Complements on Homographies

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Complements on Homographies

- **Homography** $\rightarrow$ Most general case of 2d projective transformation

$$\tilde{m}' = H\tilde{m}$$

- 8 degrees of freedom $\rightarrow$ At least four non colinear 2d points!
- Corresponds to 2 particular cases of image pairs:
  - 3d scene viewed under pure rotation around the optical centre ($\mathbf{t} = O_3$).
  - Same plane viewed under two different 3d poses.

Presentation Outline

1. Rotation around the optical centre

2. Plane viewed from different poses

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Rotation around the optical centre

In the case of a pure rotation around the optical centre ($\mathbf{t} = O_3$), the projected image transformation is a homography:

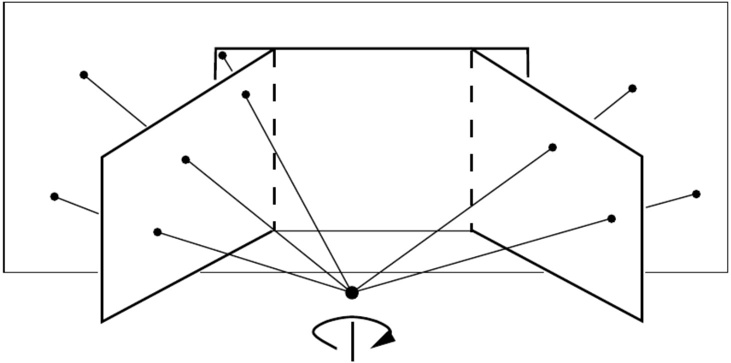


Figure from [**Hartley and Zisserman 2004**]

Rotation around the optical centre

Since $\mathbf{t} = O_3$ we get:

$$\tilde{m} = \left(\begin{array}{c|c} K & O_3 \end{array} \right) \tilde{M}$$

$$\tilde{m}' = \left(\begin{array}{c|c} K & O_3 \end{array} \right) \left(\begin{array}{c|c} R & {\color{red}O_3} \\ \hline O_3^t & 1 \end{array} \right) \tilde{M}$$

which can be written more simply:

$$\begin{array}{rcl} \tilde{m} & = & KM \\ \tilde{m}' & = & KRM = \underbrace{KRK^{-1}}_{H} \tilde{m} \end{array}$$

Rotation around the optical centre

Note the difference between rotation around the optical centre ((a) to (b)), and translation ((a) to (c)):

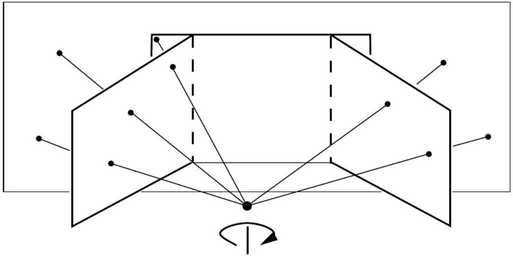
a b c

Images from [**Hartley and Zisserman 2004**]

Rotation around the optical centre

Since there is no parallax, the images can be stitched to form a mosaic:





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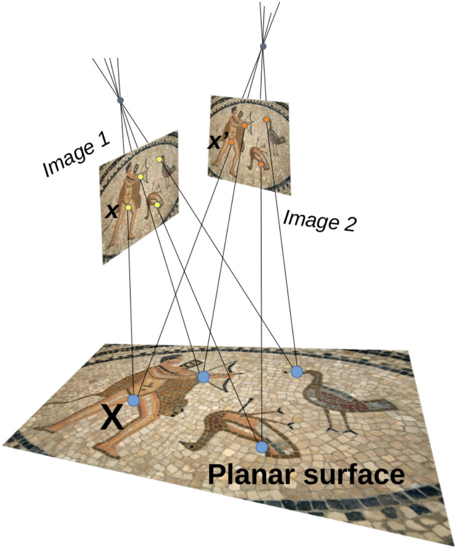
Plane viewed from different poses

$$\tilde{x} = H_{\pi,1} X$$

$$\tilde{x}' = H_{\pi,2} X$$

$$\tilde{x}' = H_{\pi,2} H_{\pi,1}^{-1} \tilde{x} = H_\pi \tilde{x}$$



Plane viewed from different poses

Let us first assume that $K = I_3$ (i.e. $f = 1, c_x = c_y = 0$). Then if the pose of the right camera is given by rotation matrix R and translation vector $\mathbf{t}$, we get:

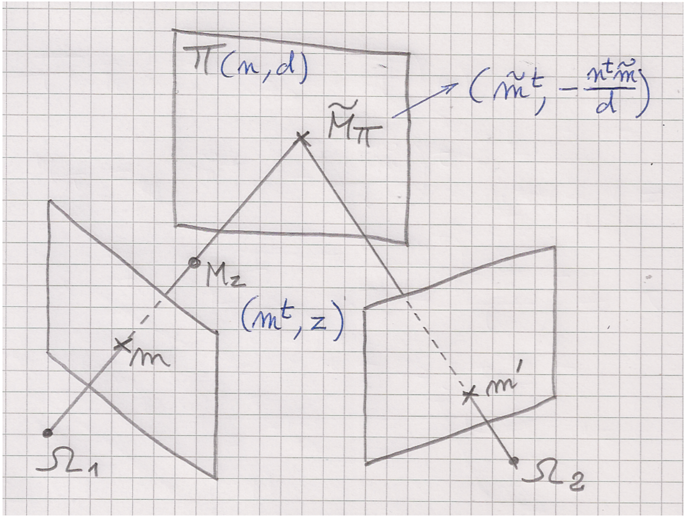
$$\tilde{m} = P\tilde{M} = \left(\begin{array}{c|c} I_3 & O_3 \end{array}\right)\tilde{M}$$

$$\tilde{m}' = P'\tilde{M} = \left(\begin{array}{c|c} R & \mathbf{t} \end{array}\right)\tilde{M}$$

Every point on the ray $M_z = (m^t, z)$ (parameterized by z) projects on m.
If the point M_z is on the plane π, it must satisfy: $\pi^t.\tilde{M}_z = 0$.
If the coordinates of the plane are given as $\pi = (\mathbf{n}^t, d)^t$, so that for points M on the plane, we have: $\mathbf{n}^t M + d = 0$,
then the point of the ray backprojected from m and intersecting plane π is:

$$\tilde{M}_\pi = \left(\tilde{m}^t, -\frac{\mathbf{n}^t\tilde{m}}{d}\right)^t$$

Plane viewed from different poses



Plane viewed from different poses

The point of the ray backprojected from m and intersecting plane π is:

$$\tilde{M}_\pi = \left(\tilde{m}^t, -\frac{\mathbf{n}^t \tilde{m}}{d} \right)^t$$

And then:

$$\begin{aligned} \tilde{m}' &= P' \tilde{M}_\pi = \left(\begin{array}{c|c} R & \mathbf{t} \end{array} \right) \tilde{M}_\pi \\ &= R\tilde{m} - \tfrac{\mathbf{t}\mathbf{n}^t}{d} \tilde{m} \\ &= \underbrace{\left(R - \frac{\mathbf{t}\mathbf{n}^t}{d} \right)}_{H_\pi} \tilde{m} \end{aligned}$$

Finally, by considering the internal parameter matrix K of a single camera moved with rotation R and translation $\mathbf{t}$, the homography related to the plane $\pi = (\mathbf{n}^t, d)^t$ is given by:

$$H = K \left(R - \frac{\mathbf{t}\mathbf{n}^t}{d} \right) K^{-1}$$