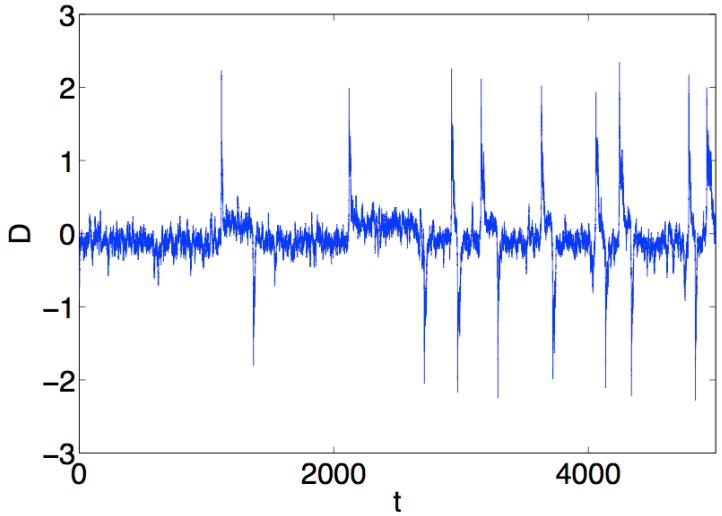
Expérience VKS



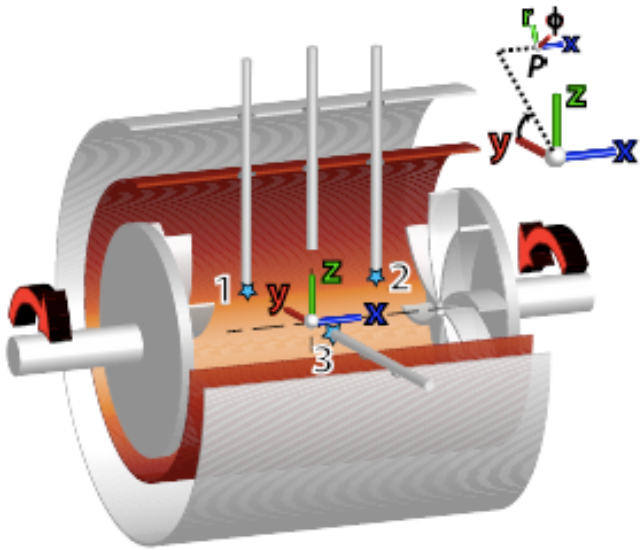


Modèle D-Q



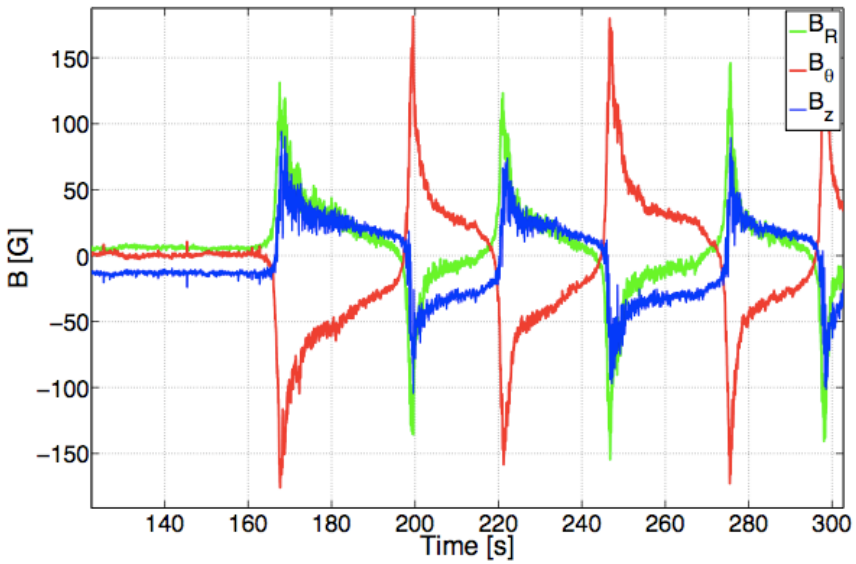


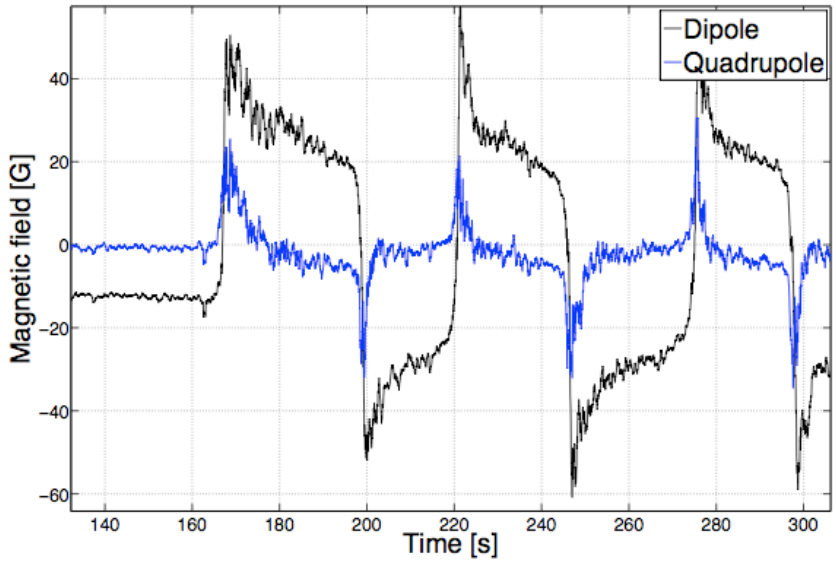
Interaction dipole-quadrupole dans l'expérience VKS



C. Gissinger, thèse 2009, PRE 2010

D : B(1) + B(2)
Q : B(1) – B(2)



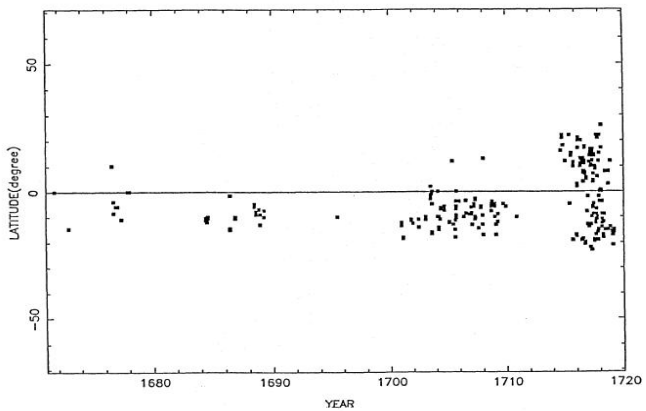


Renversements ou localisation du champ magnétique ?

Certaines dynamos astrophysiques sont hémisphériques

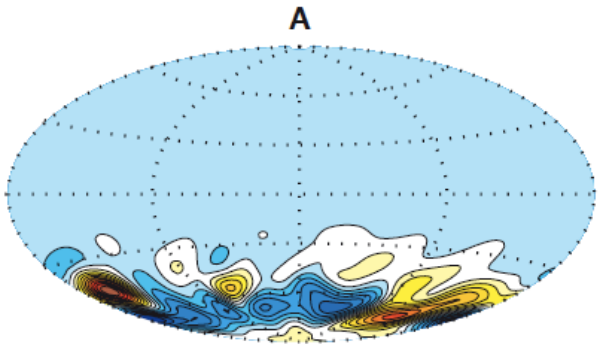
- Le soleil (minimum de Maunder)

Ribes and Nesme-Ribes, A&A 1993



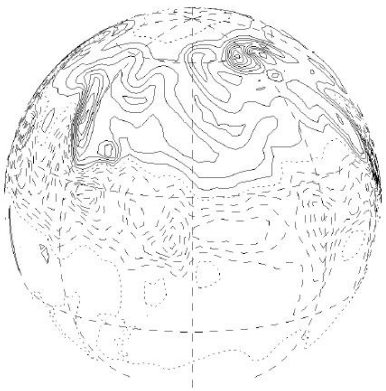
- Champ à la surface de Mars

Stanley et al. Science 2008

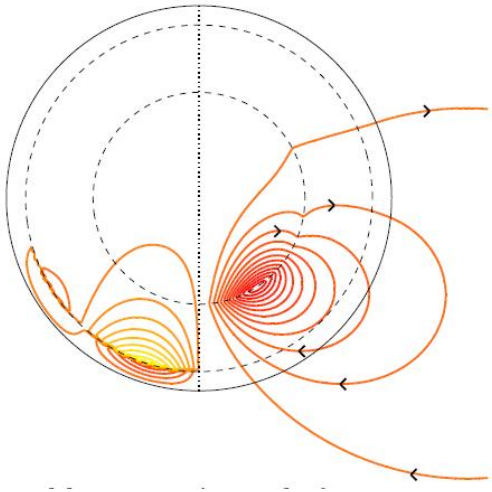


- Simulations numériques

Grote et Busse PRE 2001
Landeau et Aubert 2010



Une faible brisure de symétrie engendre une dynamo hémisphérique



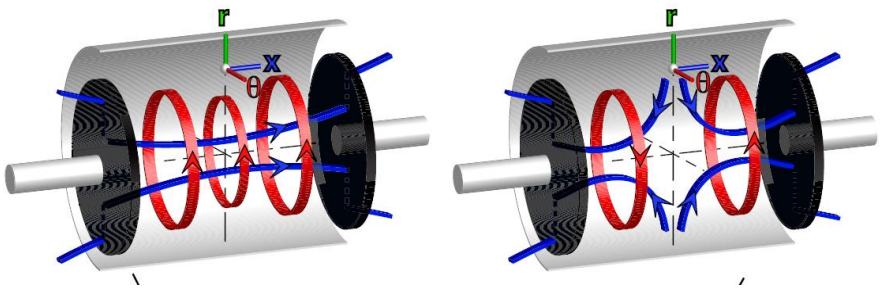
B. Gallet et F. Pétrélis, PRE 2009

Solution stationnaire: 95% de l'énergie dans l'hémisphère sud à la surface (r=R) alors que les effets qui brisent la symétrie valent moins de 1 % des effets symétriques

Modèle dipole-quadrupole: $$\dot{\theta} = \mu_i + \nu_i \cos(2\theta) - \nu_r \sin(2\theta)$$

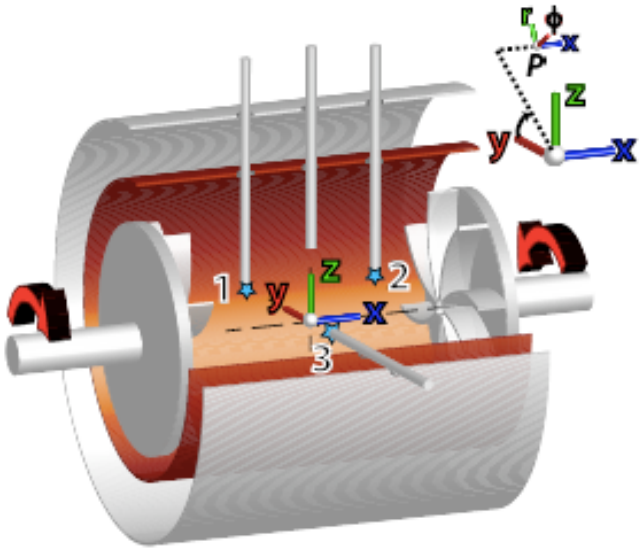
- μ_i grand: renversements
- ν_i grand: localisation, D = ± Q
- ν_r petit si D et Q

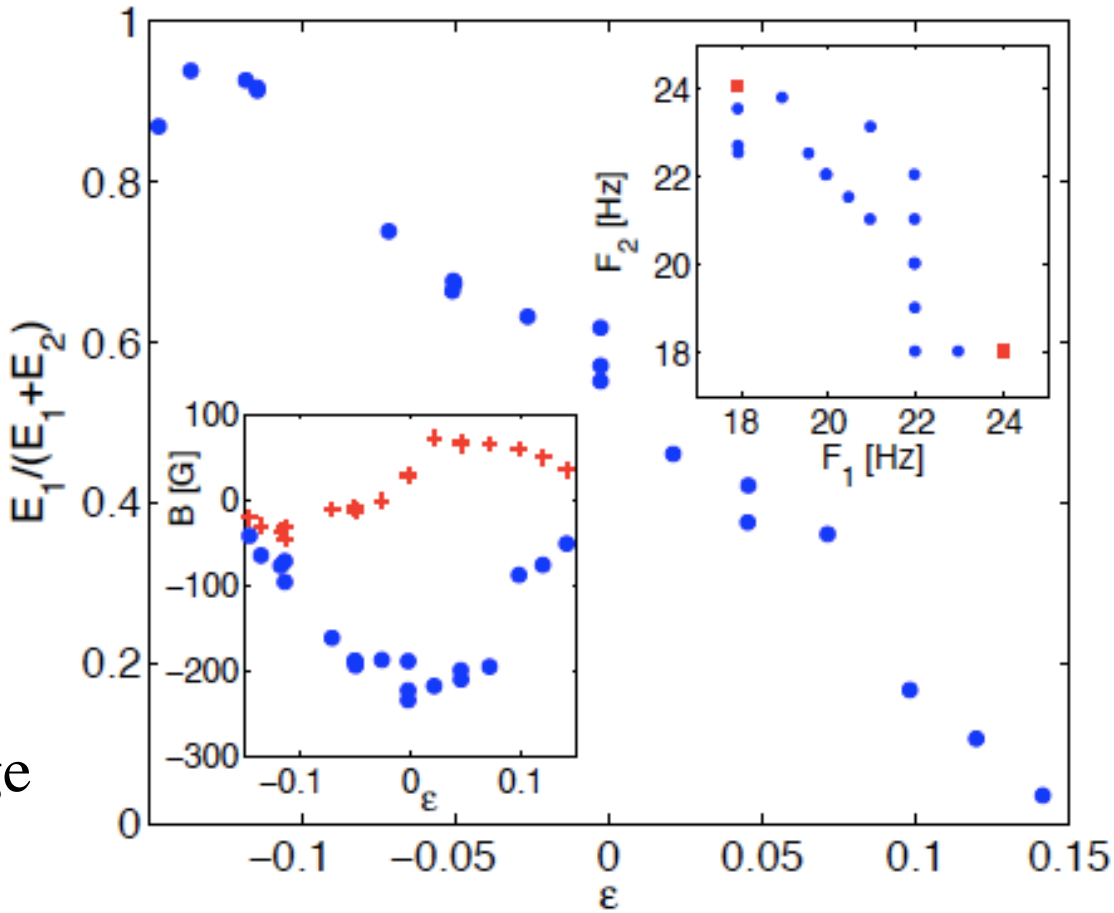
ont des seuils voisins



Dynamo localisée dans l'expérience VKS

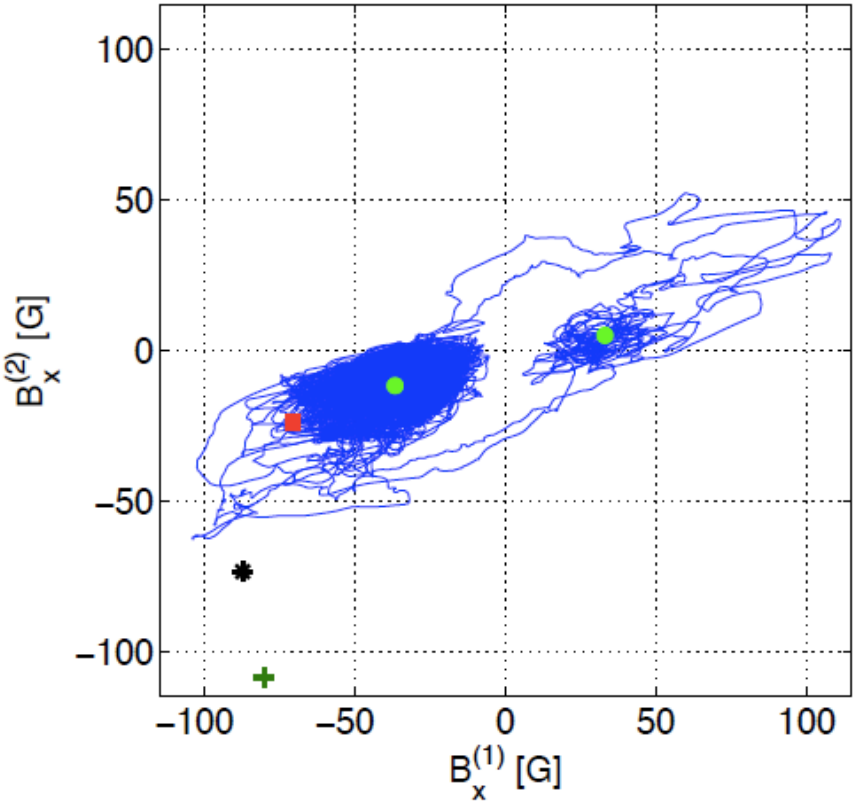
Gallet, thèse 2011

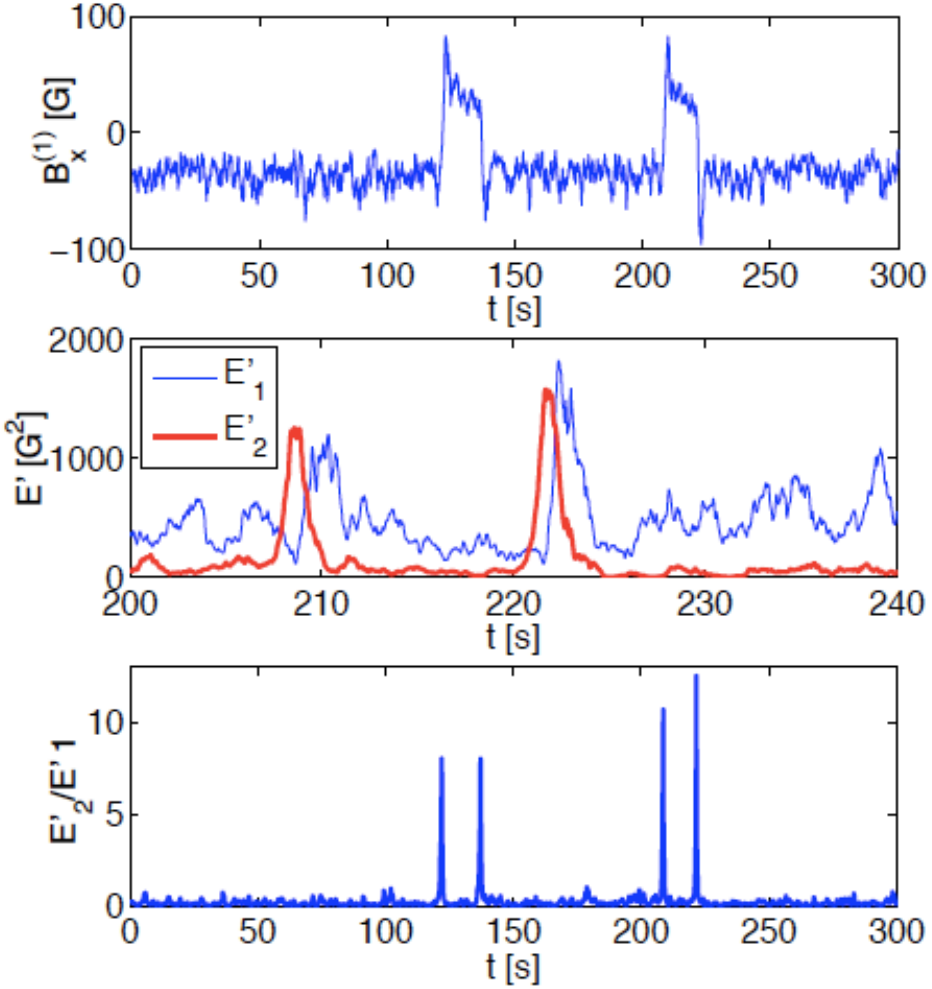




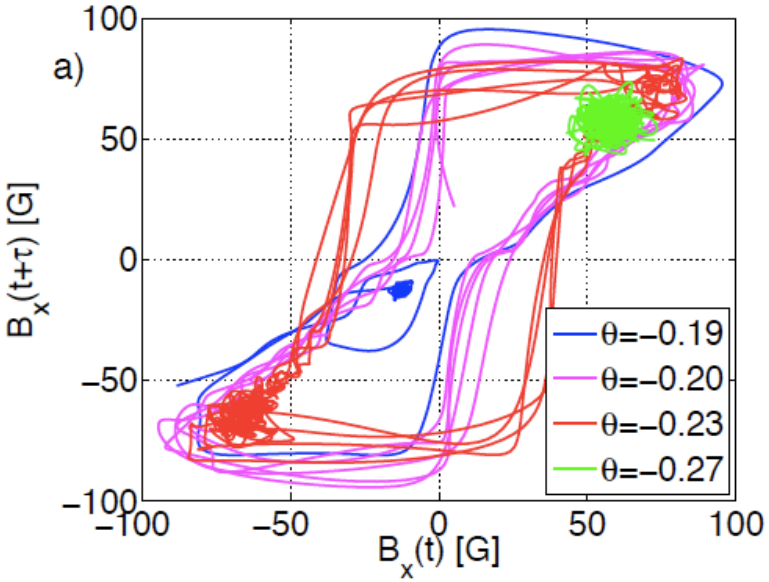
Localisation du champ magnétique au voisinage du disque le plus lent

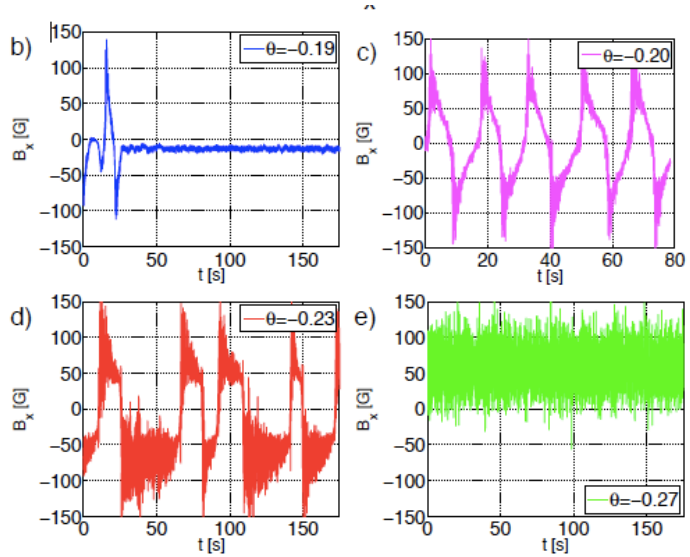
Localisation du champ magnétique durant un renversement

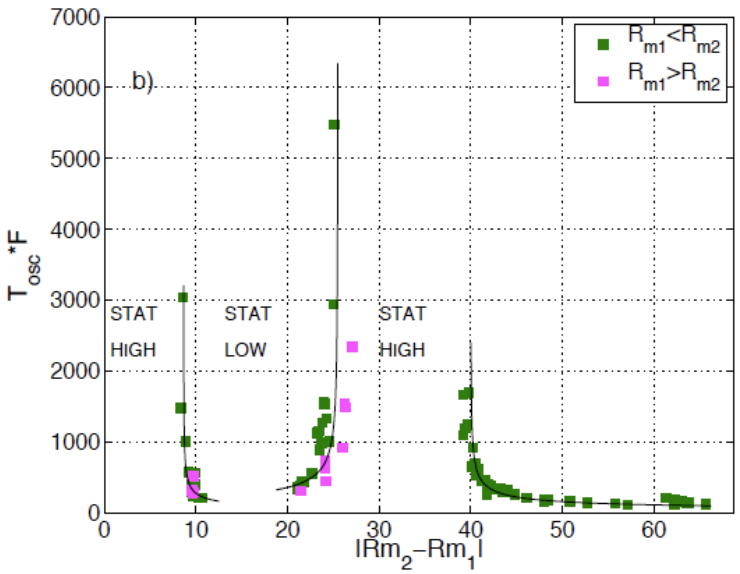


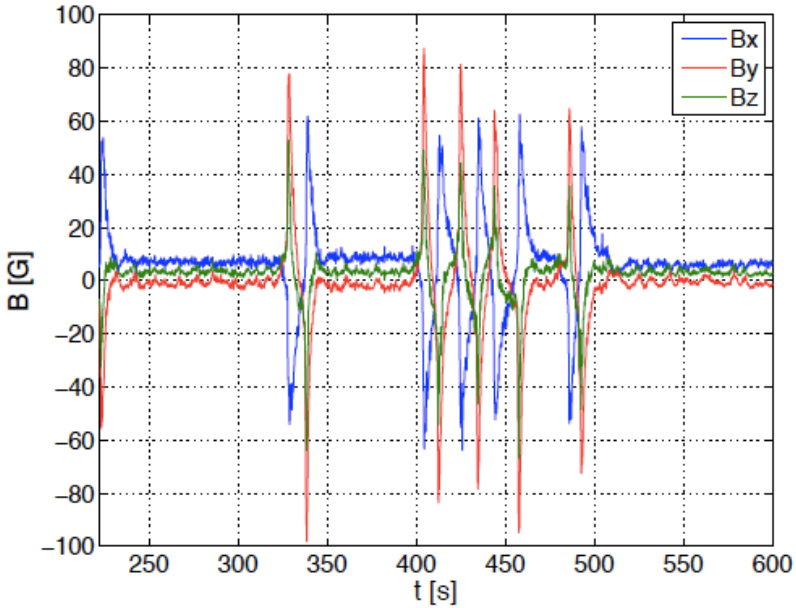


Bifurcation nœud-col ou bistabilité

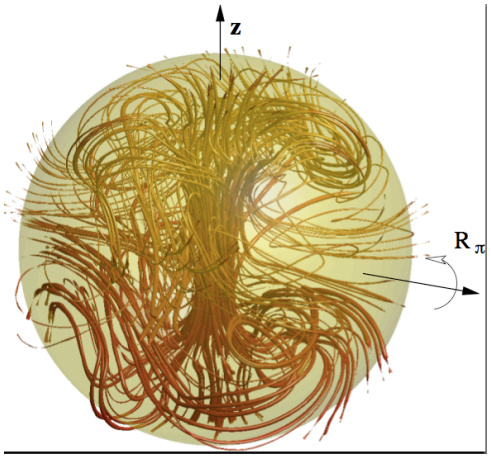


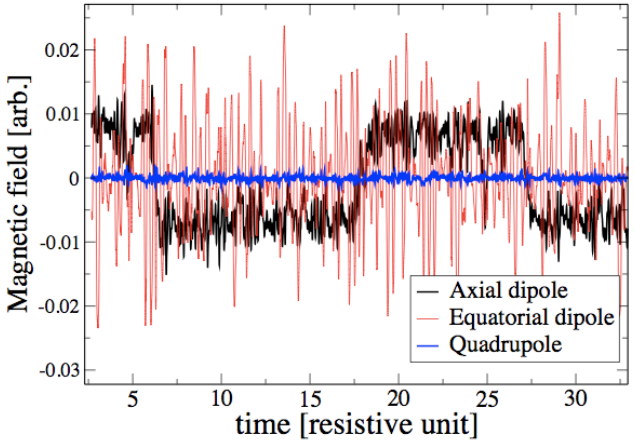




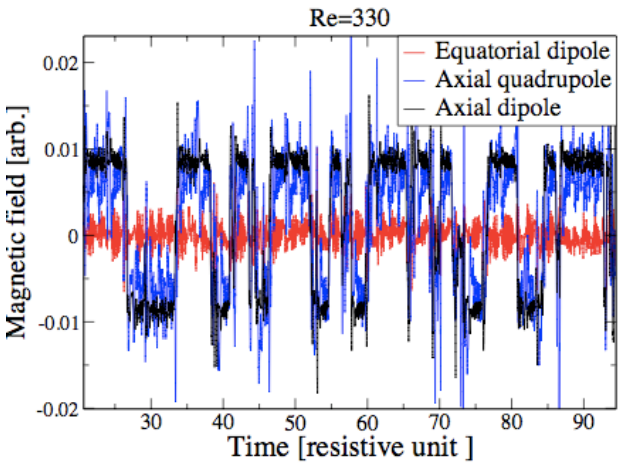


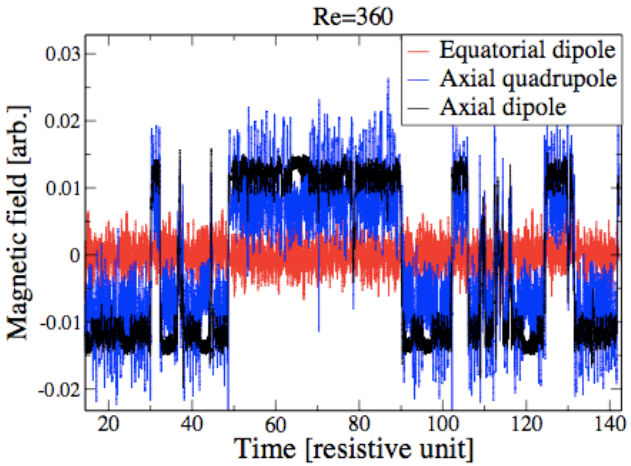
Différentes géométries de la trajectoire d'un renversement: simulations numériques directes (Gissinger et al., EPL 2010)





Pm = 1: axial and equatorial dipoles

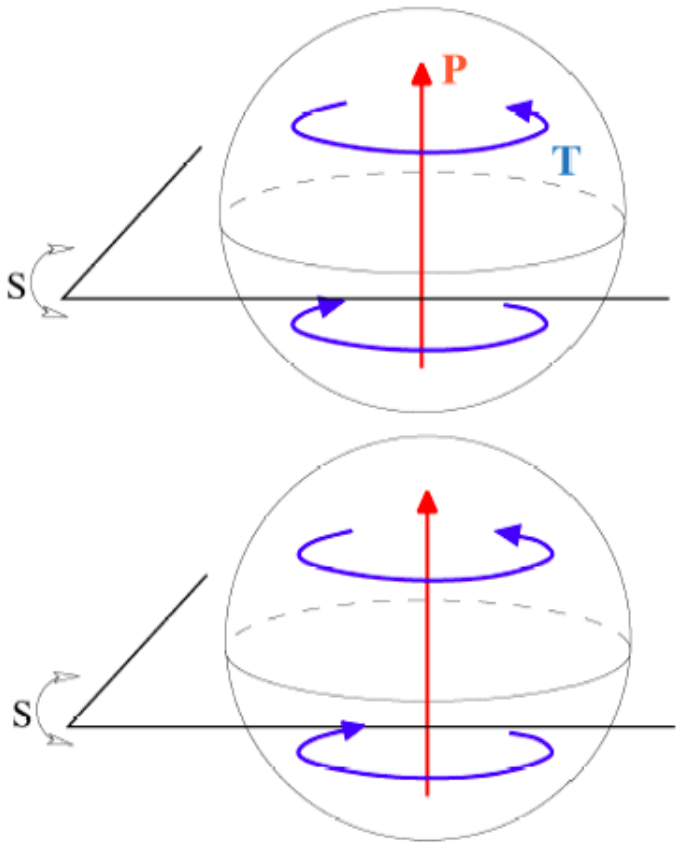




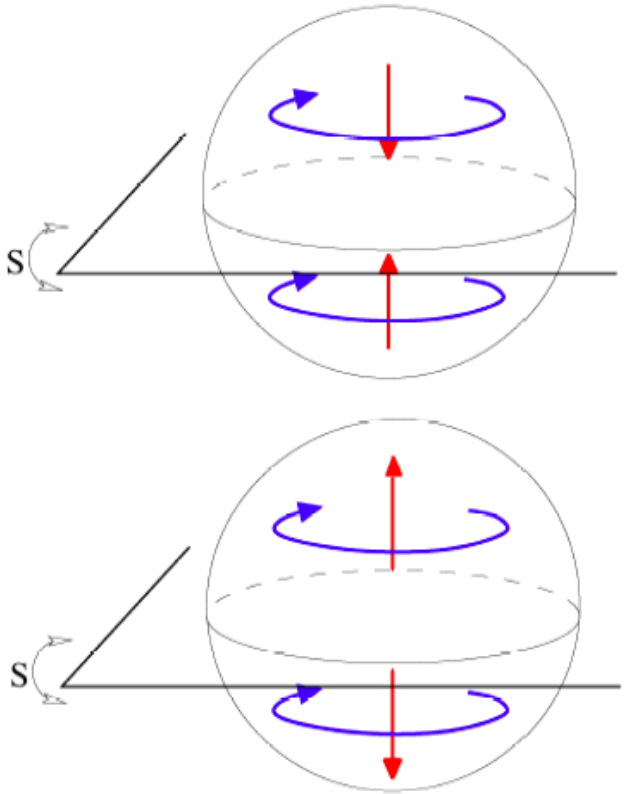
Pm = 0.5 : axial dipole and quadrupole

Les renversements du champ terrestre résultent de la brisure de la symétrie équatoriale de l'écoulement dans le noyau ?

F. Pétrélis, S. Fauve, E. Dormy, J. P. Valet,
Phys. Rev. Lett. 102, 144503 (2008)

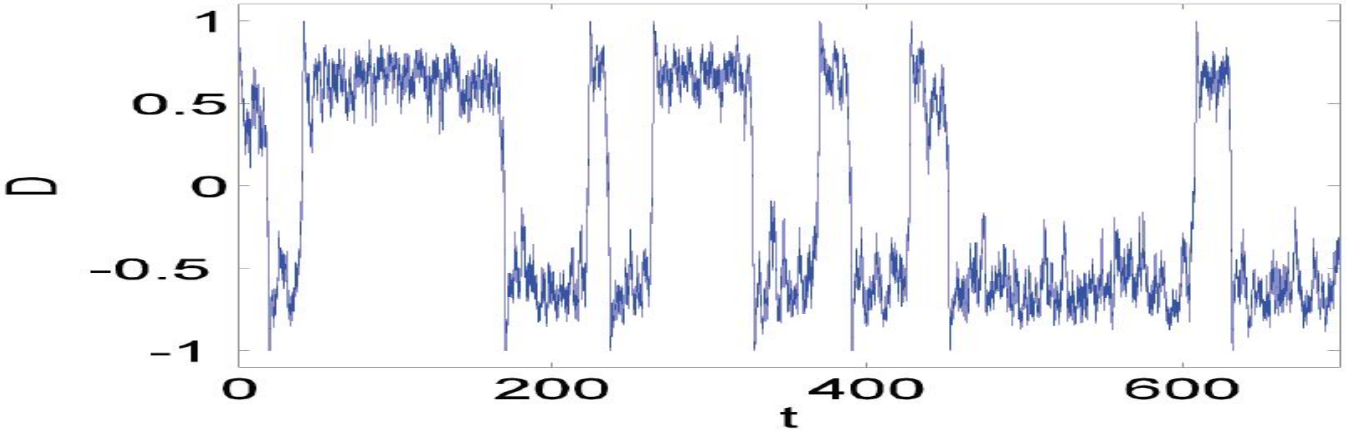


Dipole : D -> D

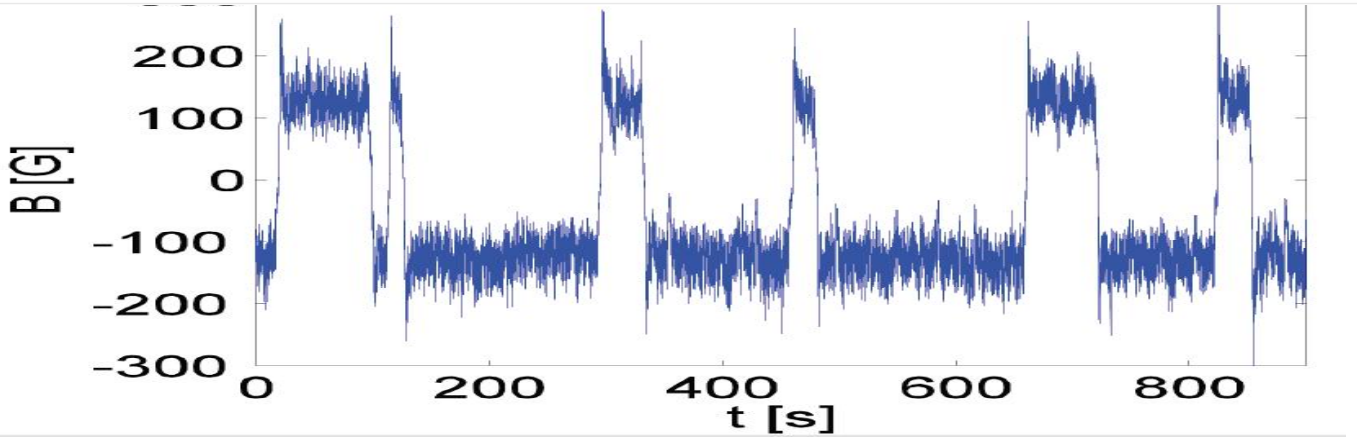


Quadrupole : Q -> - Q

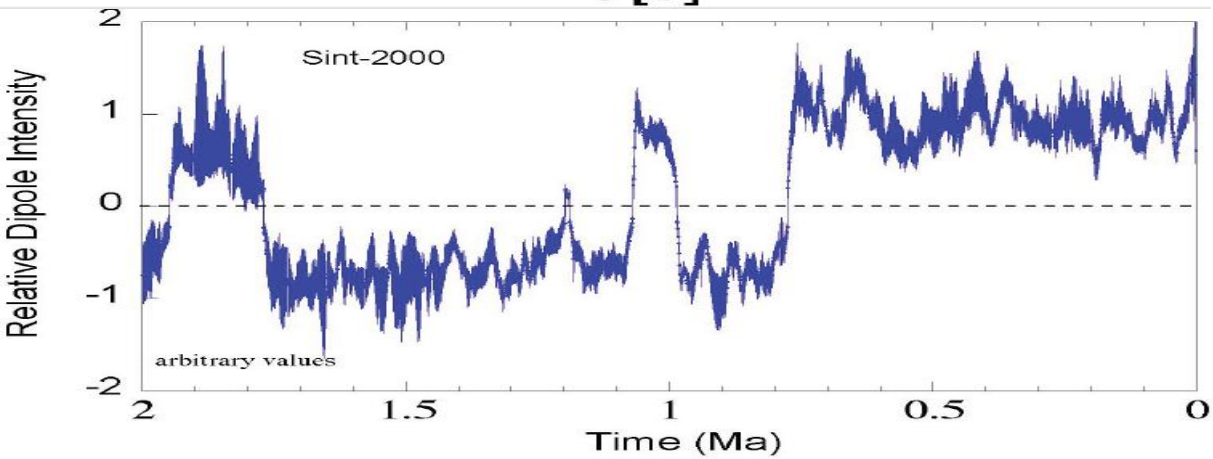
• Modèle



• VKS

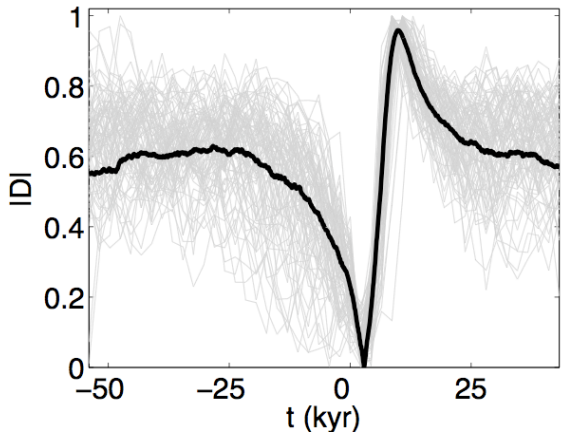


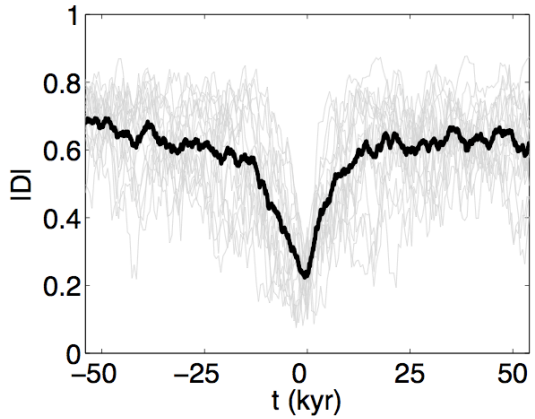
• Dipôle Terrestre



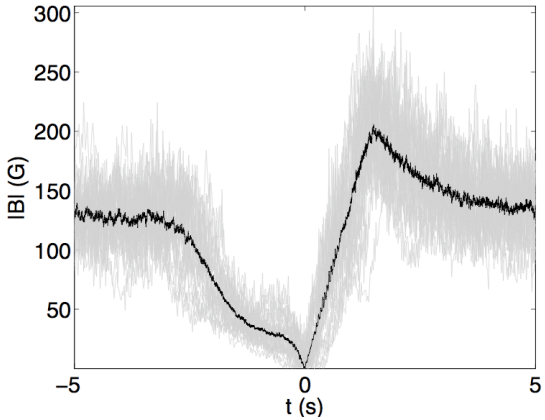
Modèle

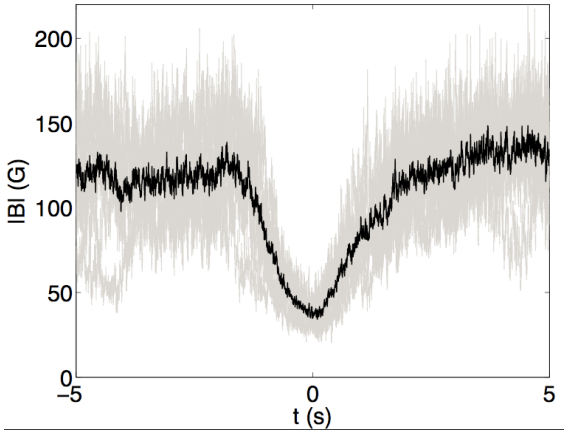
$\theta_t = \nu - \rho \sin 2\theta + \xi(t)$



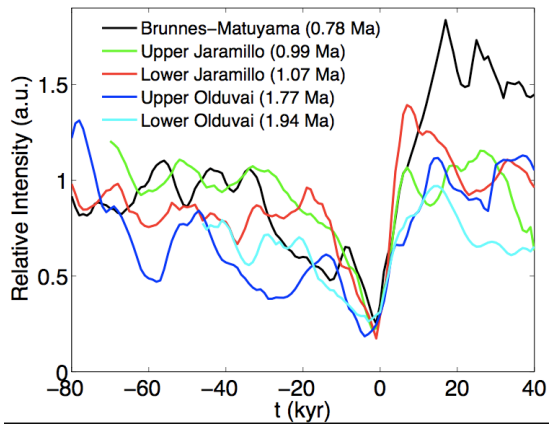


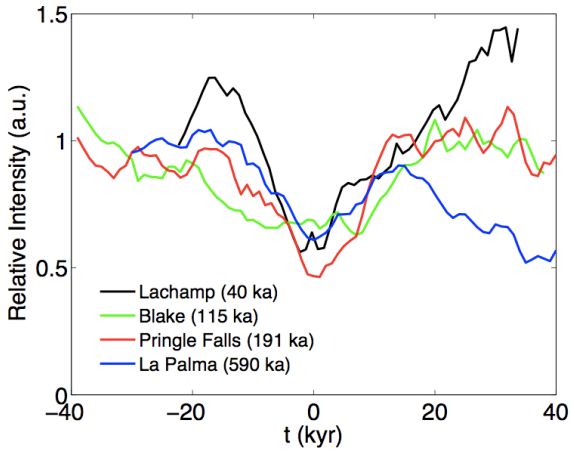
VKS



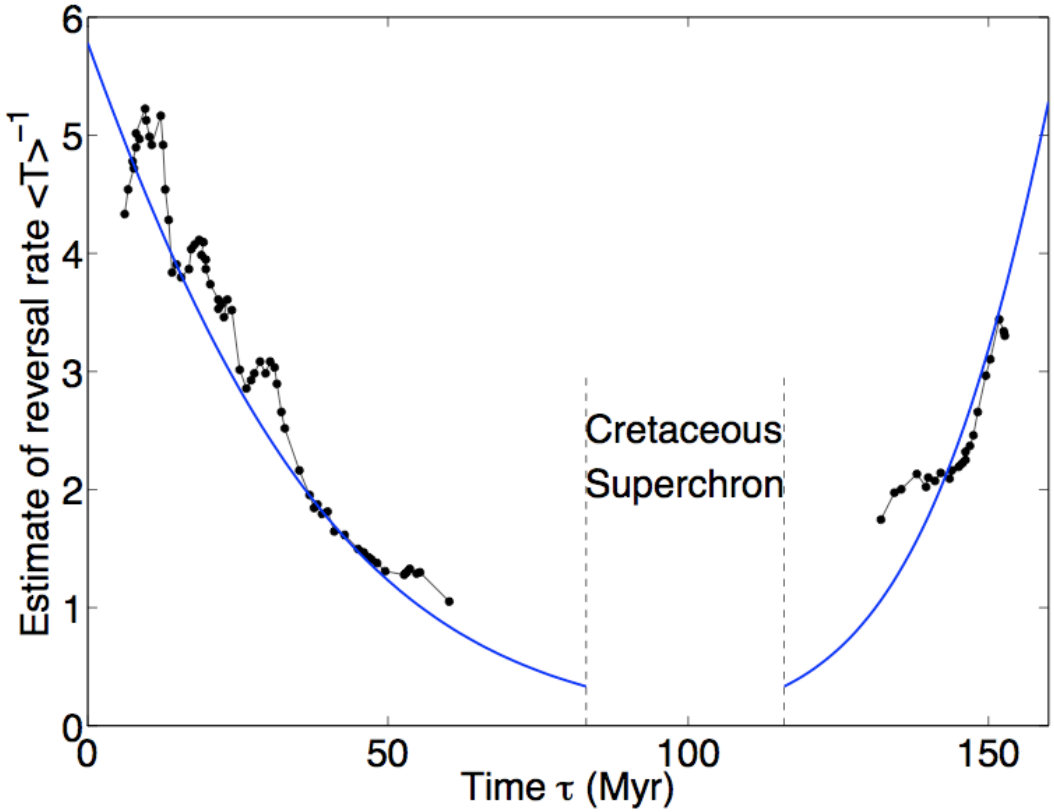


Terre (Valet et al.)





Variation de la fréquence de renversement



Données de
McFadden et al.
J. Geophys. Res. (2000)

Pétrélis et al., PRL 2010

$$\dot{\theta} = \alpha_0 + \alpha_1 \sin(2\theta) + \Delta\zeta(t)$$

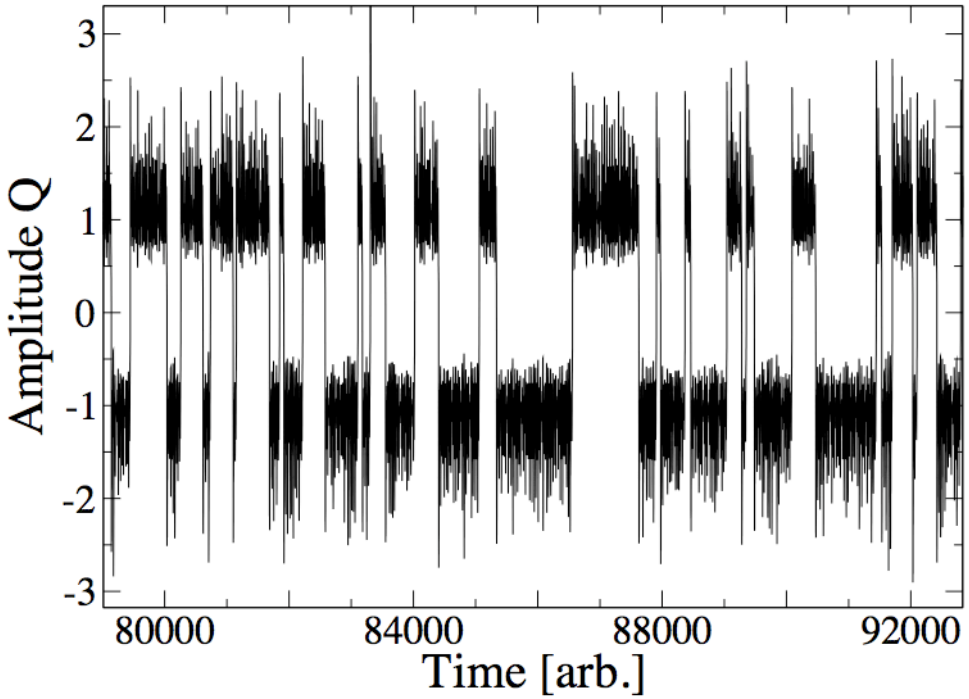
$$\langle T \rangle = \frac{\pi}{\sqrt{\alpha_1^2 - \alpha_0^2}} \exp\left[2|\alpha_1| \frac{(2(\alpha_1 + \alpha_0)/\alpha_1)^{3/2}}{3\Delta^2}\right]$$

Un modèle minimal pour les renversements

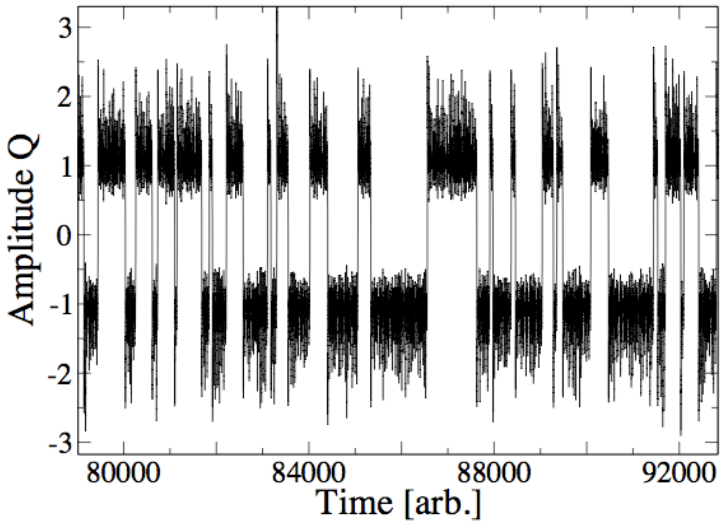
$$\dot{D} = \mu D - VQ \ ,$$
$$\dot{Q} = -\nu Q + VD \ ,$$
$$\dot{V} = \gamma - V + QD \ .$$

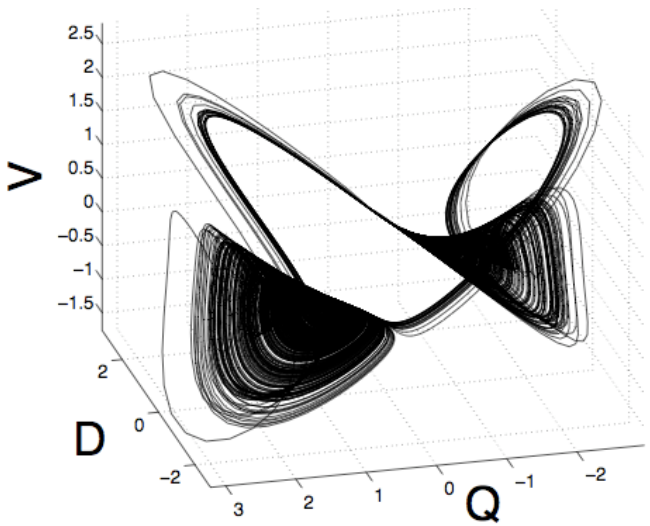
- 2 modes: Dipole et Quadrupole se transforment diff. par S
- 1 mode V brise S

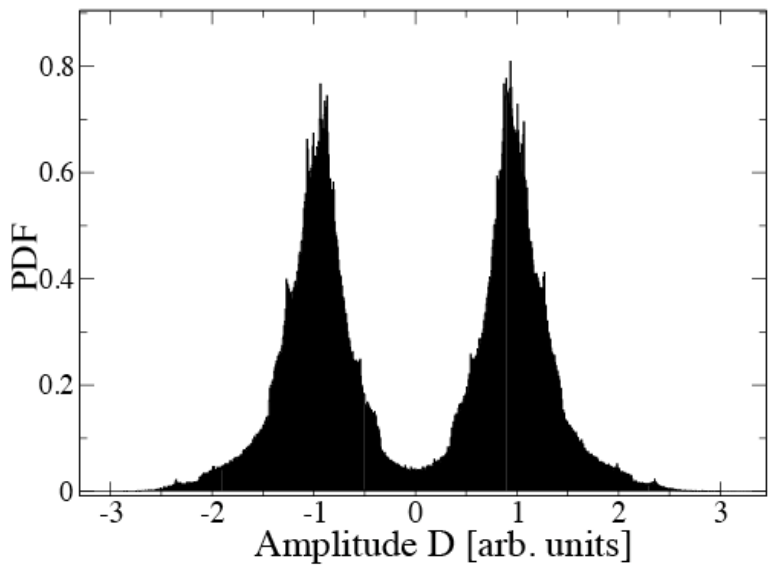
Nozières, Phys. Earth Planetary Int. 1978



Modèle minimal

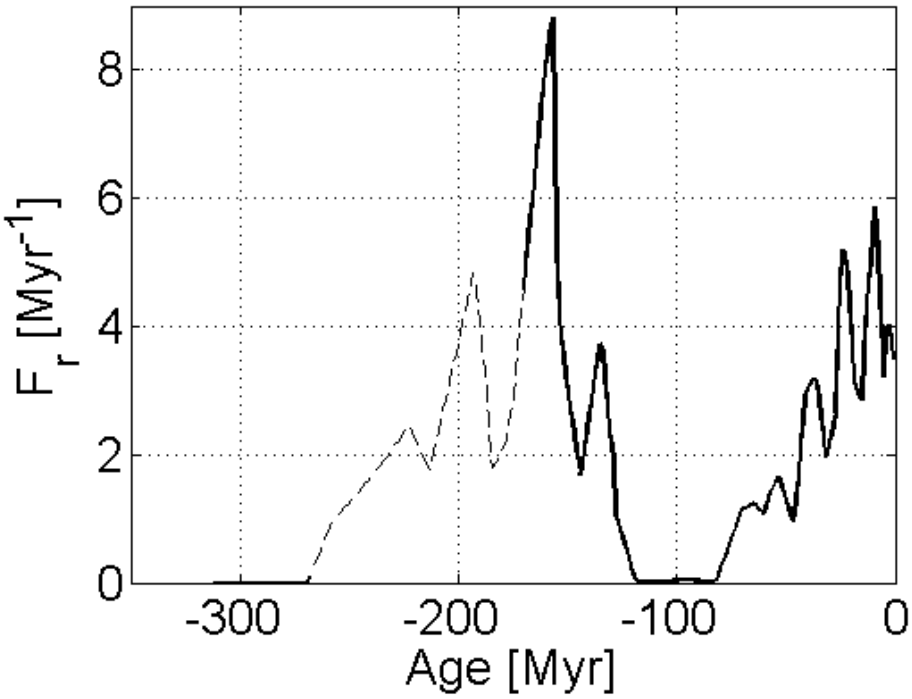






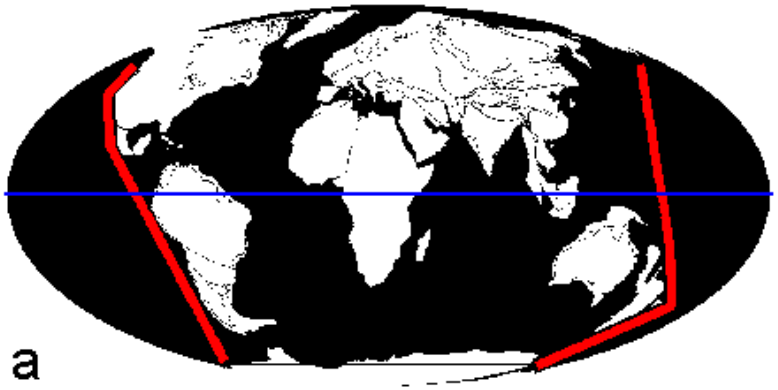
Fréquence des renversements du champ magnétique terrestre et dérive des continents

F. Pétrélis (ENS), J. Besse et J.-P. Valet (IPGP)

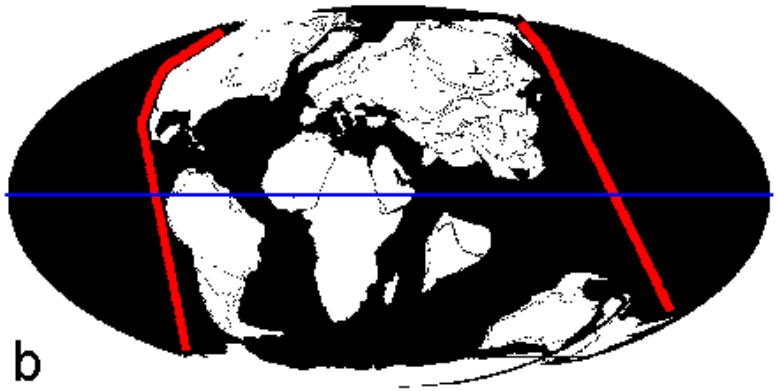


Echelle de temps comparable à celle du manteau, de la dérive des continents

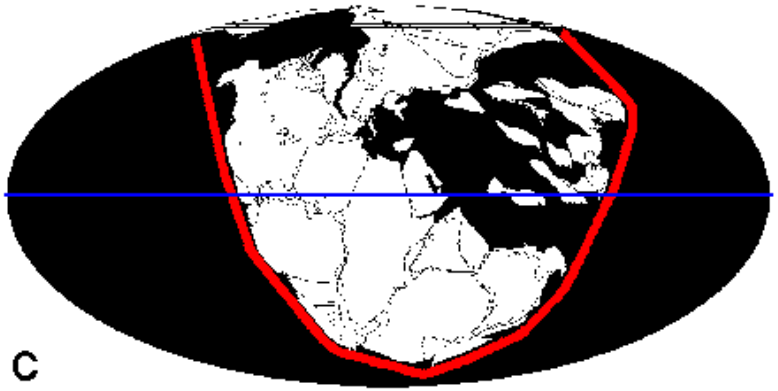
Répartition des continents



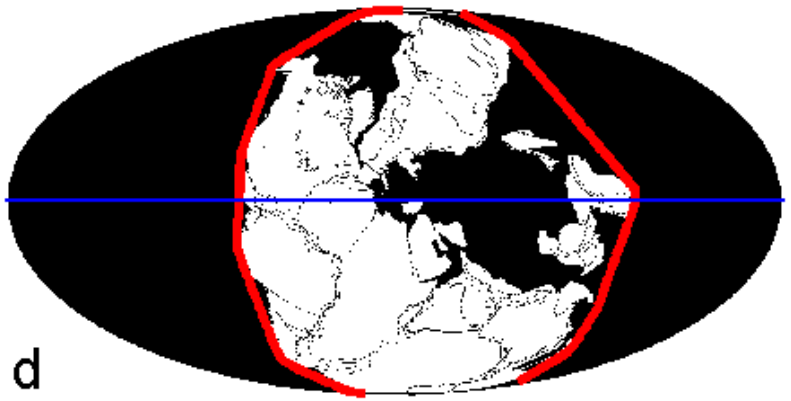
Présent



Age 65 Ma

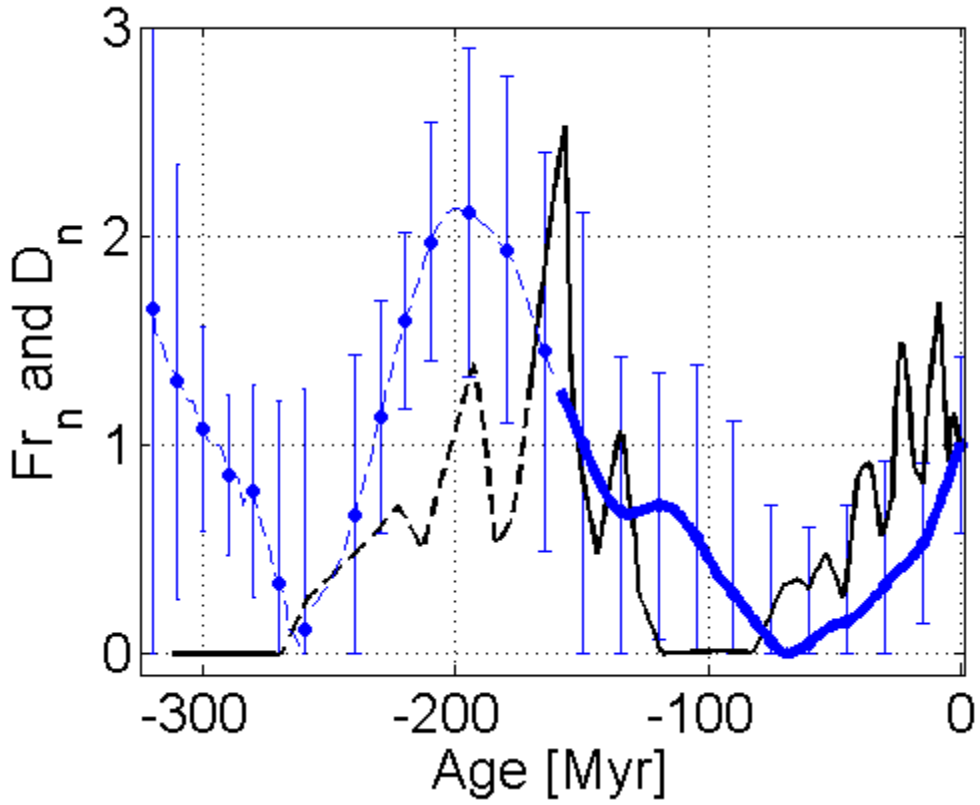


Age 200 Ma



Age 260 Ma

Fréquence des renversements du champ magnétique et symétrie de la répartition des continents par rapport à l'équateur



Conclusions

• Renversements du champ magnétique
- contrôlés par quelques modes
- peu affectés par les fluctuations turbulentes

• Pas de mécanisme externe nécessaire

• Le renversement du dipole axial implique le couplage avec un autre mode magnétique, le quadrupole.

• Pour cela l'écoulement qui crée le champ magnétique doit briser la symétrie permettant de coupler dipole et quadrupole