

VISION



Antoine Manzanera
ENSTA Paris / U2IS
Institut Polytechnique de Paris
M2 IMA – COMPUTER VISION
Sorbonne Université

COMPUTER VISION COURSE: OBJECTIVES

The objective of the COMPUTER VISION course is to present the techniques of *geometric* (3d) and *kinematic* (movement) analysis of a scene by a vision system, from the mathematical / physical / biological principles, to the algorithmic implementation.

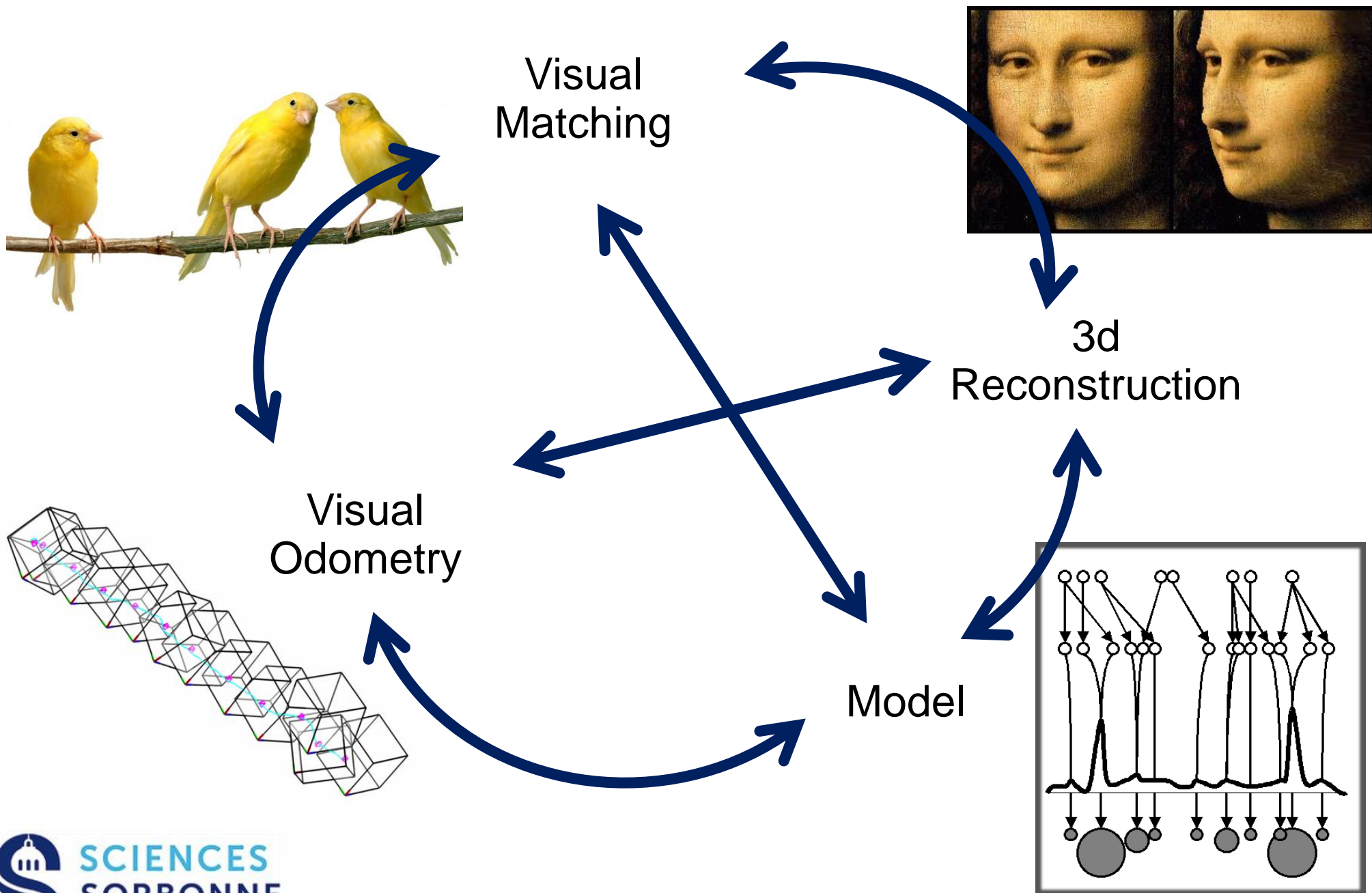
The purpose here is not to recognise the objects or to identify a scene, but to geometrically structure a scene and recover the motion of the objects and the camera.

However, *geometry and movement are fundamental cues* for image understanding.

Displacement analysis is related to 3d reconstruction by the common problem of *feature matching* between different views of the same object.

On the other hand, knowing the 3d geometry of the scene facilitates both the estimation and the interpretation of its objects movement.

KEY CONCEPTS



“COMPUTER VISION” COURSE – SCHEDULE

Theoretical Lectures:

- 3d and Motion Perception in Biological Systems (A. Manzanera)
- Co-design approaches in Computer Vision (A. Manzanera)
- Projective Geometry, Camera matrix, Panorama construction * (P. Monasse)
- Fundamental and Essential matrices, their calculus, RANSAC algorithm * (P. Monasse)
- Local Estimation of Disparity * (P. Monasse)
- Global Estimation of Disparity * (P. Monasse)
- Multi-view Stereo * (P. Monasse)
- Face detection and analysis (K.Bailly)
- 3d and Movement prediction: Machine Learning based methods (A. Manzanera)
- Motion Estimation (optical flow) (D. Béréziat)
- Motion detection and Visual Tracking (A. Manzanera)

* *Common sessions with Master MVA*

“COMPUTER VISION”: PRACTICAL ASPECTS

Lecturers (by order of appearance):

- Antoine Manzanera – U2IS, ENSTA
- Pascal Monasse, – IMAGINE, ENPC
- Kévin Bailly, – ISIR, SU
- Dominique Béréziat, – LIP6, SU

Up-to-date schedule by session, slides and other ressources:

<https://perso.ensta-paris.fr/~manzaner/Cours/IMA/VISION/>

Evaluation Modalities (TBC):

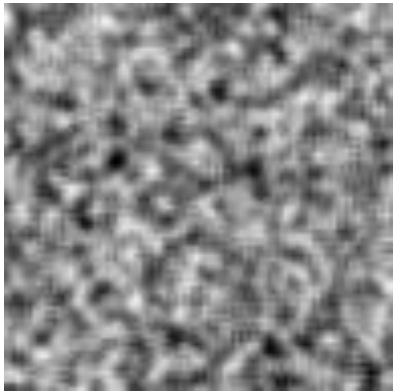
1. 2 or 3 practical projects on *Geometrical 3d Vision*
2. 1 Oral presentation on *Bio + Co-conception*
3. 1 practical work on *Faces*
4. 1 practical work on *Motion Estimation*
5. 1 practical work on *Visual Tracking*

COMPUTER VISION COURSE #1: Movement and 3d: Biological Perception

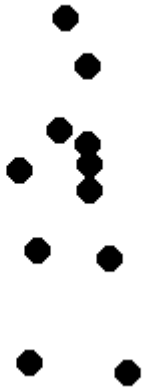
OUTLINE OF THIS LECTURE:

- ❖ The role of motion in biological vision
- ❖ Structure and functions of the retina
- ❖ Contrast perception
- ❖ Local motion perception and its adaptation
- ❖ Optical Flow: parallax, FoE, Time-before-contact
- ❖ Global motion perception
- ❖ Binocular perception of 3d
- ❖ Monocular perception of 3d
- ❖ The role of eye motion in vision

THE ROLE OF MOTION IN NATURAL VISION



Well, a texture...

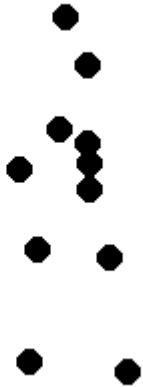
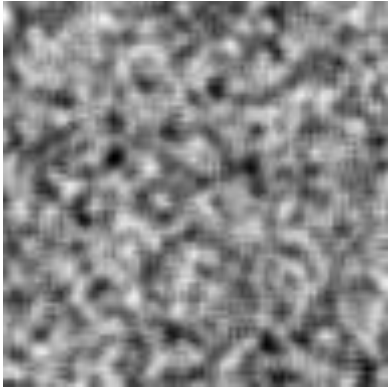


Hmm, a point cloud?



Ugh, an over-segmented image!!!

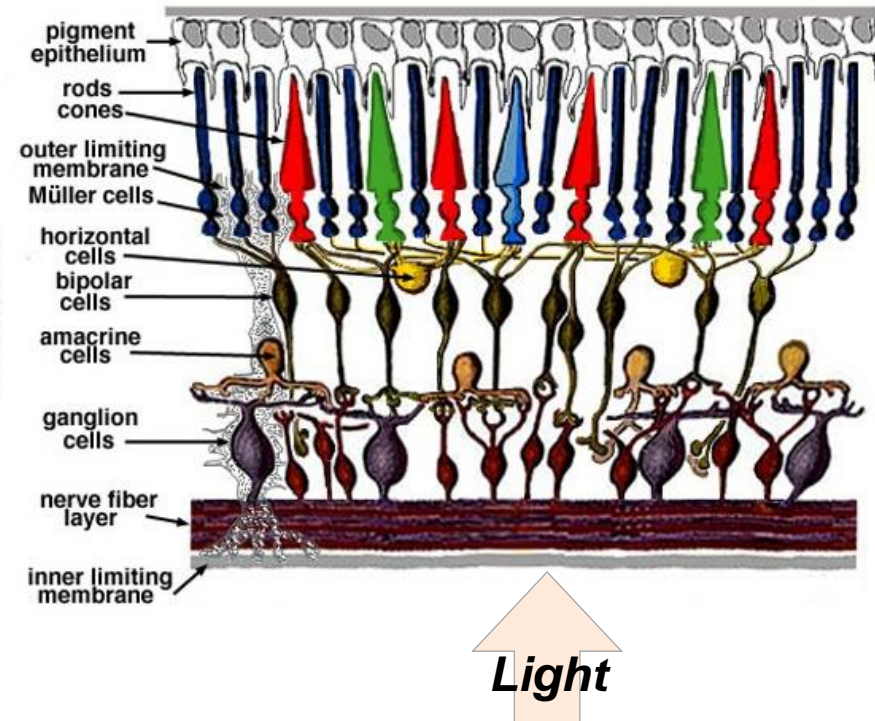
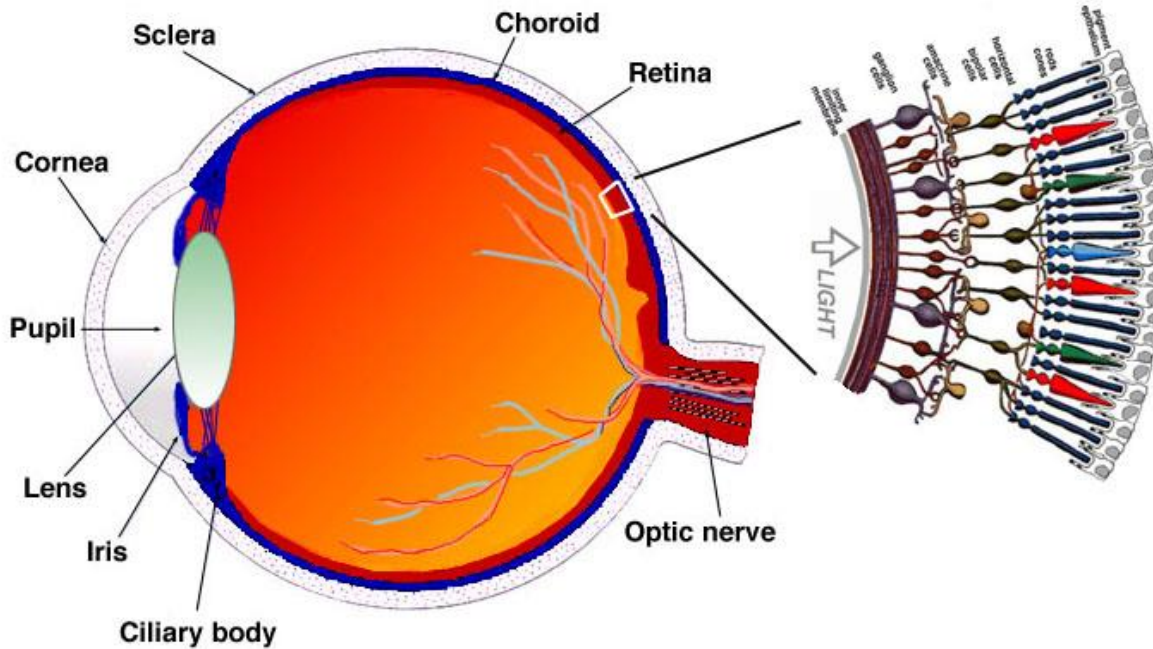
THE ROLE OF MOTION IN NATURAL VISION



[PhD Fabio Martínez 2014]

Motion is a primary cue for visual interpretation...

STRUCTURE OF THE HUMAN RETINA



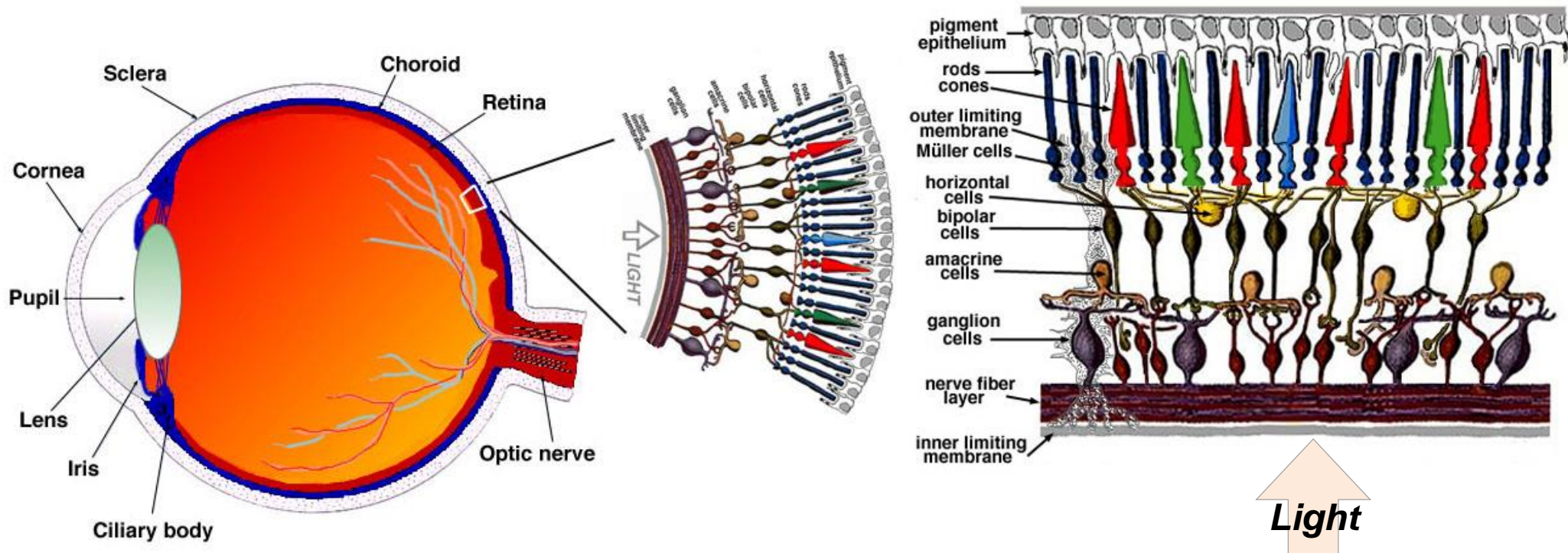
Complex organisation in *layers*:

- (1) Photoreceptors (cones and rods)
- (2) Bipolar and Horizontal Cells
- (3) Ganglion Cells (converge to the optical nerve) and Amacrine Cells.

In terms of information, there are about 100 millions of photoreceptors, for only 1 million axons in the optical nerve...

[Figures from Helga Kob, Webvision]

FUNCTION(S) OF THE RETINA

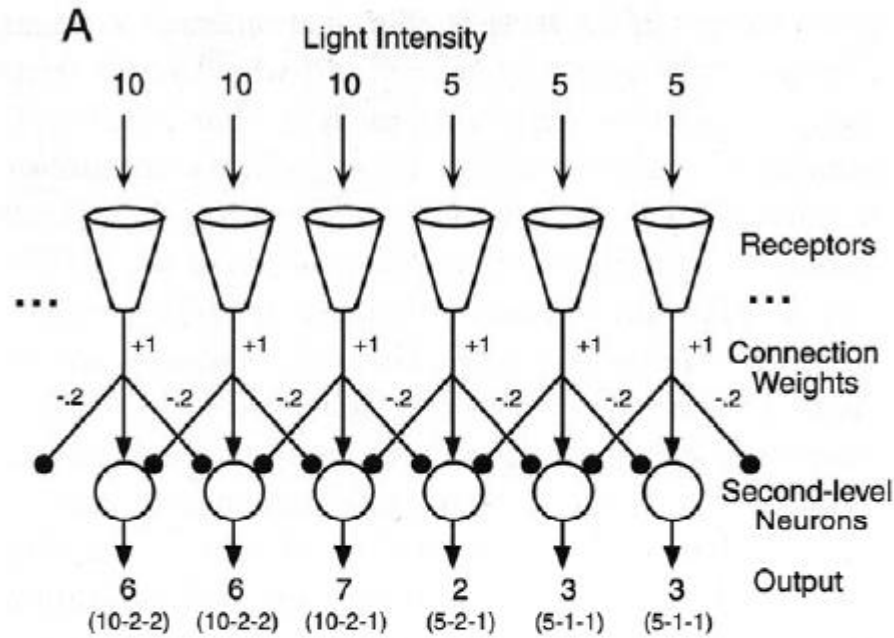


...the retina *summarises and structures* the visual information transmitted to the cortex via the optical nerve, by emphasizing essential properties of the image:

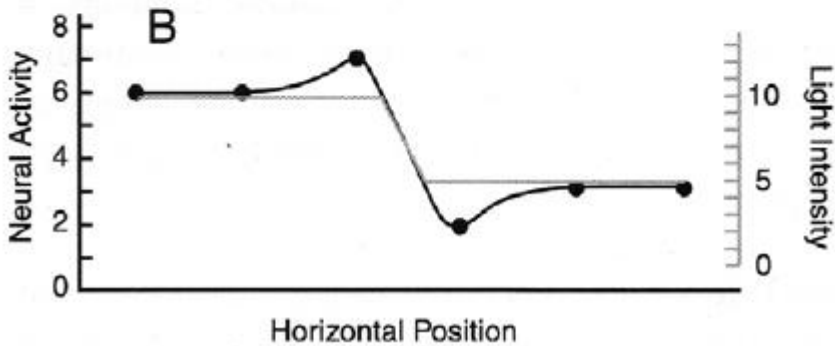
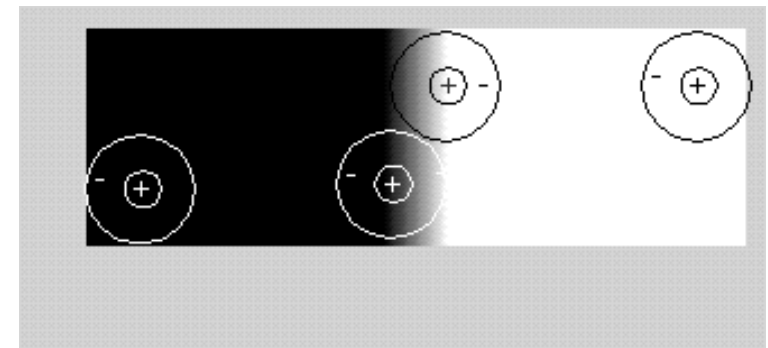
- **Contrast** / Colour / Shape
- **Movement**
- Spatial frequencies

[Figures from Helga Kob, Webvision]

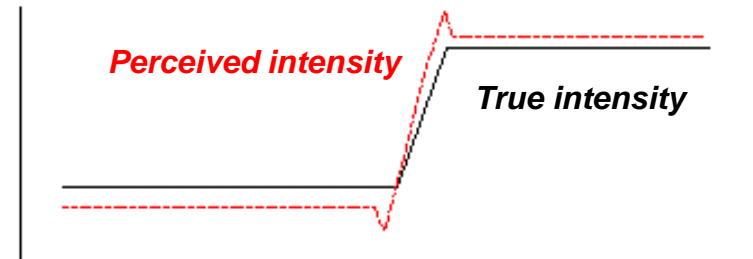
CONTRAST AND LATERAL INHIBITION



"Mach Bands" illusion



Lateral Inhibition in photoreceptors

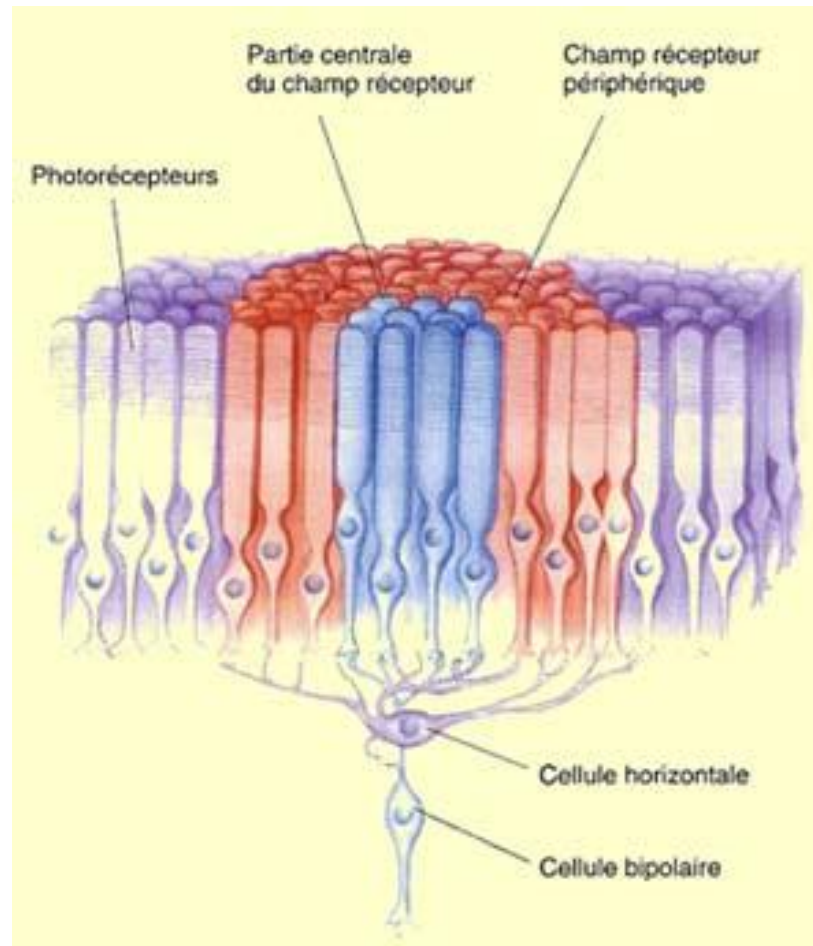


[from M.A. Giese 2006]

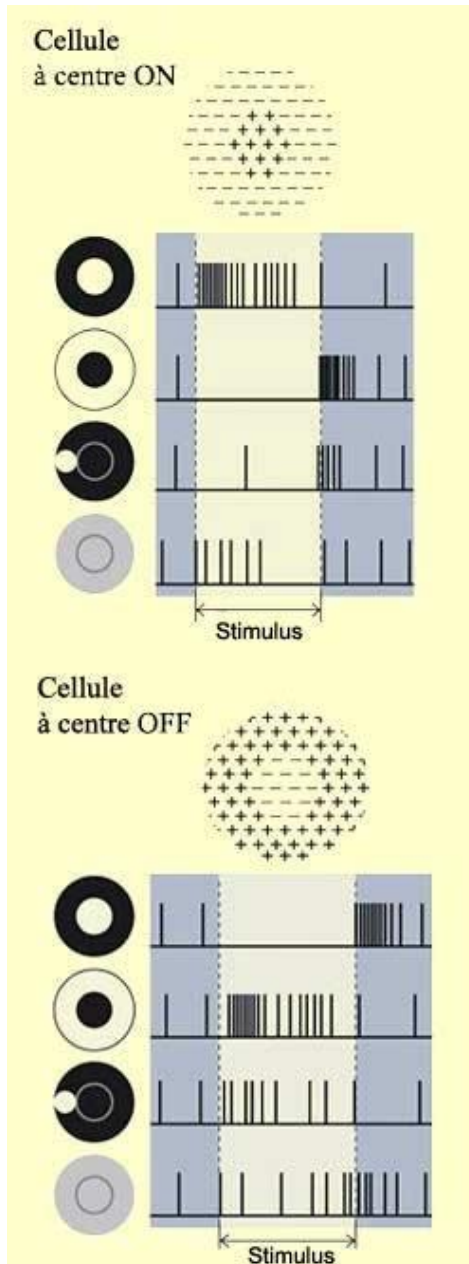
CONTRAST PERCEPTION: COMPLEX CELLS

Complex cells like bipolar (graded response), or ganglion cells (spiking response potentials, on the right), aggregate the responses provided by a certain set of photoreceptors.

The set of photoreceptors connected to a complex cell is called its *receptor field*.



[ill. <http://lecerveau.mcgill.ca>]

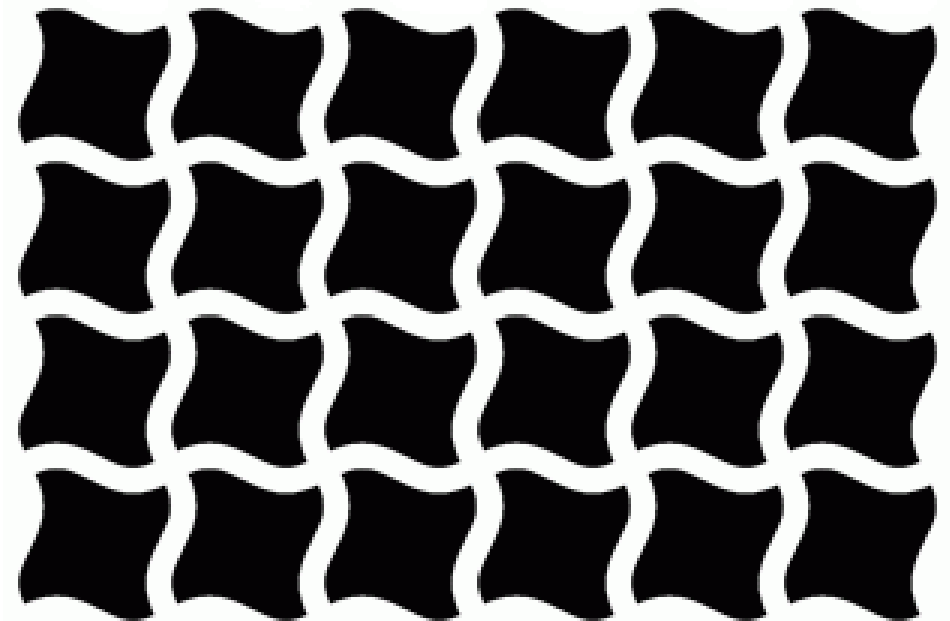
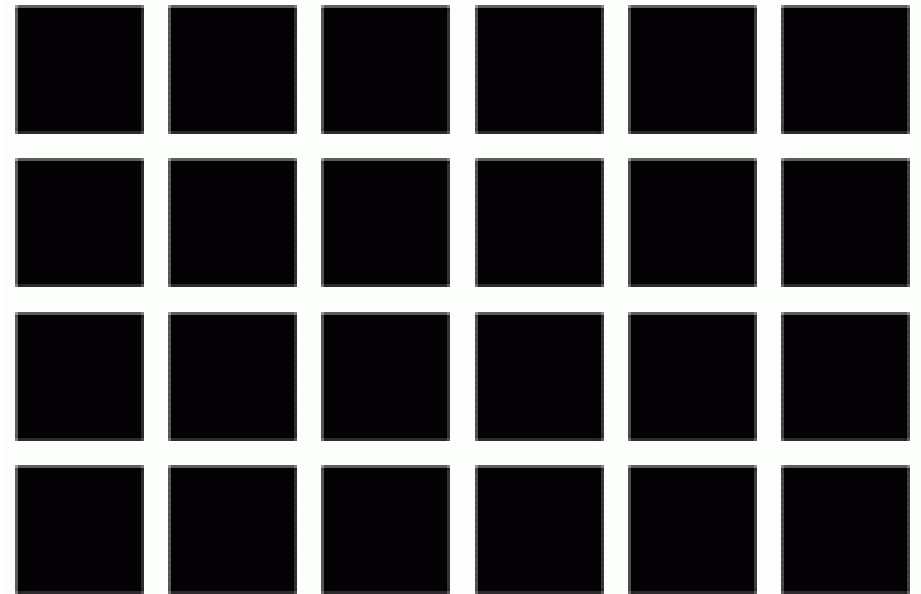


LATERAL INHIBITION AND HERMANN GRID ILLUSION??

Lateral inhibition and local contrast enhancement in receptor fields are also widely invoked to explain the “Hermann Grid” illusion (top).

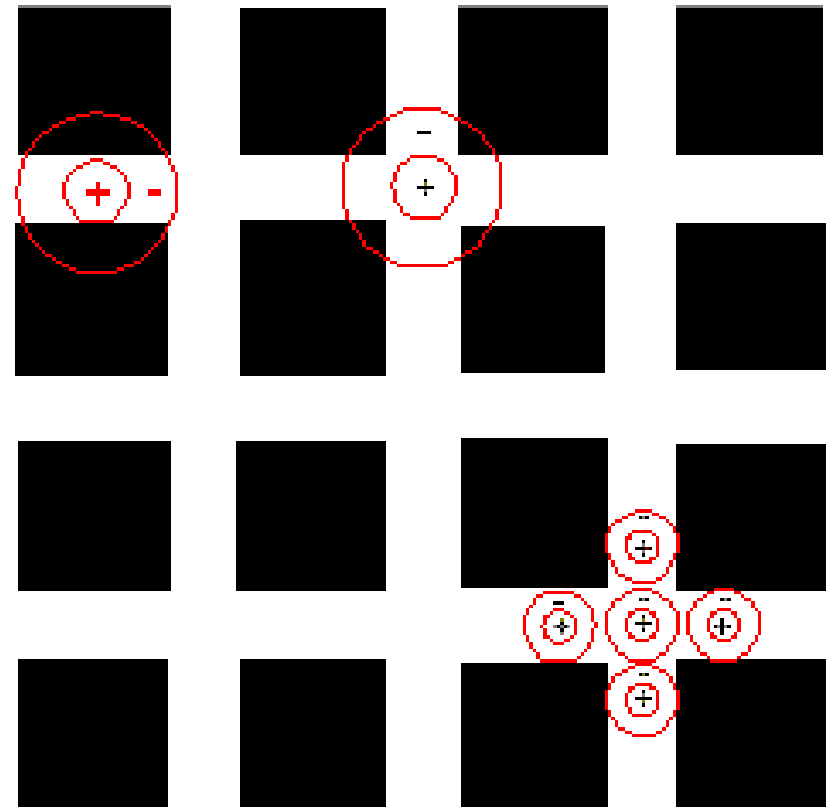
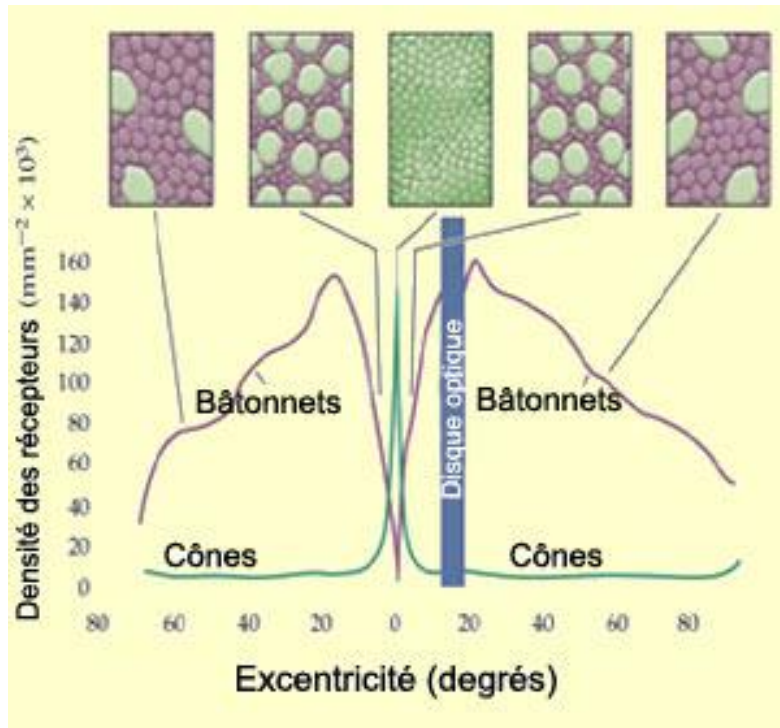
However this explanation is controversial, since a moderate deformation of the bands dramatically reduces the illusion (bottom).

Anyway, why do we not perceive a black spot at the gaze position?



FOVEAL AND PERIPHERAL VISION

The phenomenon is explained by the *difference in size of receptor fields* between the gaze position (fovea) and the periphery, also related to the spatial distribution of photoreceptors in the retina surface:

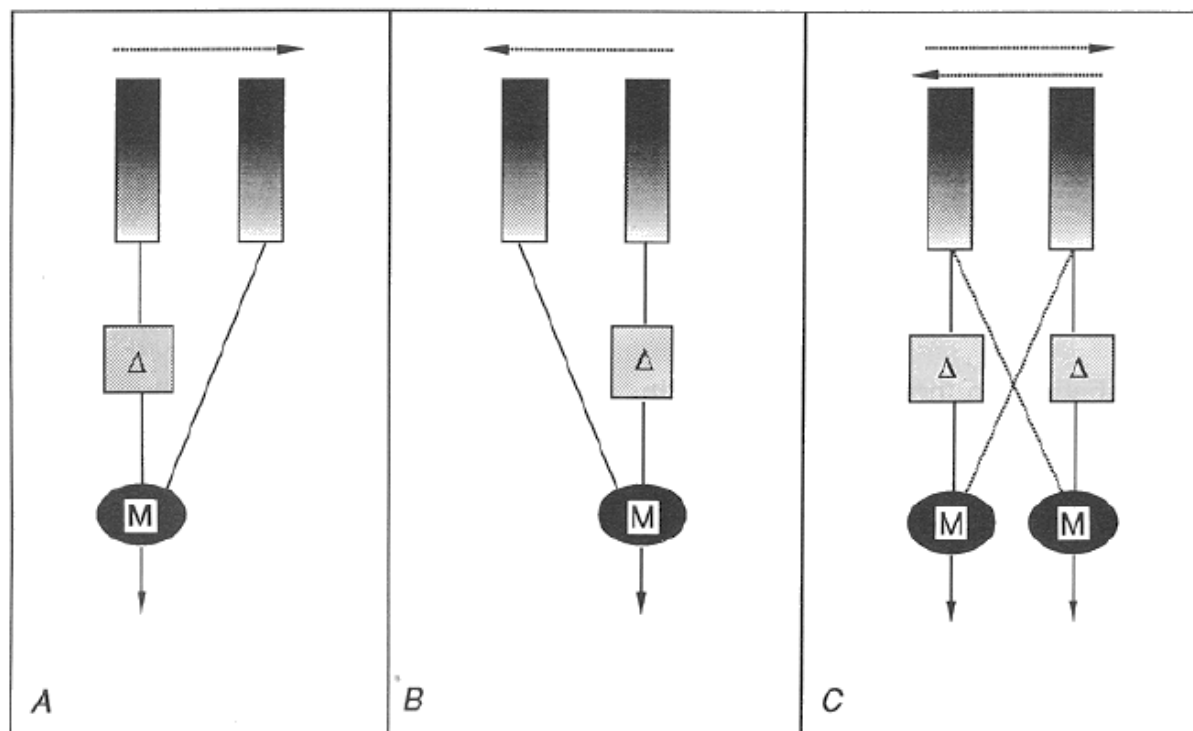


Specialisation of retina areas:

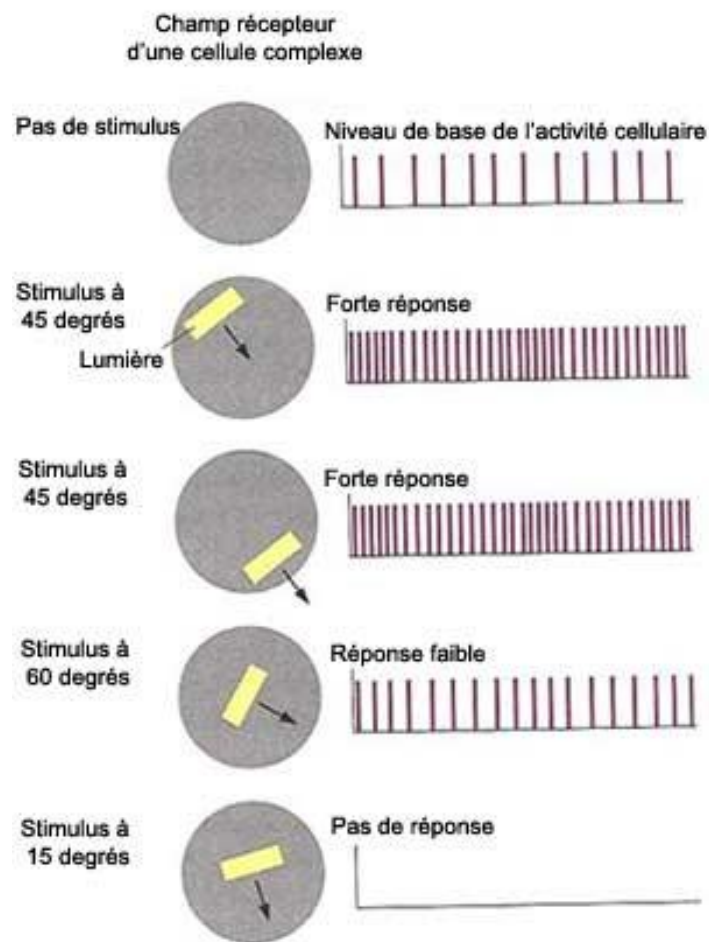
- Vision of details
 - Vision of colours
- } → Gaze position: fovea
- Vision of movement
- Pre-attention area: periphery

PERCEPTION OF LOCAL MOVEMENT

As well as for contrast, local motion perception is performed by elementary retina mechanisms, right at the output of photoreceptors (Reichardt model, on the left), and at the level of complex cells (on the right).



Reichardt Model (1956)
 Δ : Delay ; M: Multiplication



[ill. <http://lecerveau.mcgill.ca>]

THE CASE OF THE TOAD

The toad's visual acuity is strongly dependent on the movement.

His perception is specialised for distinguishing different kinds of movement:



[thanks to Peter Melzer]



No movement: no reaction



Movement in principal direction: attack

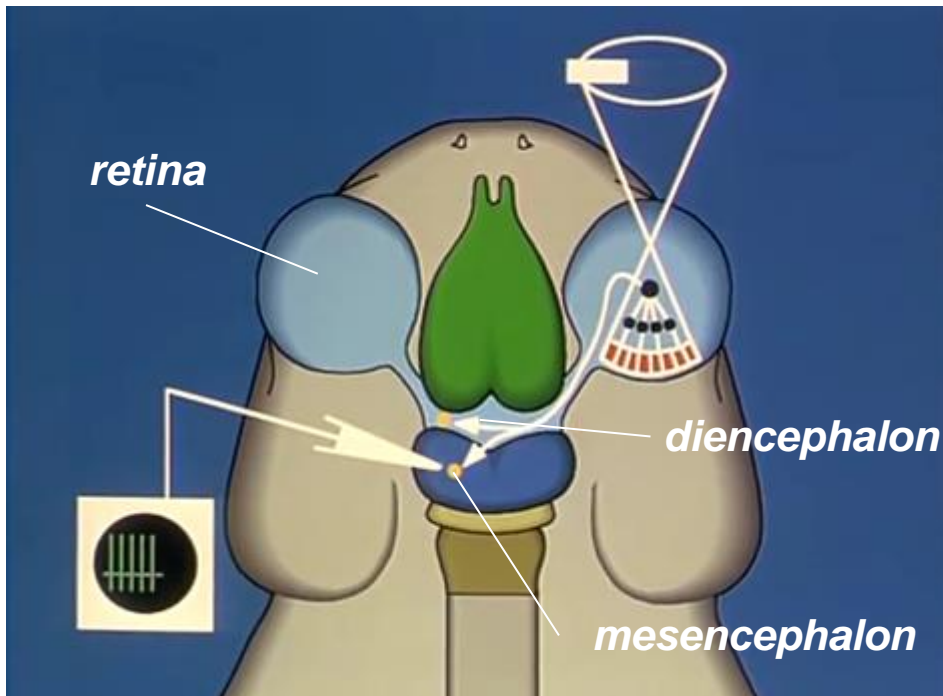


Movement in orthogonal direction: no reaction

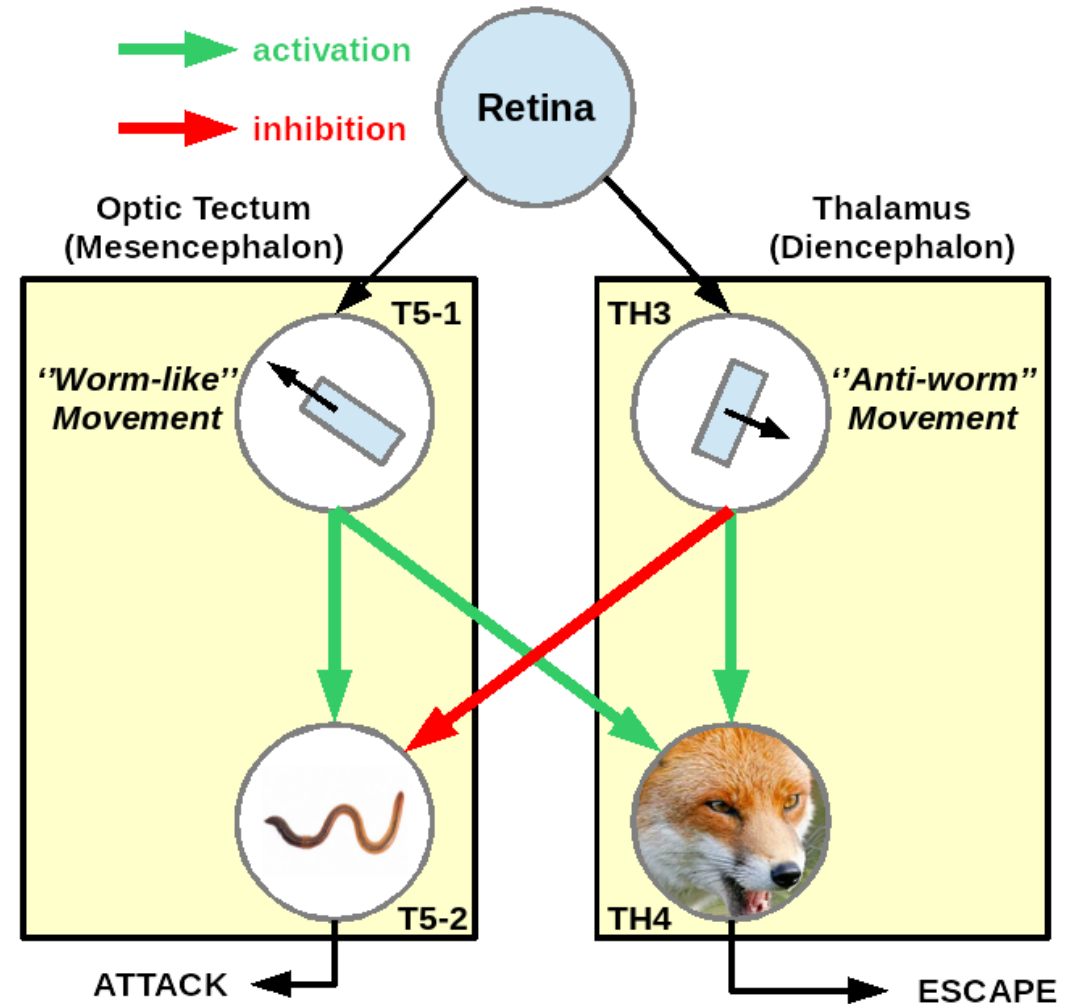
[in J.P. Ewert, Neuroethology of toads, 1993]

THE CASE OF THE TOAD

The toad's visual system involves *extremely short sensorimotor pathways* that generate behaviours essential to his survival:



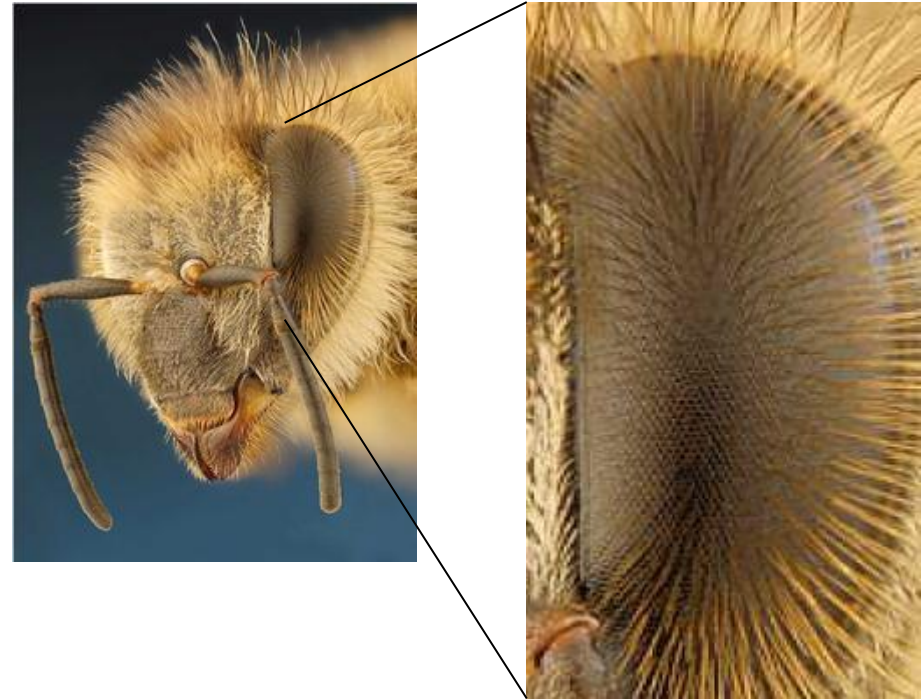
[Jörg-Peter Ewert 1993]



THE FLIGHT OF THE BUMBLEBEE

The bees have a pair of eyes that are approximately hemispheric, made of an hexagonal mesh of photoreceptors.

Their visual system provides remarkable navigation and communication tools, by the mean of a polar representation of their environment, that allows very precise orientation and distance measures.

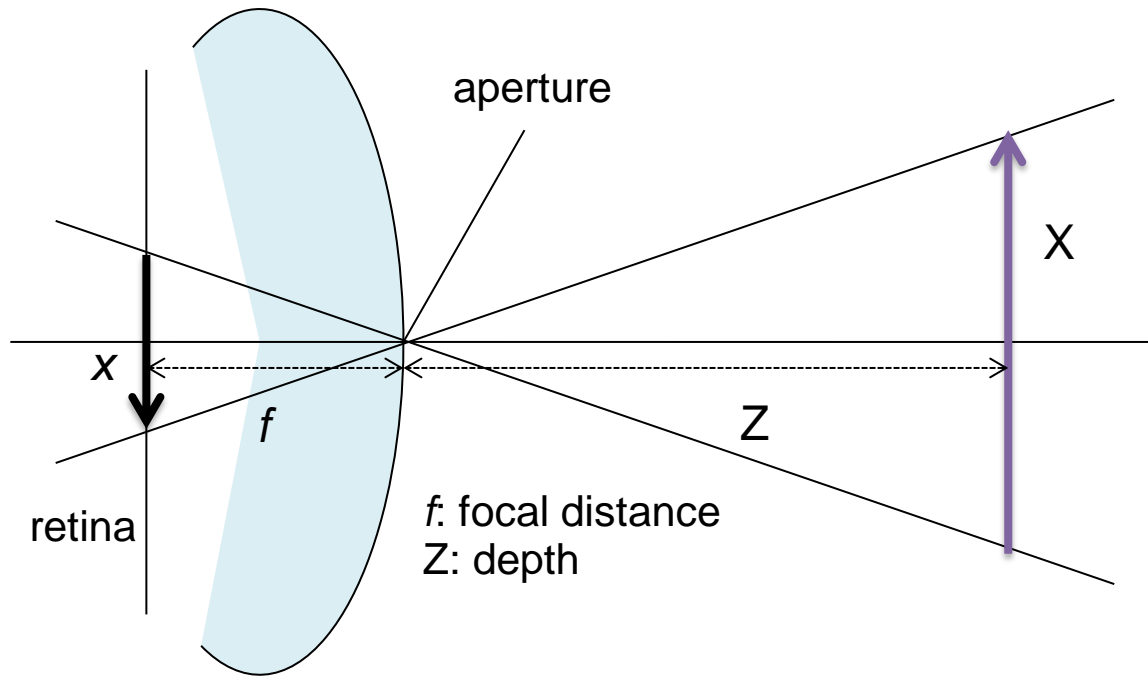


The vision fields of their two eyes are practically disjoint (i.e. no stereovision).

On the other hand, they perceive very well the direction and magnitude of apparent velocities (optical flow).

Spatial averaging of the optical flow allows the bees to estimate relative depths and to avoid obstacles.

OPTICAL FLOW: PARALLAX



(O,X,Y,Z) real 3d coordinates

(O,x,y) 2d retina coordinates

Perspective equation (pinhole model):

$$x = \frac{f X}{Z}$$

Time derivative (optical flow):

$$\dot{x} = f \left(\frac{\dot{X}}{Z} - \frac{X \dot{Z}}{Z^2} \right)$$

And in the case of a pure translation along OX axis (horizontal travelling, $\dot{Z} = 0$; $\dot{X} = Cte$) :

$$\dot{x} = \frac{f \dot{X}}{Z}$$

and then:

$$Z = \frac{f \dot{X}}{\dot{x}}$$

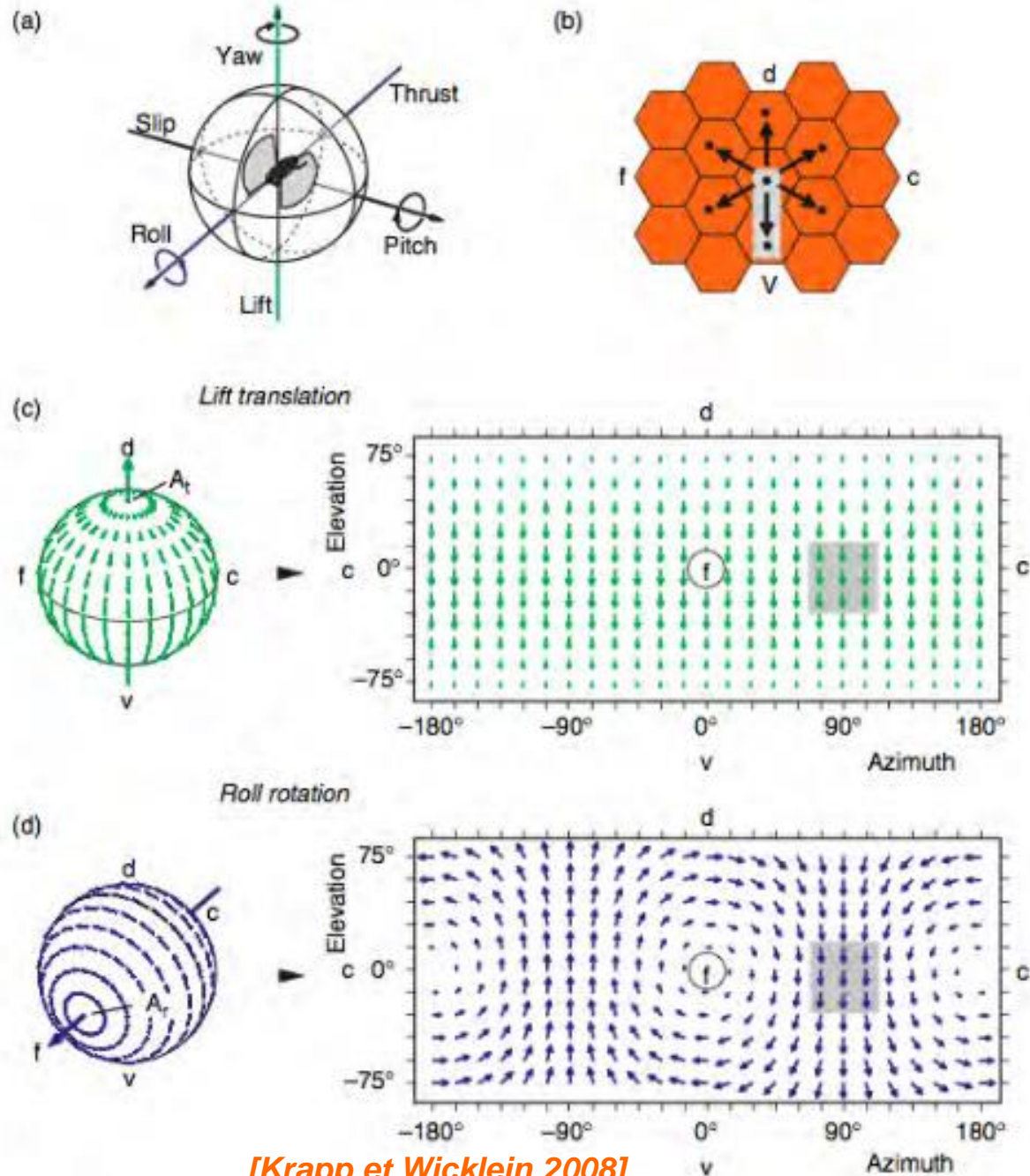
Depth is inversely proportional to the magnitude of apparent velocity

THE FLIGHT OF THE BUMBLEBEE



[Jürgen Tautz 2008]

The bees are able to navigate in small corridors by controlling the direction of their flight from the balance between the spatial averages of the optical flows perceived by their left and right eye, respectively.



[Krapp et Wicklein 2008]

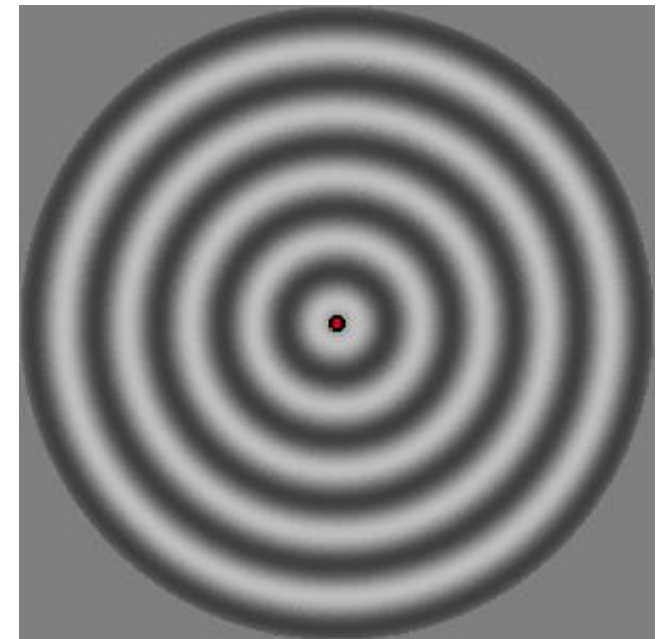
MOTION PERCEPTION: FAST ADAPTATION

The retina cells adapt quickly (a few seconds) to a certain environment, making perception relative to this environment.

Those retina-cortex (or intra-retina) feedback mechanisms allow a fast adaptation to a changing context and are typical of the *strongly differential character of perception*.



Complementary coloured perception of afterimage

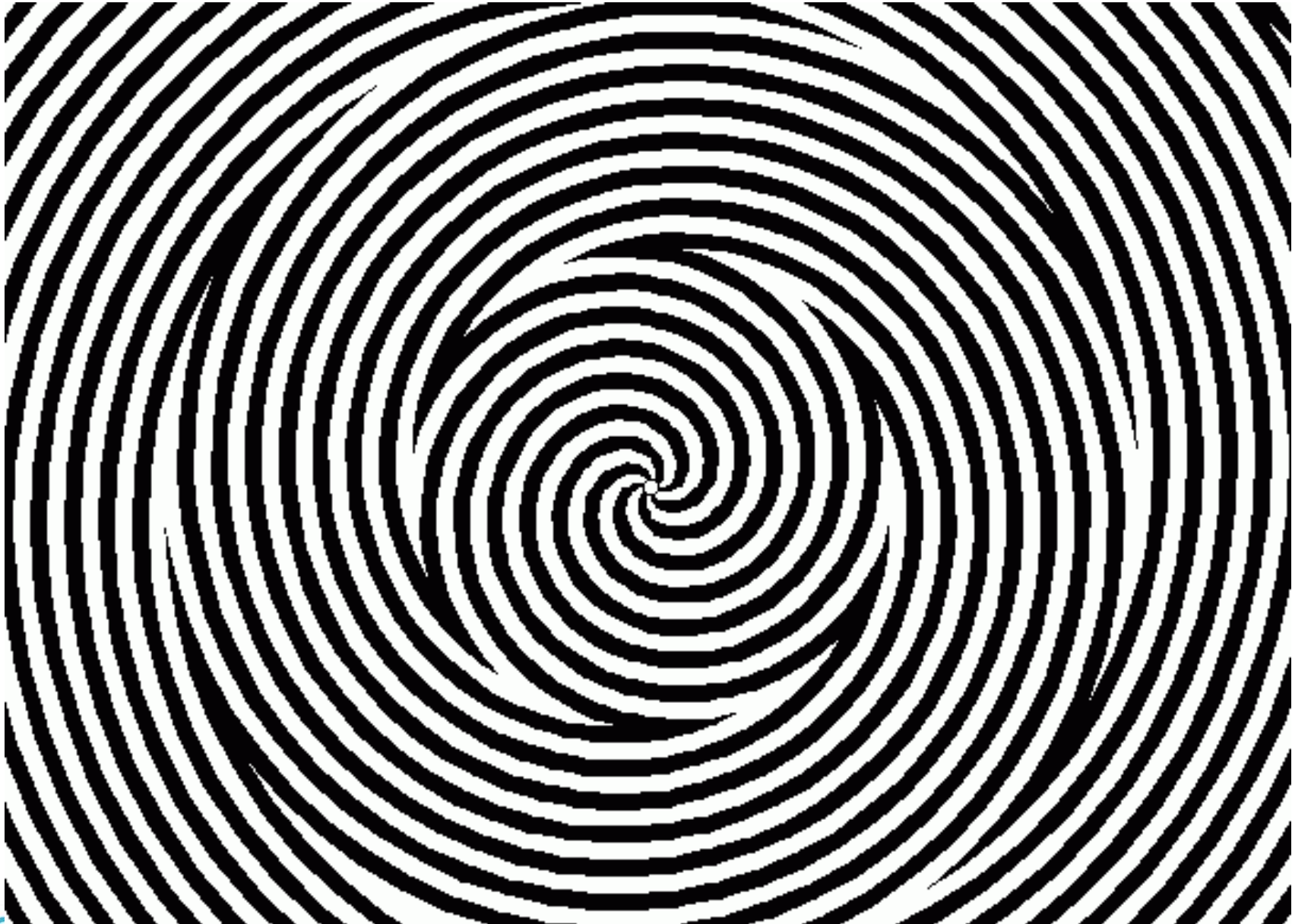


After-motion Perception

MOTION PERCEPTION: FAST ADAPTATION



MOTION PERCEPTION: FAST ADAPTATION



MOTION PERCEPTION: FAST ADAPTATION

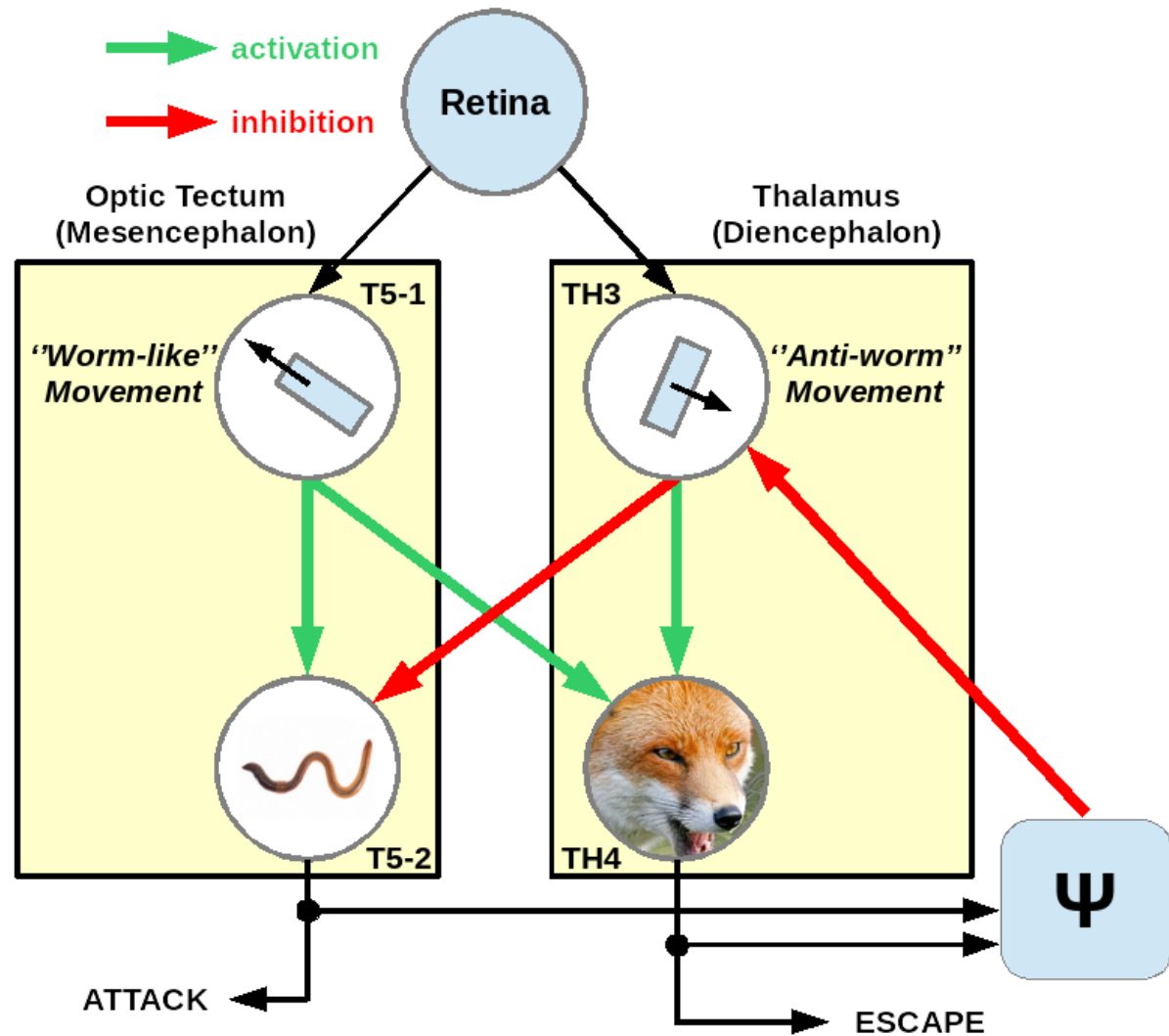


SLOW ADAPTATION: BACK TO THE TOAD



The toad's sensorimotor behaviour can be modified by training.

[Jörg-Peter Ewert 1993]



OPTICAL FLOW: INTERPRETATION OF THE FoE

(O,X,Y,Z) real 3d coordinates ; (O,x,y) 2d retina coordinates

Perspective equations
(pinhole model) :

$$\begin{cases} x = \frac{f X}{Z} \\ y = \frac{f Y}{Z} \end{cases}$$

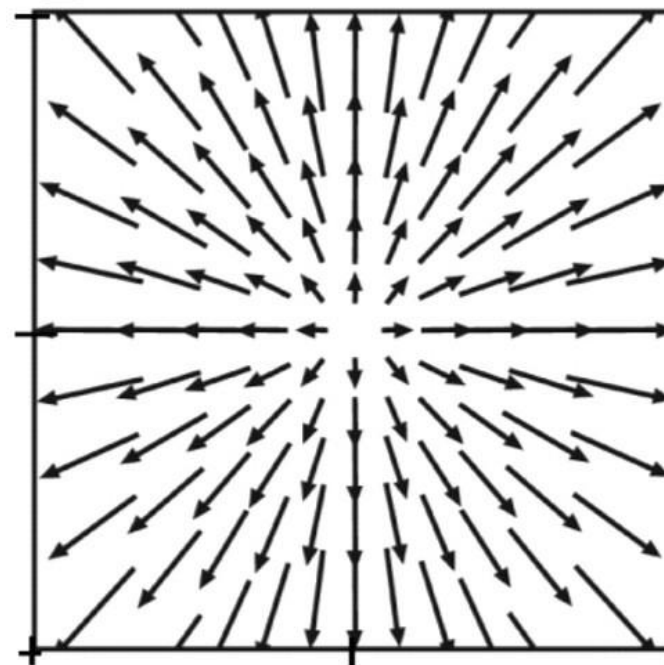
Time derivatives
(optical flow) :

$$\begin{cases} \dot{x} = f \left(\frac{\dot{X}}{Z} - \frac{X \dot{Z}}{Z^2} \right) \\ \dot{y} = f \left(\frac{\dot{Y}}{Z} - \frac{X \dot{Y}}{Z^2} \right) \end{cases}$$

Case of a pure translation along the optical axis OZ ($\dot{X} = \dot{Y} = 0$; $\dot{Z} = Cte$) :

$$\begin{cases} \dot{x} = -f \frac{X \dot{Z}}{Z^2} = -\frac{x \dot{Z}}{Z} \\ \dot{y} = -f \frac{X \dot{Y}}{Z^2} = -\frac{y \dot{Z}}{Z} \end{cases}$$

The displacement field forms a zoom on the image with a Focus of Expansion (FoE) at the optical centre O.



OPTICAL FLOW: INTERPRETATION OF THE FoE

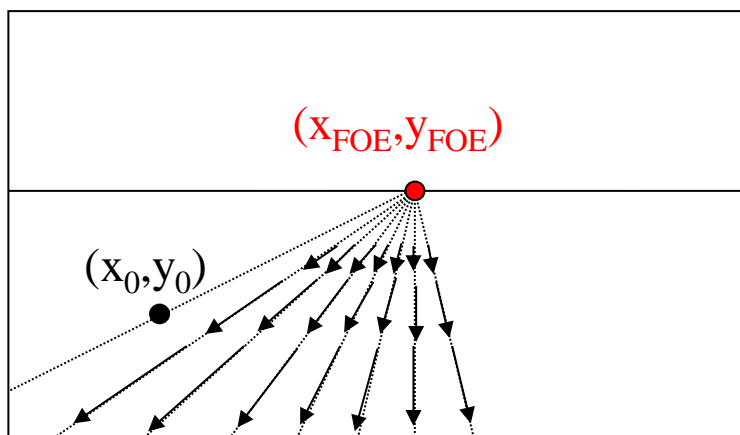
More generally, during a uniform translation within a static scene, velocity directions of points projected onto the retina plane converge toward a point from the projective plane called the Focus of Expansion (FoE).

Let T the 3d translation vector:

$$T = (-\dot{X}, -\dot{Y}, -\dot{Z})$$

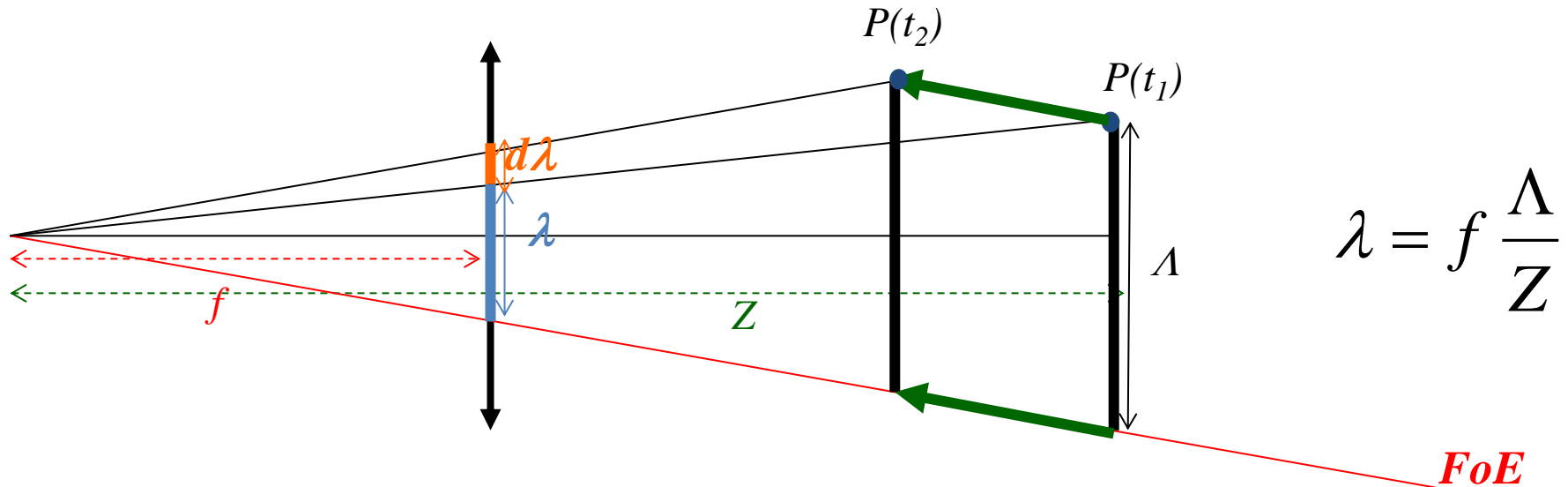
Let (X_0, Y_0, Z_0) a point of the scene. After a time t , it projects on the image at point (x_t, y_t) , with:

$$(x_t, y_t) = \left(f \frac{X_0 + t\dot{X}}{Z_0 + t\dot{Z}}, f \frac{Y_0 + t\dot{Y}}{Z_0 + t\dot{Z}} \right)$$



$$\begin{aligned} (x_{FOE}, y_{FOE}) &= \lim_{t \rightarrow -\infty} (x_t, y_t) \\ &= \left(f \frac{\dot{X}}{\dot{Z}}, f \frac{\dot{Y}}{\dot{Z}} \right) \end{aligned}$$

OPTICAL FLOW: TIME BEFORE CONTACT



$$\lambda = f \frac{\Delta}{Z}$$

movement in a static scene:

$$\dot{\lambda} = -\frac{f\Delta}{Z^2} \dot{Z}$$

and then

$$\frac{\lambda}{\dot{\lambda}} = -\frac{Z}{\dot{Z}}$$

time before contact

OPTICAL FLOW: TIME BEFORE CONTACT

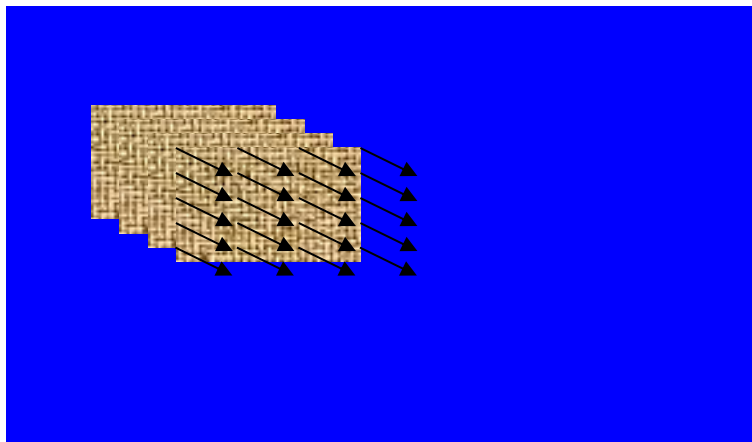
In most animal species, including humans, an escape reflex is observed when facing the 2d projection of an expanding object.

Some birds (e.g. pigeons) have neurons whose response seems highly correlated to time-before-contact.



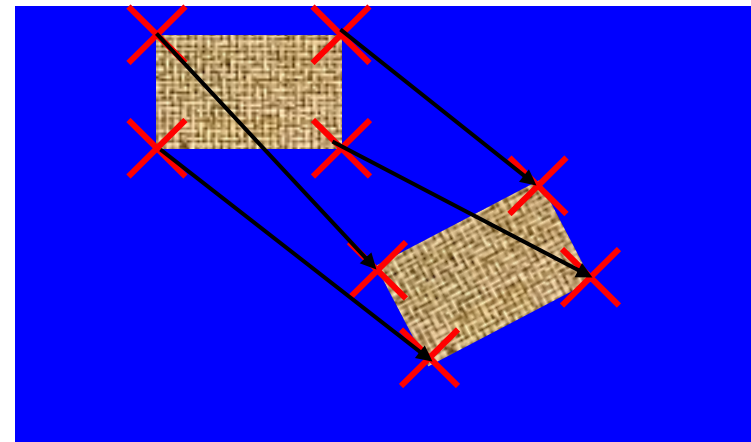
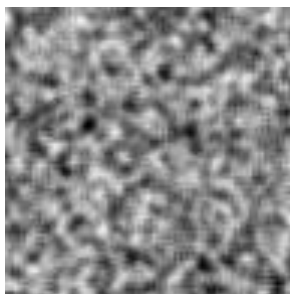
*L'arrivée d'un train en gare de La Ciotat
L. & A. Lumière (1895)*

MATCHING, ESTIMATION, TRACKING?



Motion Estimation (optical flow):

- . Dense estimation
- . Continuous motion
- . Point matching
- . Focus on the observations
- . Essentially *bottom-up*



Visual Tracking:

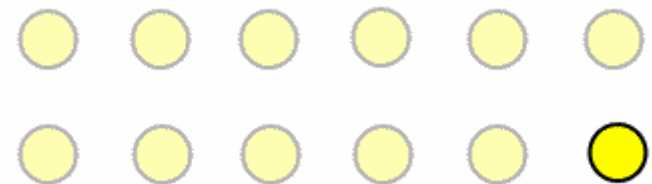
- . Sparse estimation
- . Discontinuous motion
- . Pattern matching
- . Focus on the model (prediction)
- . Essentially *top-down*



MATCHING AND MOTION PERCEPTION

Interstimulus interval	Perception	
$T < 30$ ms		Simultaneity (no motion)
30 ms $< T < 60$ ms		Continuous motion (Phi)
60 ms $< T < 200$ ms		Displacement (Beta)
$T > 200$ ms		Events succession (no motion)

[from M.A. Giese 2006]

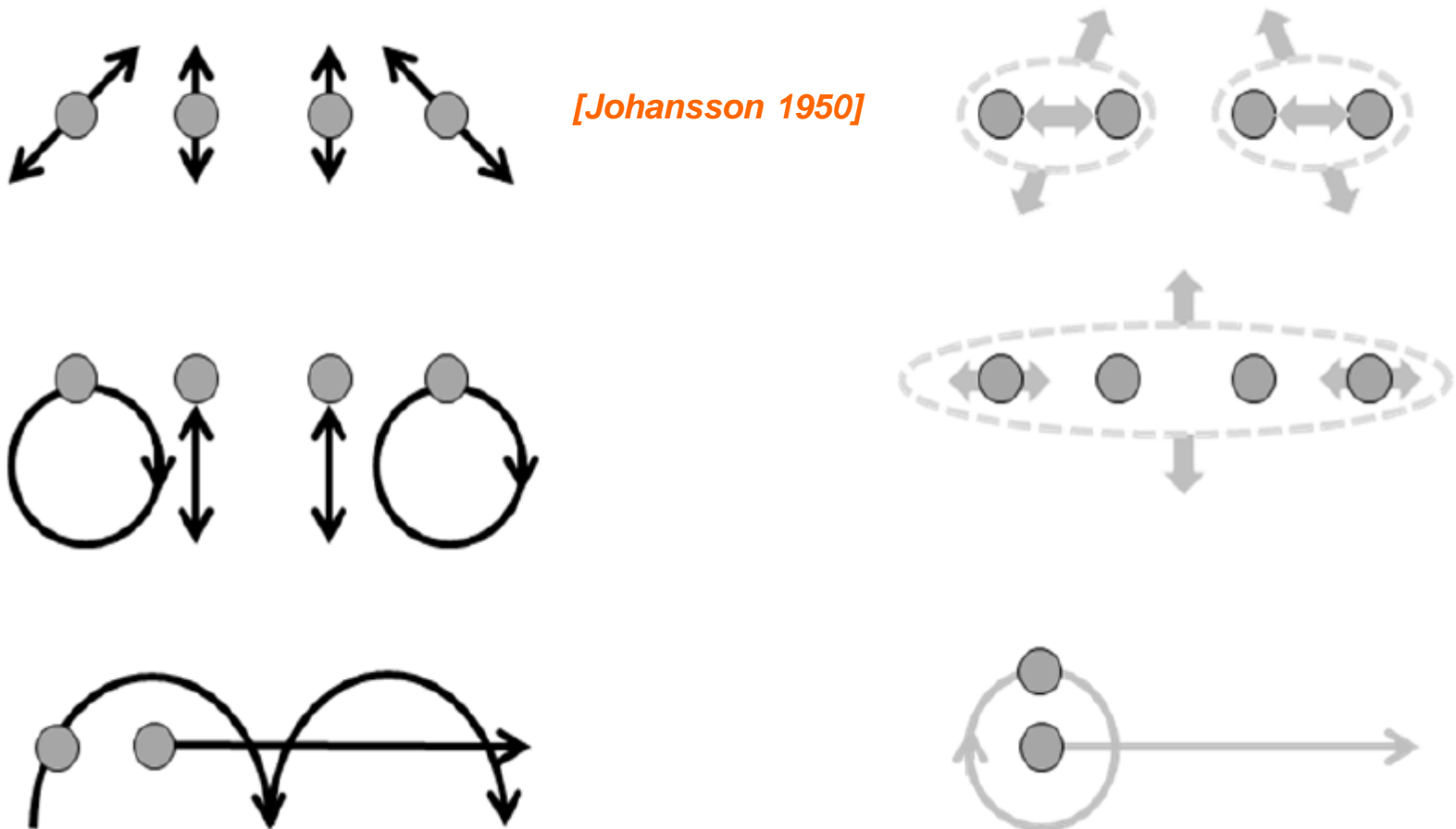


Beta motion

MOTION PERCEPTION AND *GESTALT*

Like for shape perception, there exist *perceptual grouping* mechanisms for motion, based on different criteria (proximity, simplicity, closure to familiar shape, common fate, etc.) that imply a hierarchical perception of movement:

(1) Global movement → (2) Movement of parts



PERCEPTION OF MOVEMENT: THE APERTURE EFFECT

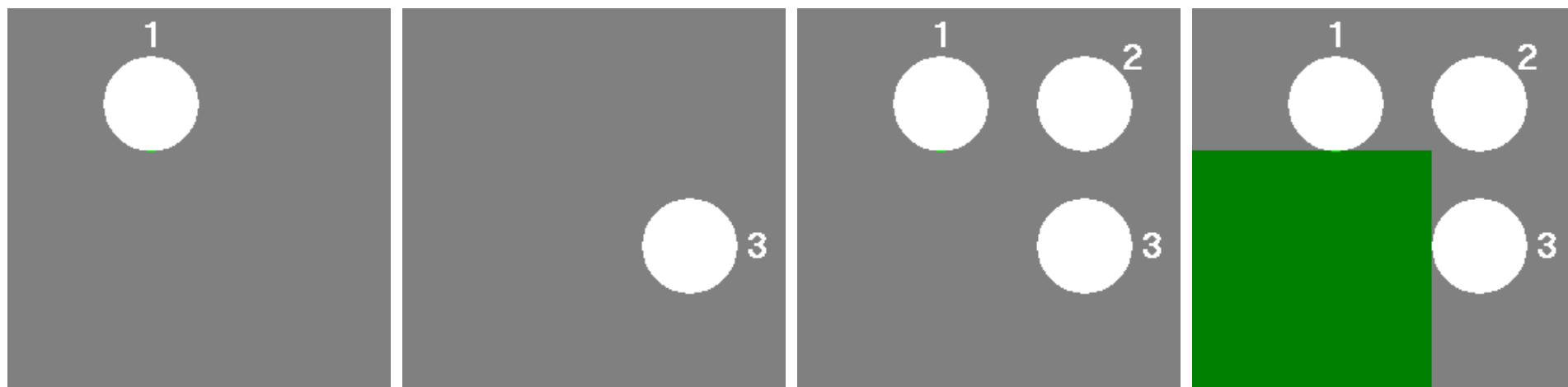


Barber pole

A partial vision of the movement (i.e. through an aperture) often creates a deceptive perception of motion.

This phenomenon is related to a fundamental constraint for estimating the optical flow: locally, the velocity can only be estimated along the spatial gradient direction.

See lecture on *optical flow*

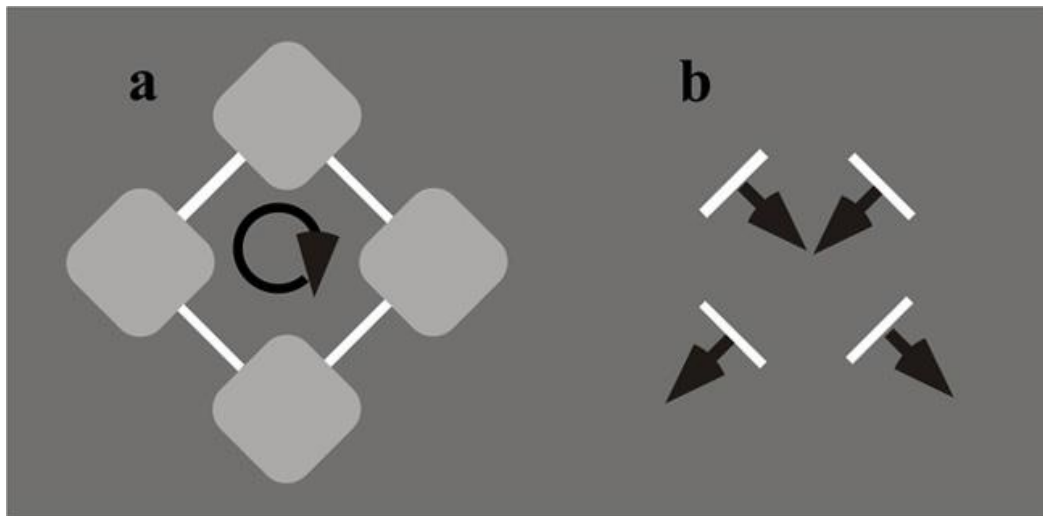


APERTURE EFFECT VS GESTALT

In the *motion binding* illusion, the perceptual grouping is found at two levels:

- (1) Without occlusion, the association based on same translation of parallel segment pairs (*similarity + common fate*) wins.
- (2) With occlusion, the *simplicity* criterion (square) wins, hence allowing the perception of the real movement.

Unlike the “classical” aperture effect, it is the occlusion that allows to correctly interpret the movement in this case!



See the Flash demo:



square-figure2-medium.swf

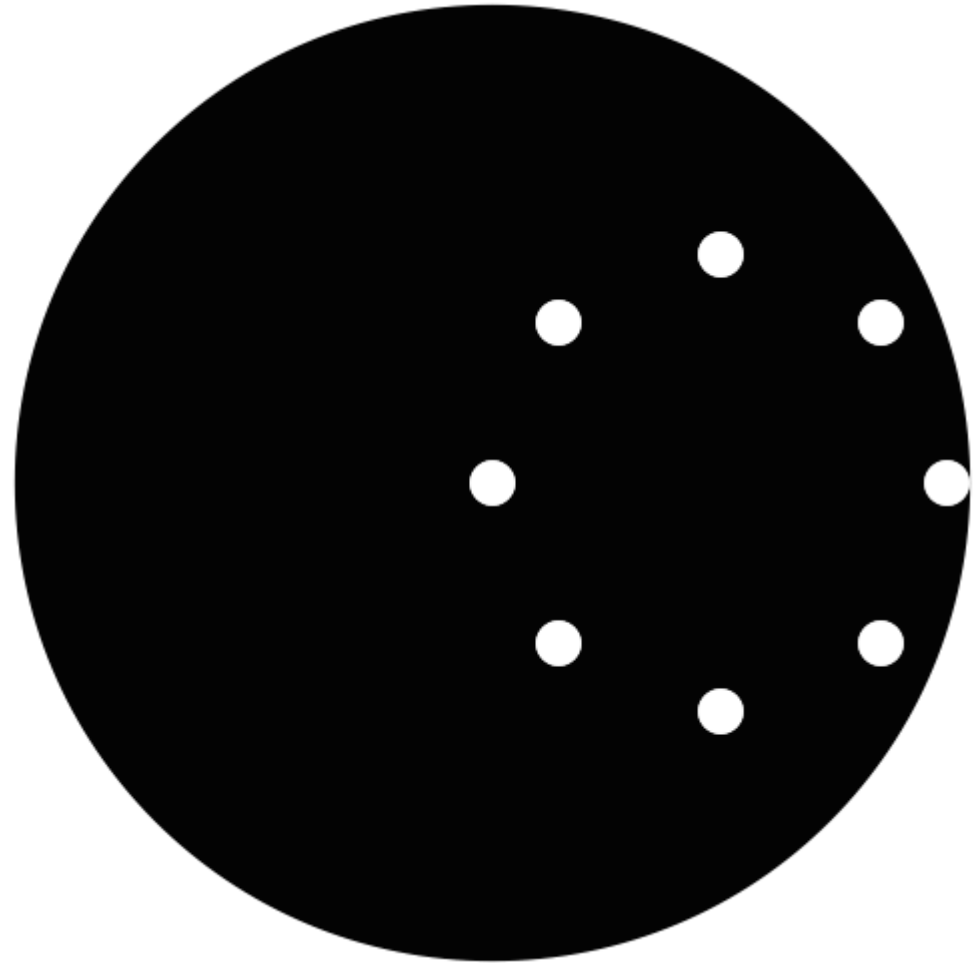
<http://web.mit.edu/persci/demos/Motion&Form/square/square-figure2-medium.html>

[Mc Dermott, Weiss, Adelson 2001]

MOTION PERCEPTION AND *GESTALT*

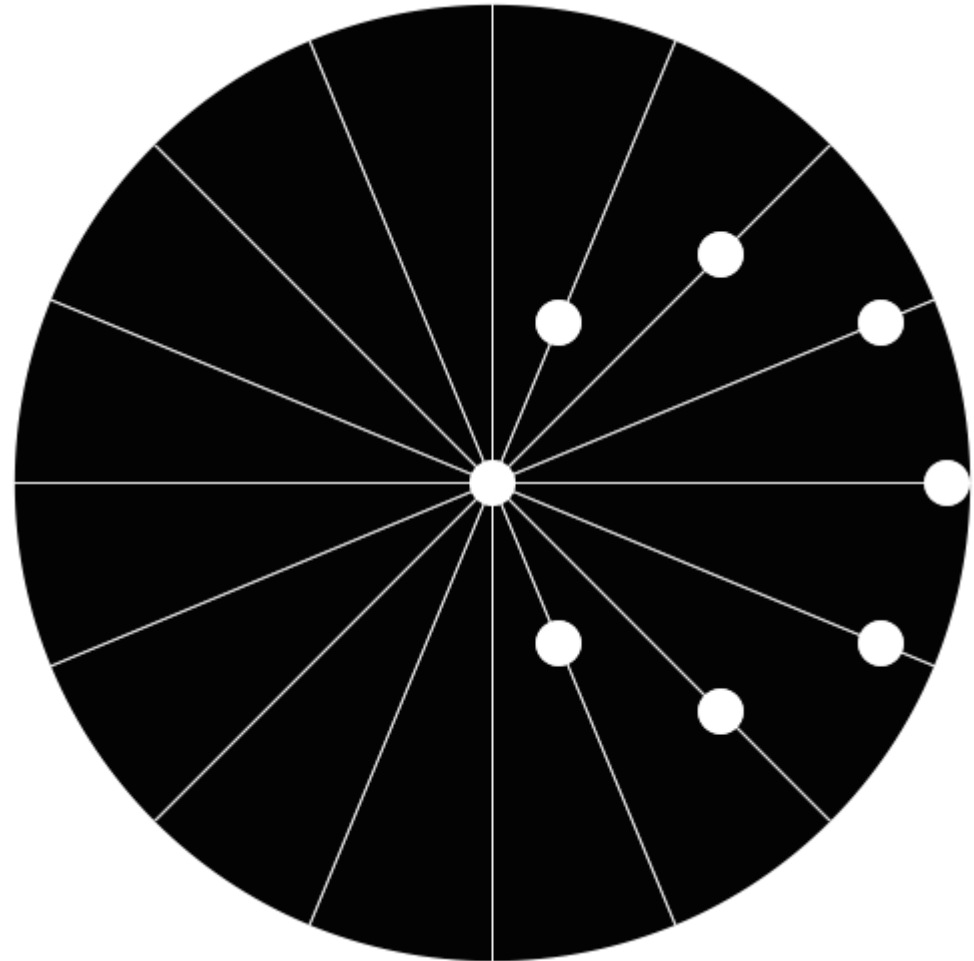
Another funny example where the perception of a global motion takes precedence over the tracking of individual objects, even very simple:

The Tusi motion

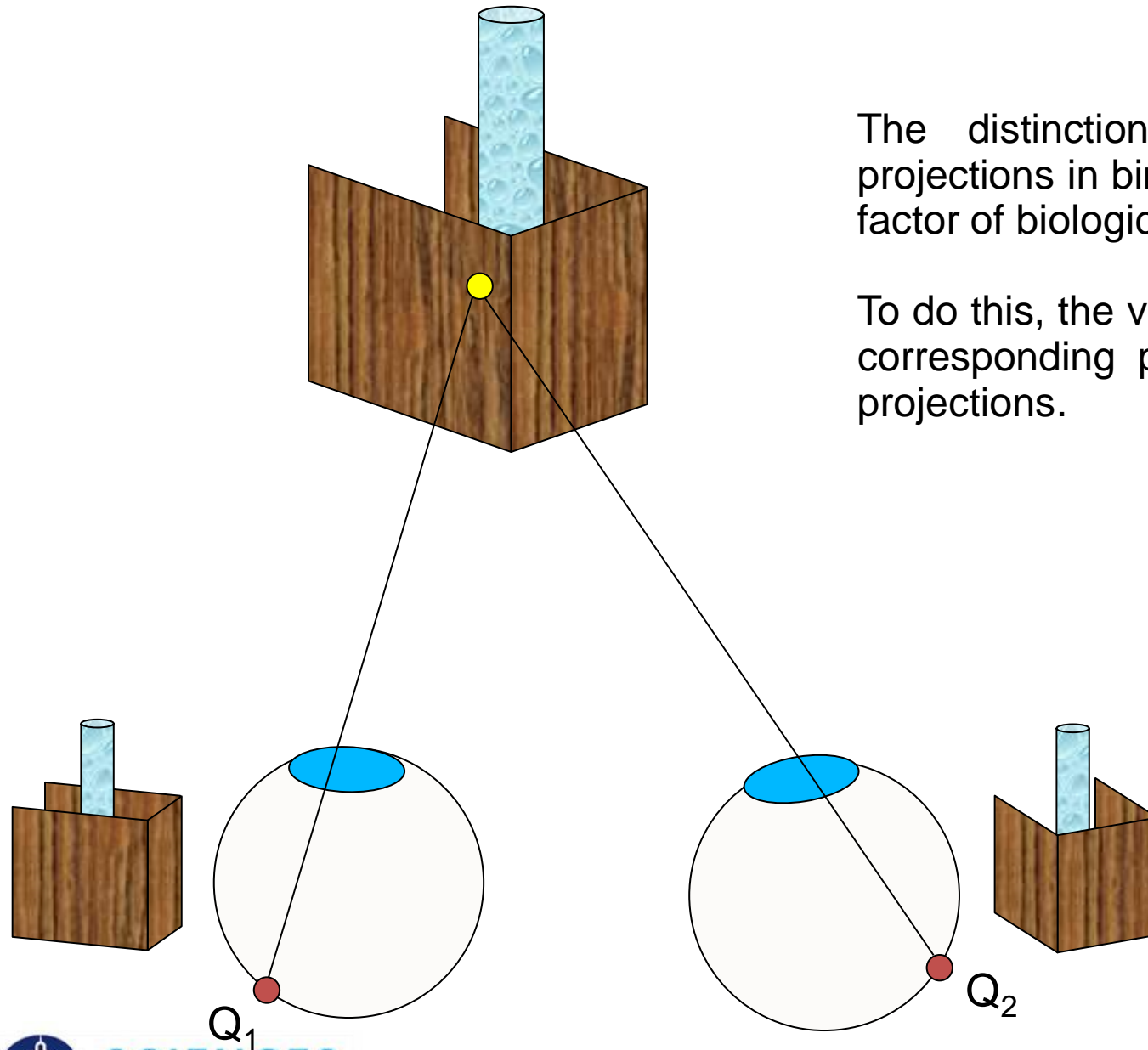


MOTION PERCEPTION AND *GESTALT*

Does the presence of diameter lines change the global perception?



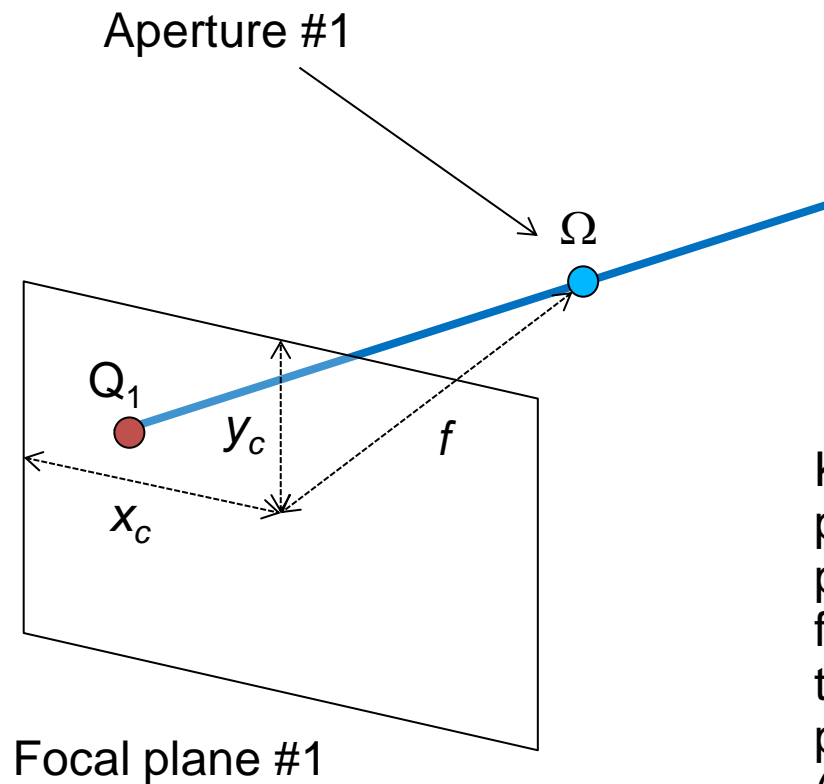
VISUAL MATCHING AND 3D PERCEPTION



The distinction between left and right projections in binocular vision is an essential factor of biological 3d perception.

To do this, the visual system must match the corresponding points of the two perceived projections.

STEREOVISION AND STRUCTURE FROM MOTION



Knowing the geometry of the first focal plane, i.e. the position of the optical centre (x_c, y_c) , which is the projection of the aperture on the focal plane, and the focal distance f , which is the distance of the aperture to the focal plane, the optical path of every point Q_1 projected on the focal plane can be back-traced (back-projection of point Q_1 in blue).

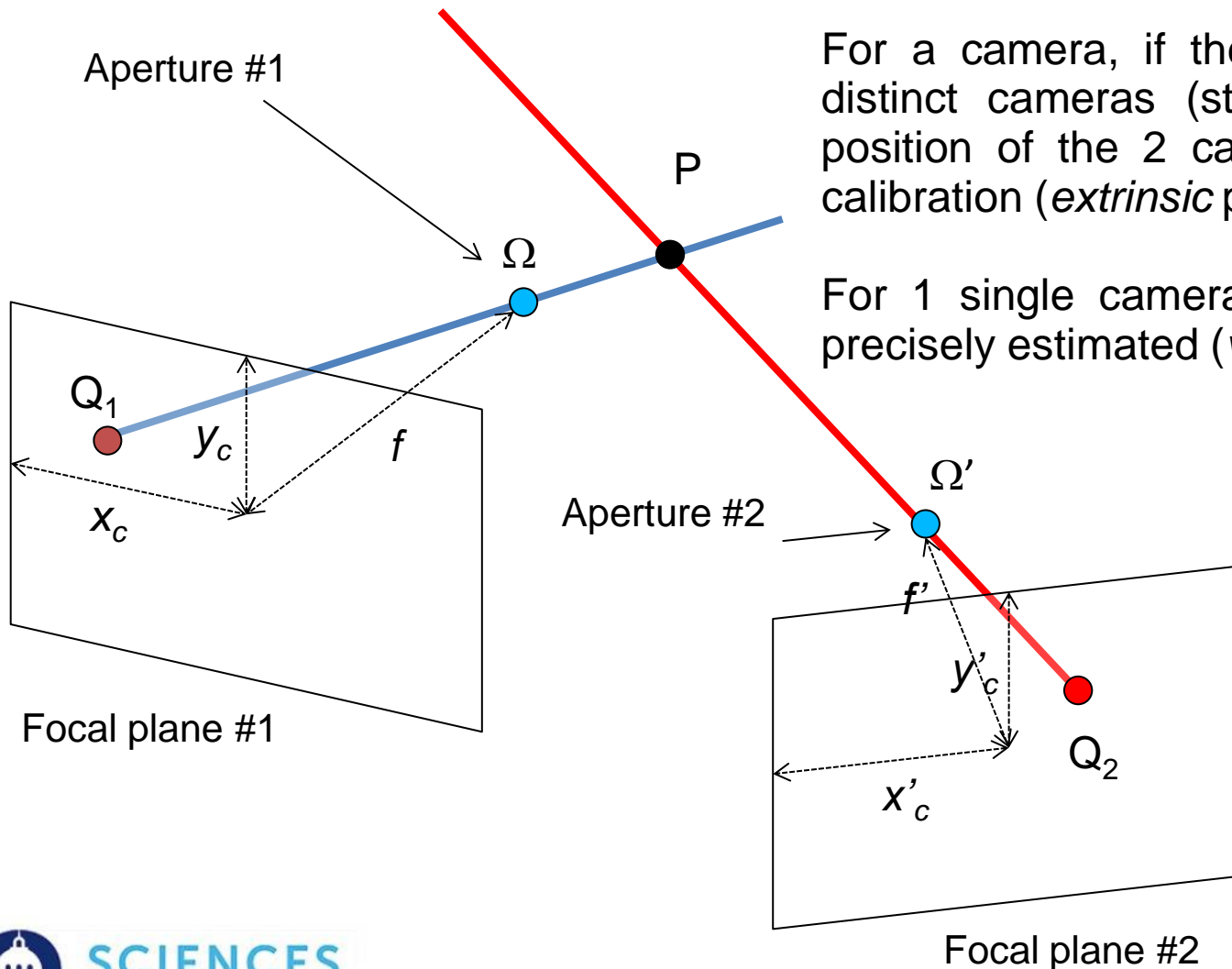
For a camera, these so-called *intrinsic* parameters are estimated by calibration.

STEREOVISION AND STRUCTURE FROM MOTION

The same applies for a 2^d focal plane. If Q_1 and Q_2 correspond to the same point (matching) then their back-projections intersect at this point P .

For a camera, if the 2 focal planes belong to 2 distinct cameras (stereovision), then the relative position of the 2 cameras must be estimated by calibration (*extrinsic* parameters).

For 1 single camera, its displacement has to be precisely estimated (*visual odometry*).



We then get:

$$\Omega P = \Omega \Omega' \frac{\sin \hat{\Omega}'}{\sin \hat{P}}$$

$$\Omega' P = \Omega \Omega' \frac{\sin \hat{\Omega}}{\sin \hat{P}}$$

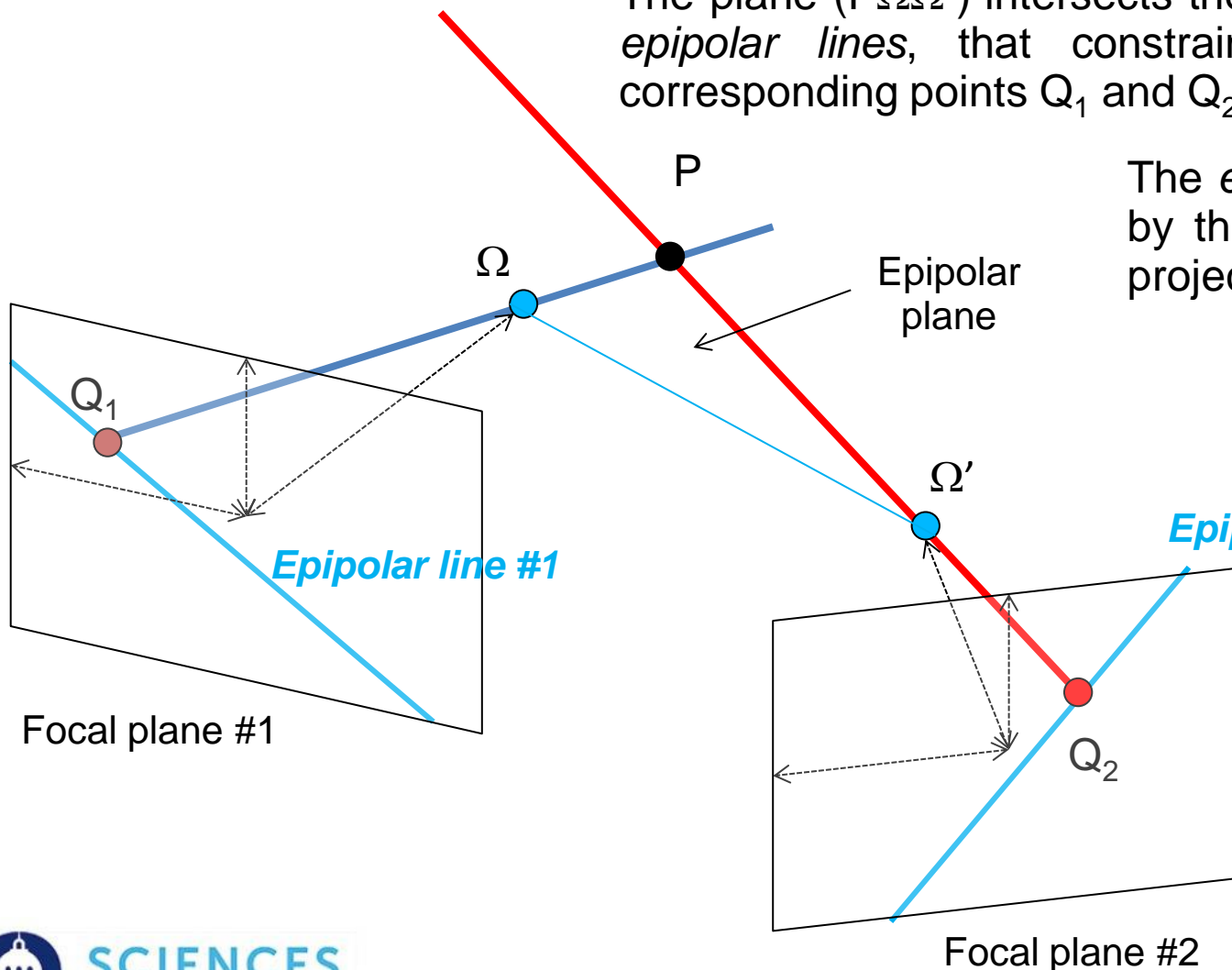
STEREOVISION AND STRUCTURE FROM MOTION

The two lines corresponding to the optical paths are intersecting, and then coplanar.

The plane $(P\Omega\Omega')$ intersects the two focal planes on so-called *epipolar lines*, that constrain the possible positions of corresponding points Q_1 and Q_2 .

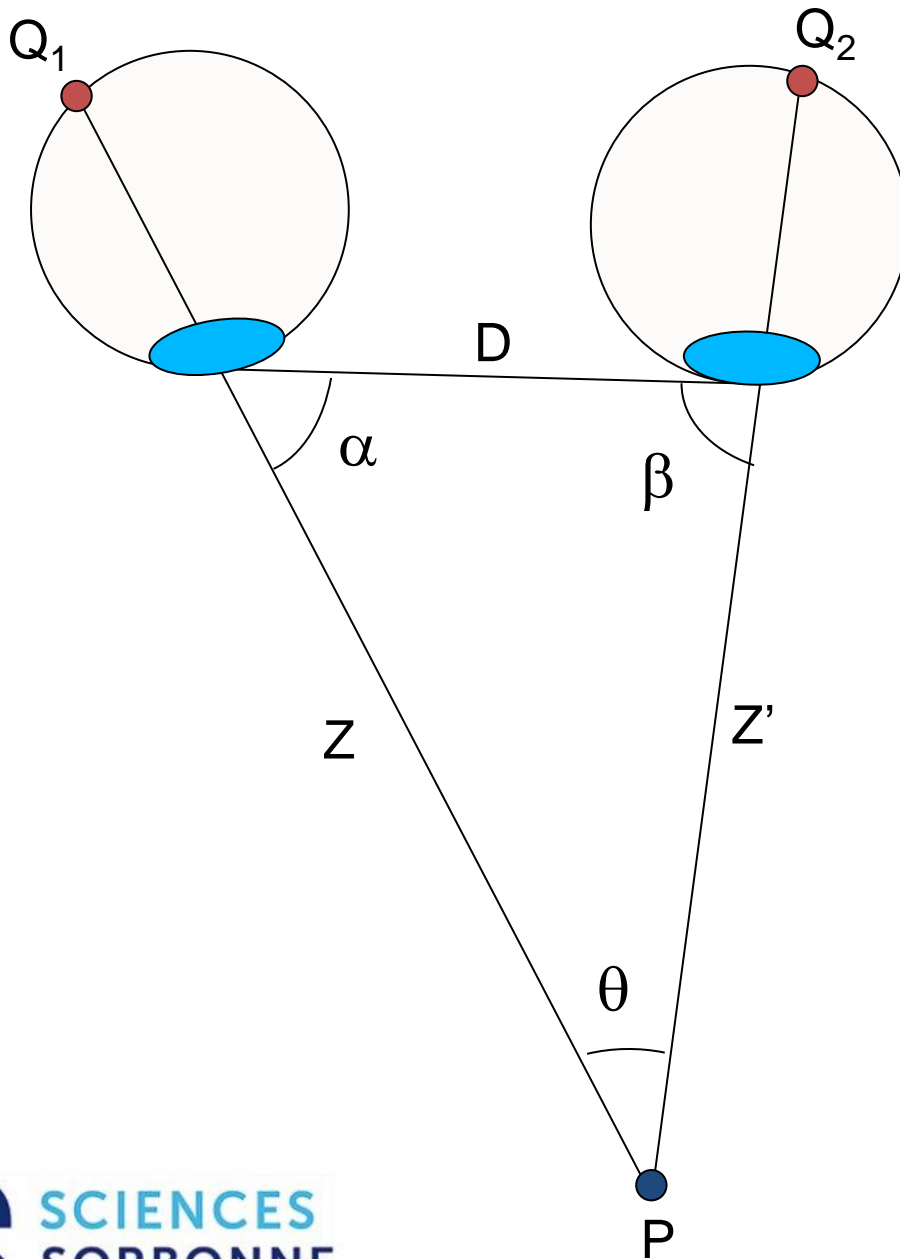
The *epipolar constraint* is expressed by the *fundamental matrix* F in the projective geometry framework:

$$Q_1 F Q_2 = 0$$



- $Q_1 F$: Epipolar line #2
- $F Q_2$: Epipolar line #1

DEPTH AND THE BINOCULAR VERGENCE



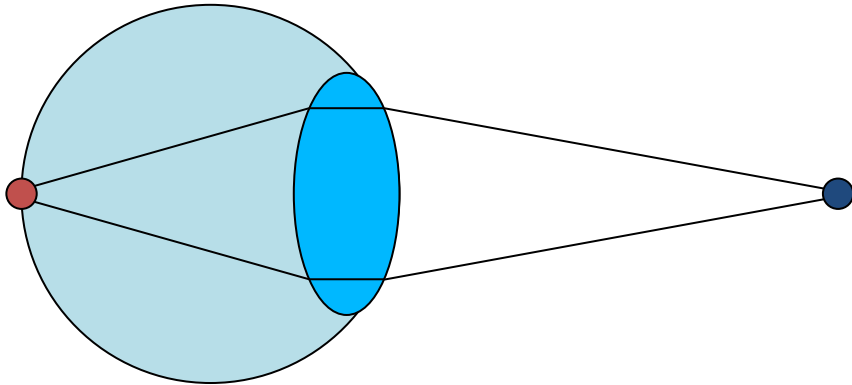
Triangulation principle:

$$Z = D \frac{\sin \beta}{\sin(\alpha + \beta)}$$
$$Z' = D \frac{\sin \alpha}{\sin(\alpha + \beta)}$$

Vergence angle: $\theta = \pi - (\alpha + \beta)$

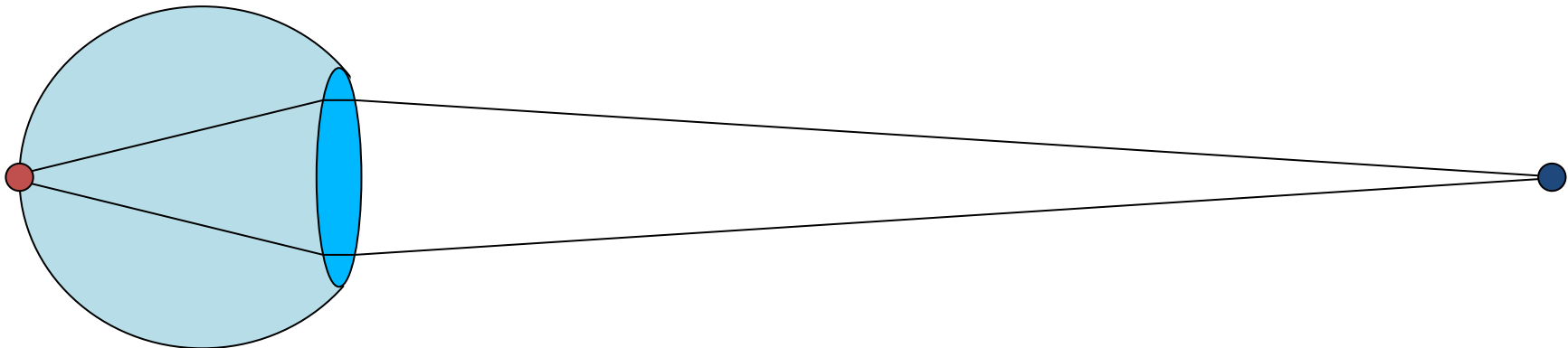
$$Z = D \frac{\sin \beta}{\sin \theta}$$
$$Z' = D \frac{\sin \alpha}{\sin \theta}$$

DEPTH AND ACCOMMODATION (MONOCULAR)



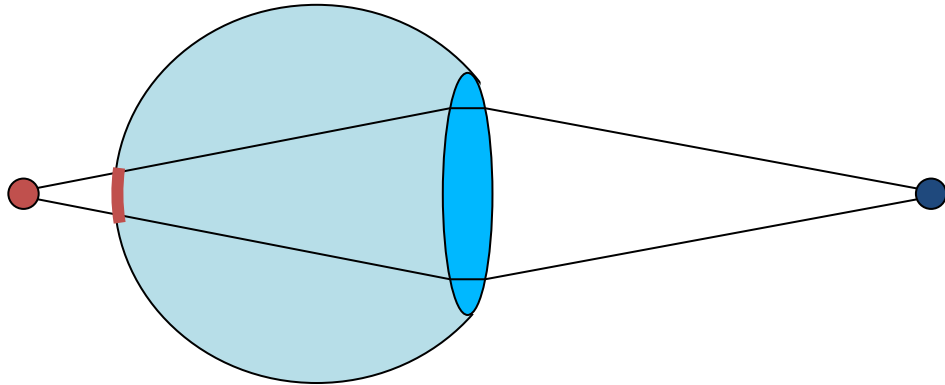
The accommodation mechanism consists in deforming the eye lens to adjust its focal in such a way that the image of the focalised object appears sharp on the retina.

Short focal: near object appears sharp on the retina plane



Long focal: far object appears sharp on the retina plane

DEPTH AND ACCOMMODATION (MONOCULAR)

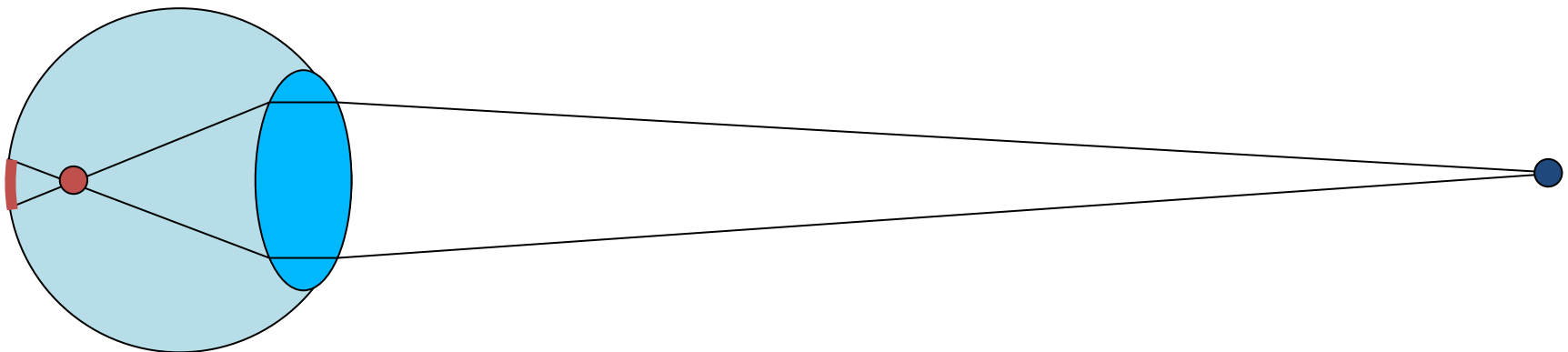


On the contrary, the points out of the focalisation plane P_f form an image whose level of blur is proportional to their distance to P_f .

See: *depth from defocus*

(Note the ambiguity of the position due to the blur symmetry with respect to P_f).

Focal too long: near object appears blurred on the retina plane

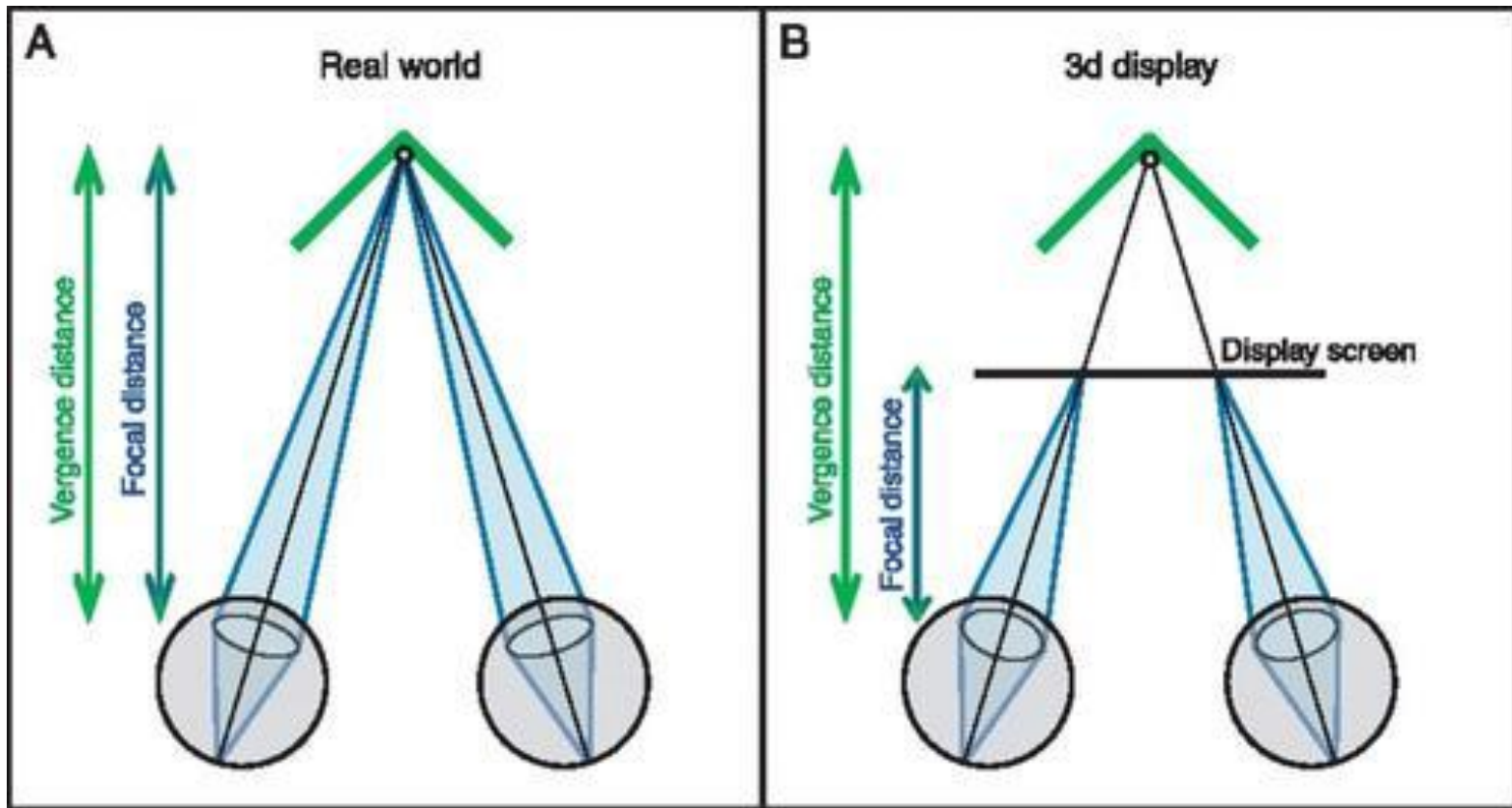


Focal too short: far object appears blurred on the retina plane

STEREOPSIS VS 3D DISPLAY

In natural binocular vision (stereopsis), vergence and accommodation are congruent (left diagram).

It is however possible to put – more or less deliberately – in conflict these two functions (right diagram). Thanks to this mechanism, it is possible to get a sharp 3d perception using a 2d display.



3D DISPLAY: ANAGLYPHS



Šibenik City
Hotel Hall
(Anaglyph)



3D DISPLAY: AUTOSTEREOGRAMS

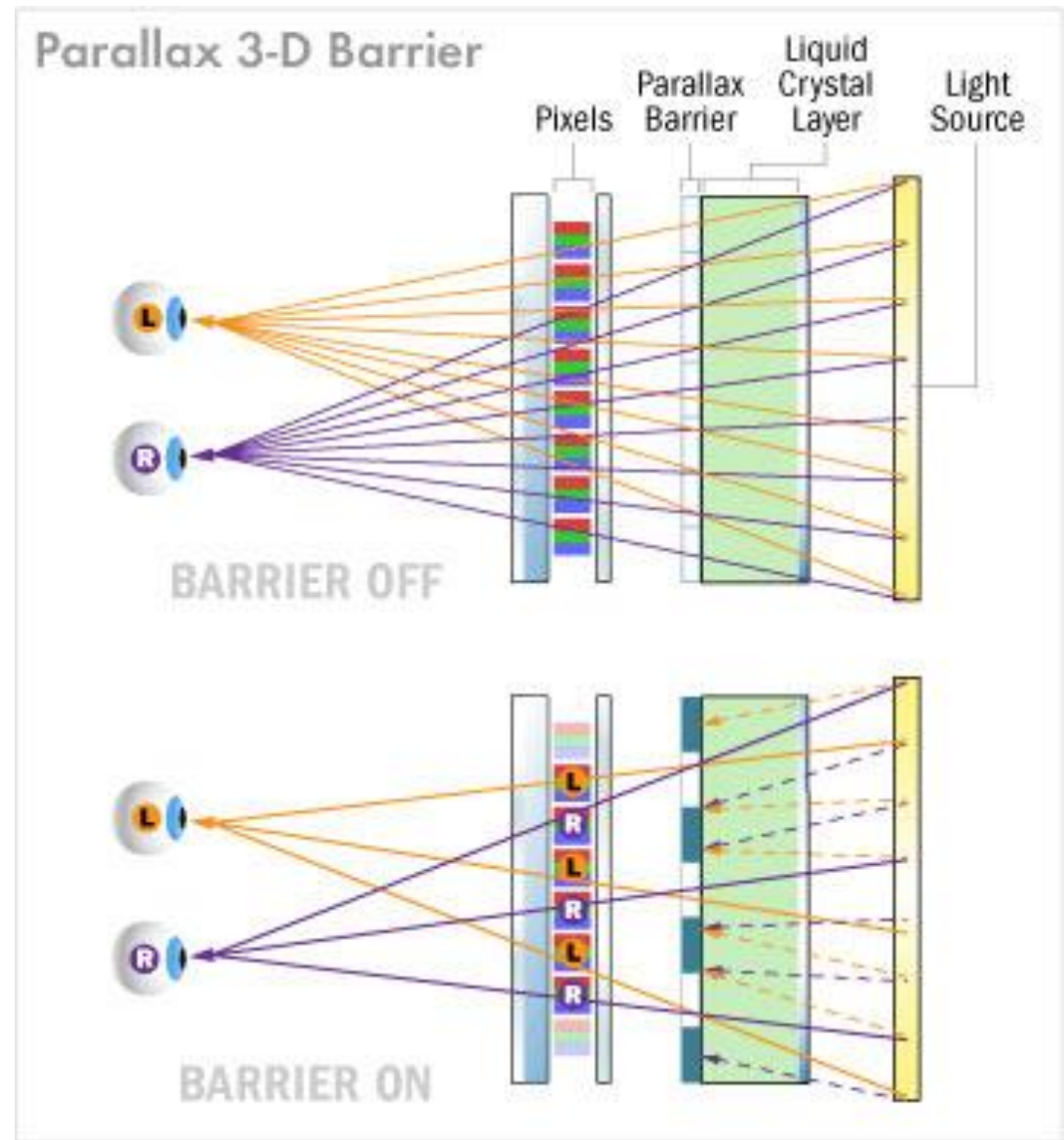


L.M. Rutherford, Full Moon, stereo pair (1864)

3D DISPLAY: PARALLAX BARRIER 3D SCREEN

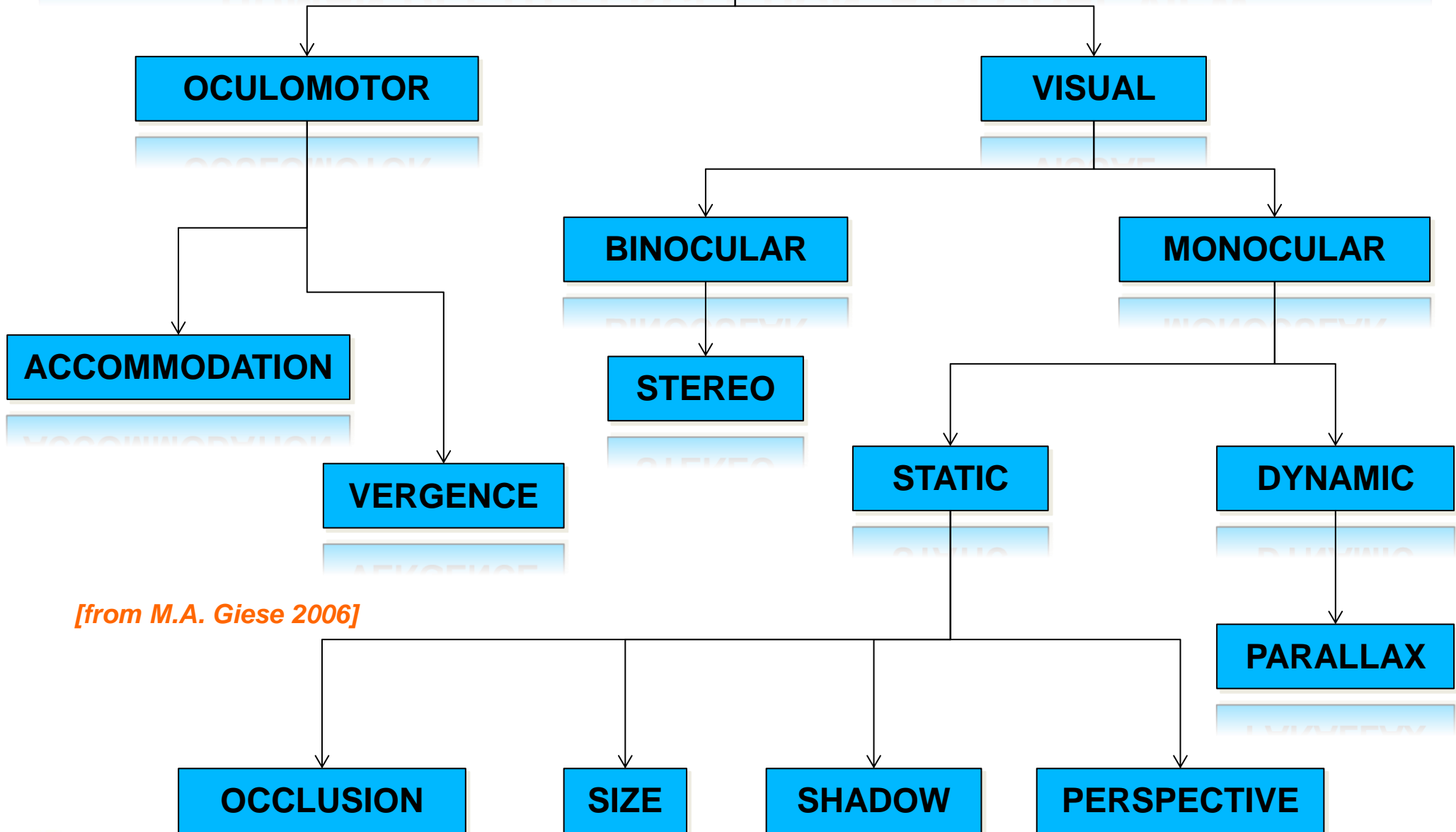
In parallax barrier 3d screens, an opaque vertical grid is positioned between the liquid crystal layer and the colour filters (pixels), in such a way to separate by parallax, pixels seen by the left eye from those seen by the right one.

Ex: Nintendo 3DS



[Ill. howstuffworks.com]

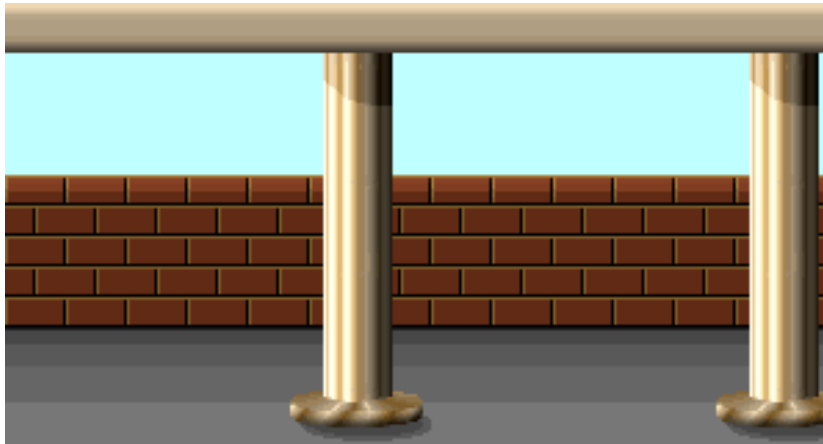
HUMAN DEPTH PERCEPTION: A GLOBAL VIEW



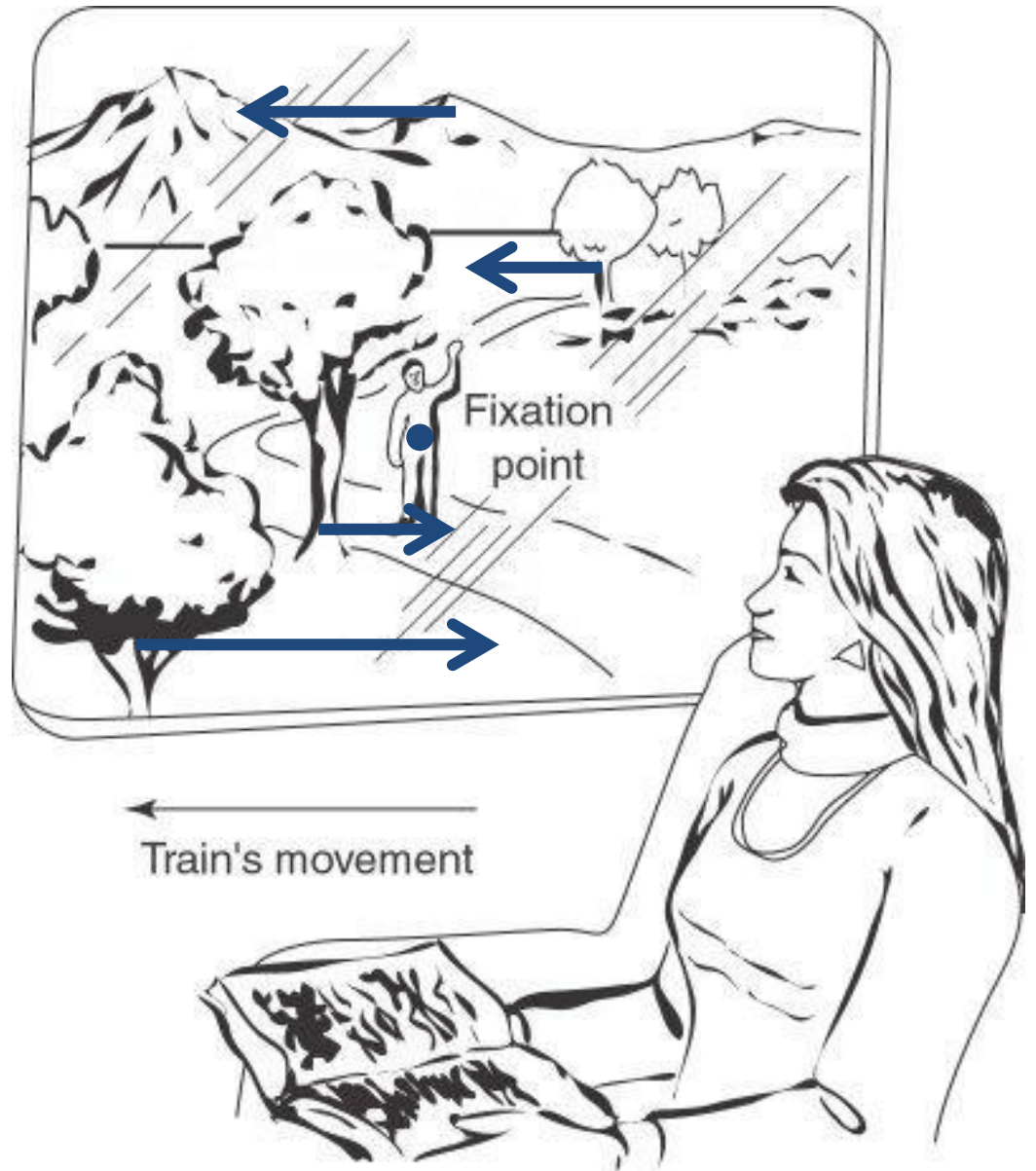
[from M.A. Giese 2006]

DYNAMIC MONOCULAR 3D: PARALLAX

$$Z = \frac{f \dot{X}}{\dot{x}}$$



[© nvnews.net]



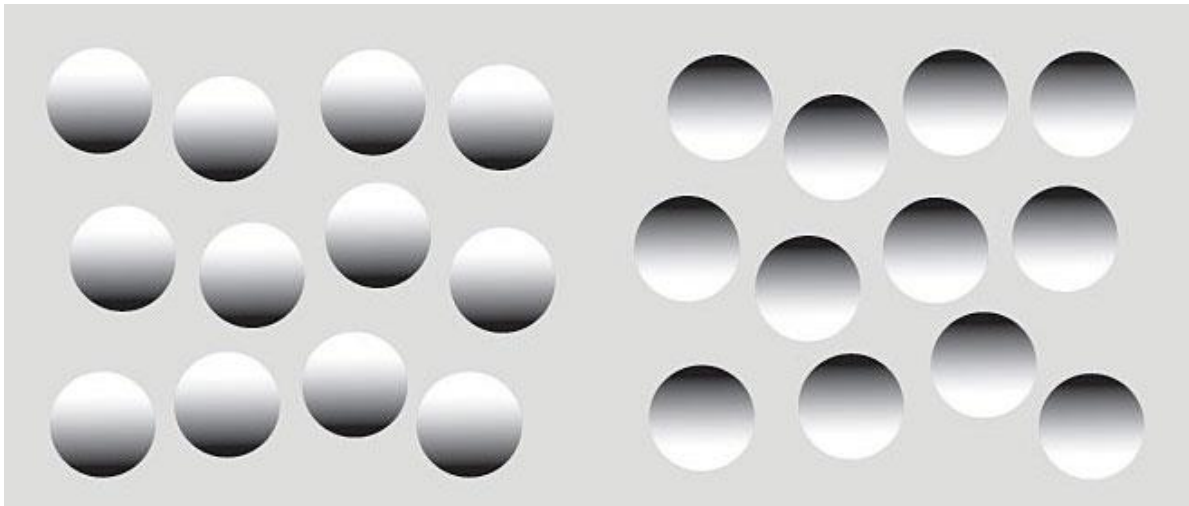
[Kenneth M. Steele 2014]

STATIC MONOCULAR 3D: OCCLUSIONS



*Giotto –
Pentecoste
(c. 1305)*

STATIC MONOCULAR 3D: SHADOWS



Self shadowing is a strong but ambiguous depth cue (light source position vs concavity).

Without shape prior, the concavity is determined by a prior of top lighting (left image).

When the shape prior is strong (face then convex), the concavity prior dominates the lighting prior (top-down effect, animation on the right).

See shape from shading



STATIC MONOCULAR 3D: SIZES



Georges Seurat – *Un dimanche après-midi à l'Île de la Grande Jatte* (1884-86)

STATIC MONOCULAR 3D: PERSPECTIVE

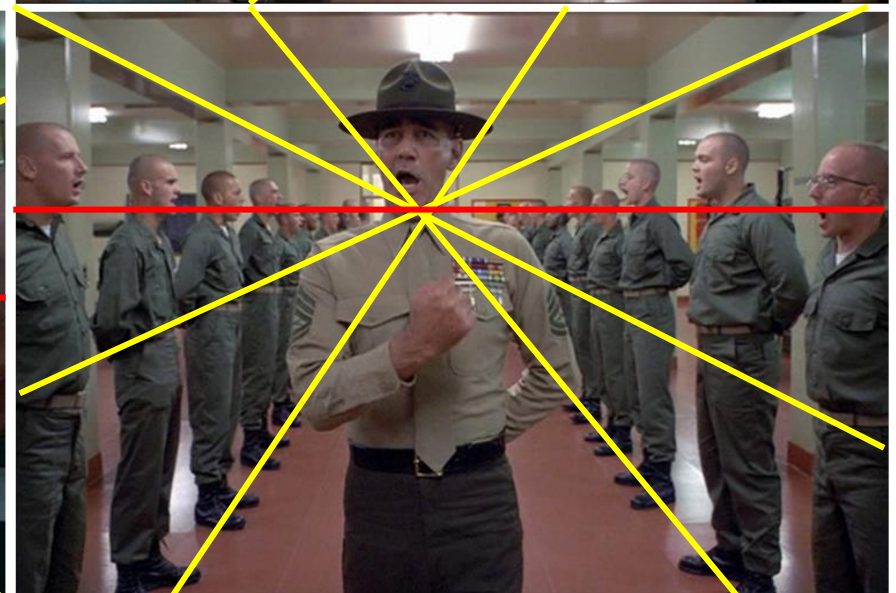
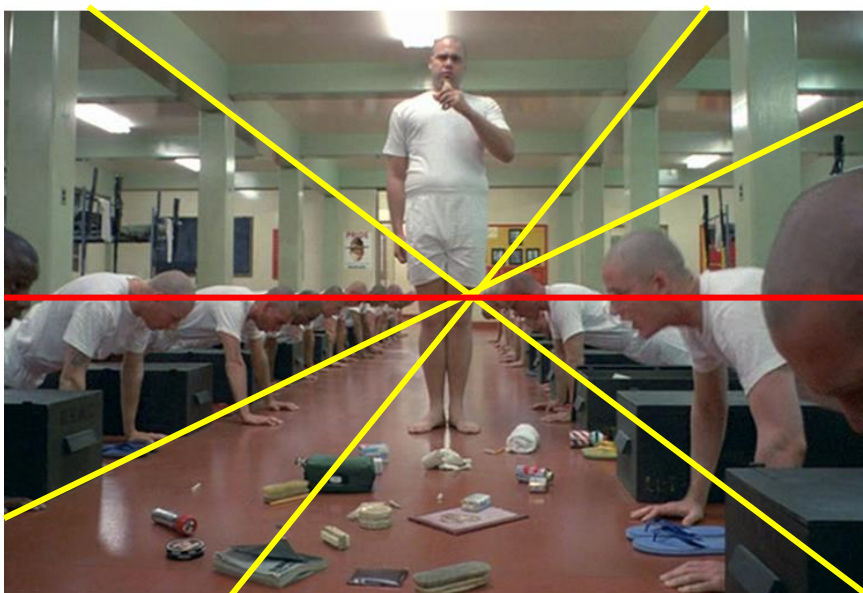


Stanley Kubrick – The Shining (1980)

STATIC MONOCULAR 3D: SIZES AND PERSPECTIVE

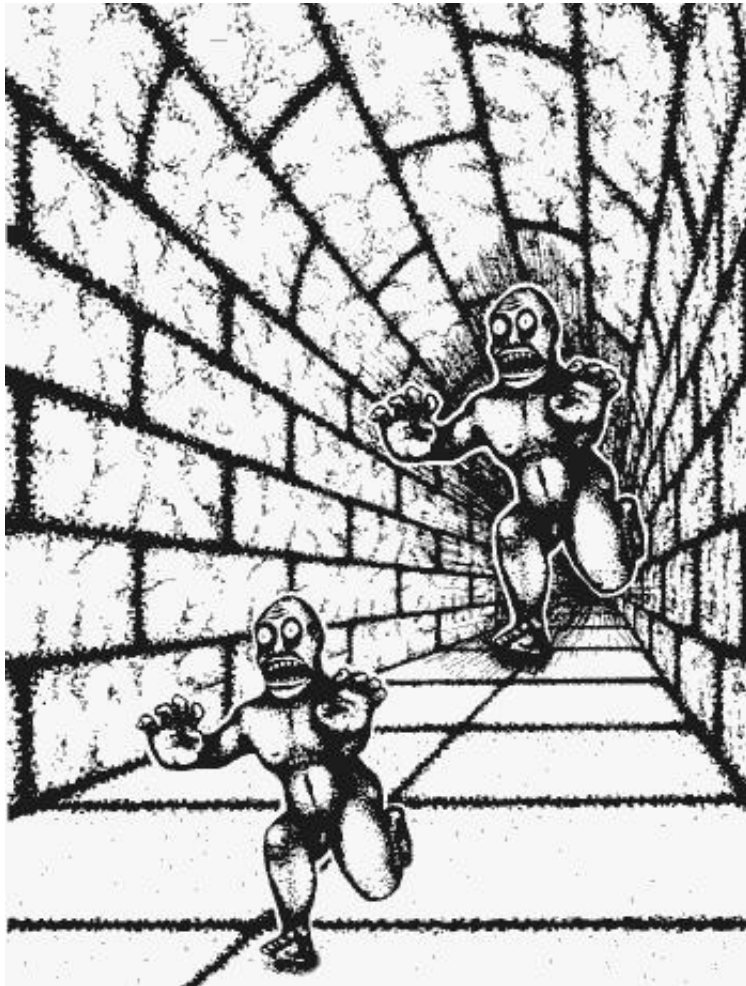


PERSPECTIVE: HORIZON AND VANISHING POINT



STATIC MONOCULAR 3D: SIZES VS PERSPECTIVE

Two popular examples of conflict between monocular depth cues:



The Hallway illusion



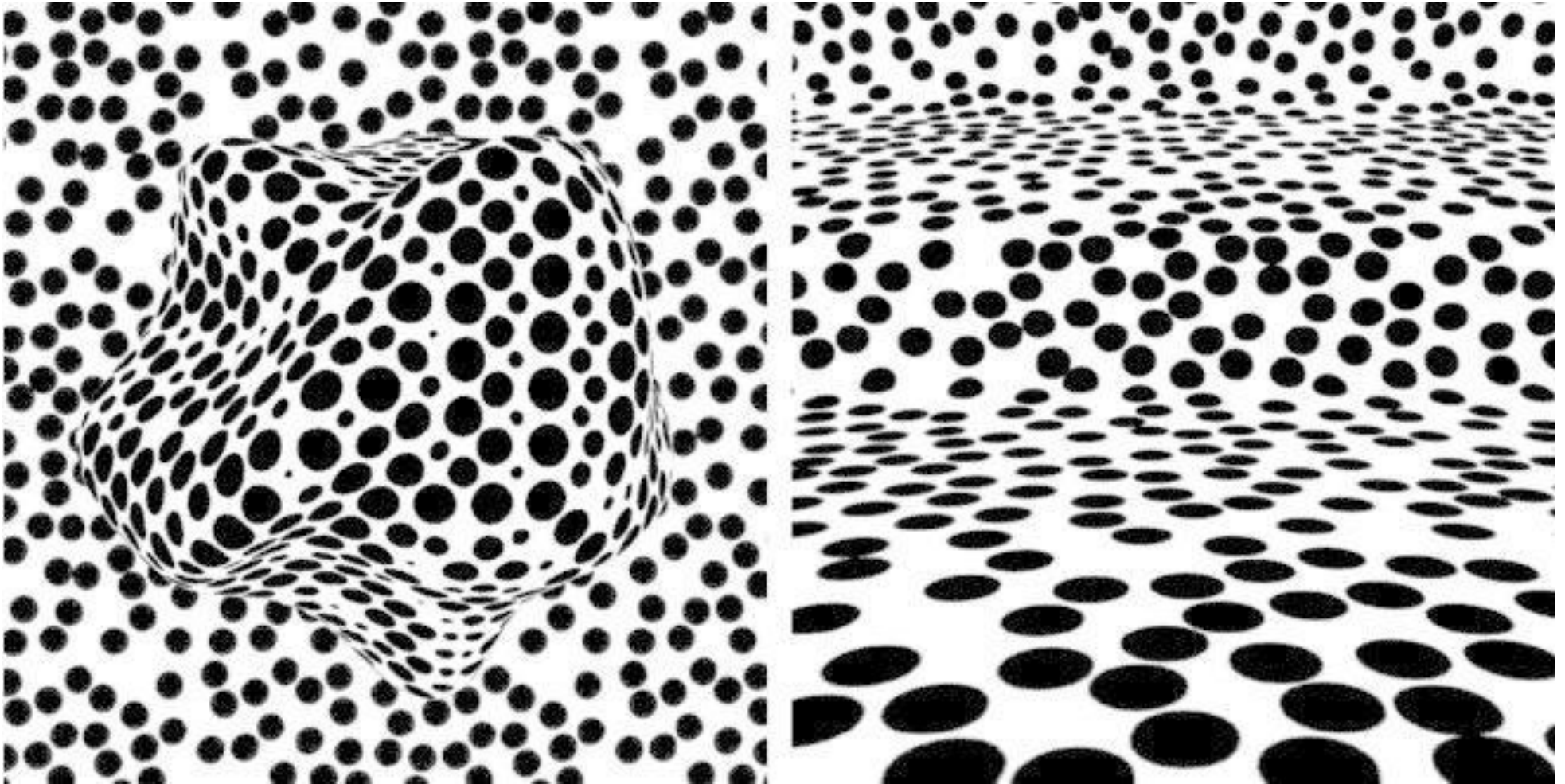
The Ames room

STATIC MONOCULAR 3D: SIZES VS PERSPECTIVE



Avignon TGV station: illusory space amplification created by accelerated perspective

STATIC MONOCULAR 3D: TEXTURE GRADIENTS



[Ill. DrThomas @ Studyblue]

TEXTURE GRADIENTS, SIZES AND PERSPECTIVES



Gustave Caillebotte – Rue de Paris, temps de pluie (1877)

PERSPECTIVE: HORIZON AND VANISHING POINTS



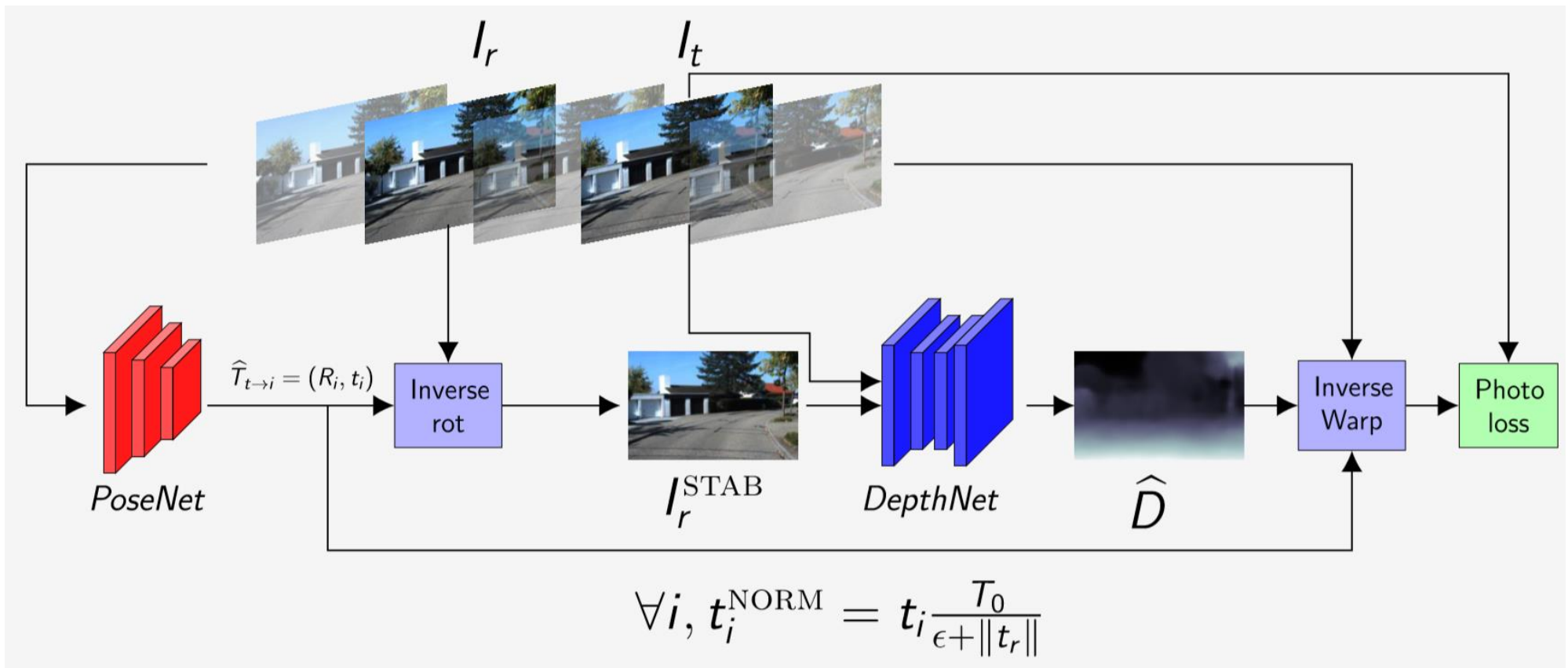
Gustave Caillebotte – Rue de Paris, temps de pluie (1877)

(TEMPORARY) CONCLUSION ON 3D: EXPLAIN THE DNNs?

What cues do the end-to-end depth prediction networks exploit? There is no formal proof yet, but they must be multiple:

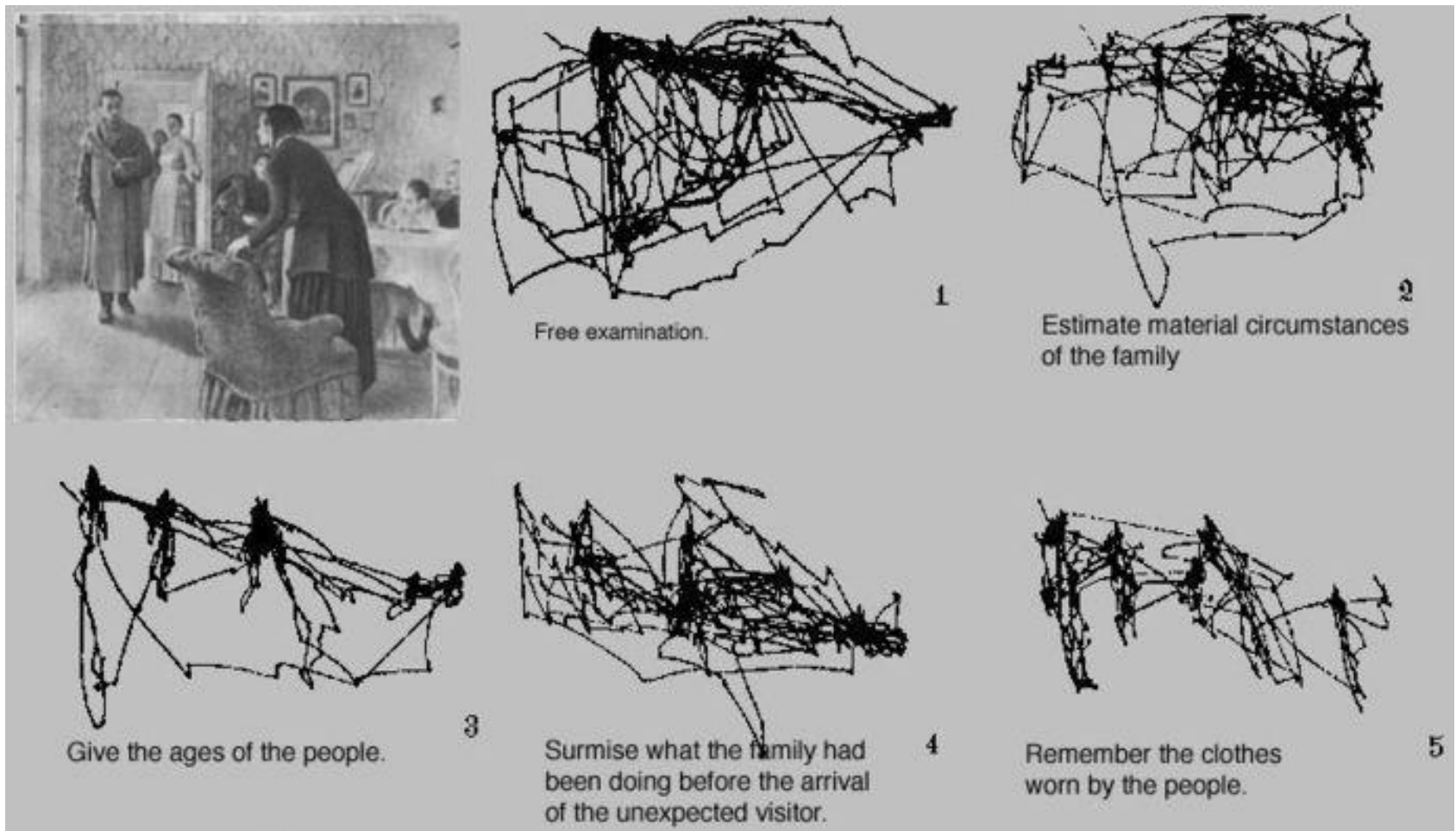
- Single-view
- Multi-views
- Photometric
- Géométric
- Context-based
- .../...

See lecture on learning-based 3d!



EYE SACCADES AND VISUAL SCAN

The analysis of our eye movements witnesses a *very partial scan* of the image (even without a priori, see image 1 below), and this scan seems to play a significant role in image interpretation.



EYE SACCADDES AND SACCADIC MASKING

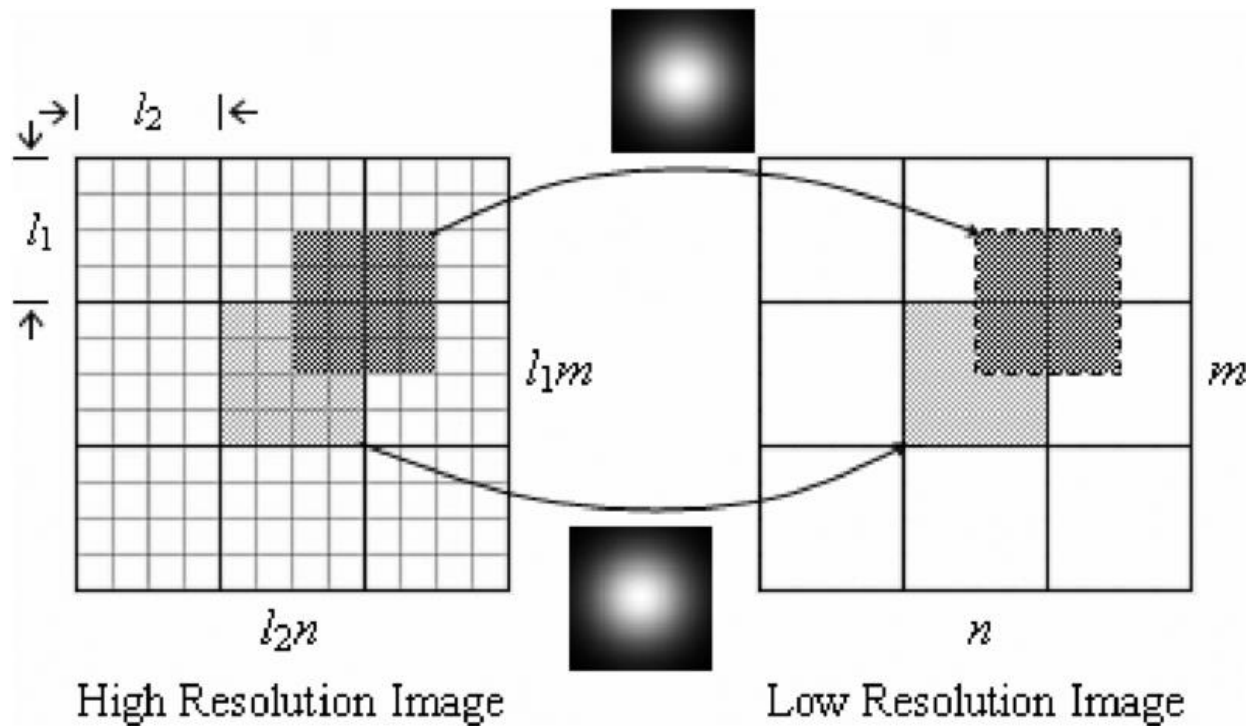
Our visual system *masks sudden eye movements*, so that it is impossible to perceive our own saccades in direct in a mirror, although they can be seen by another observer.



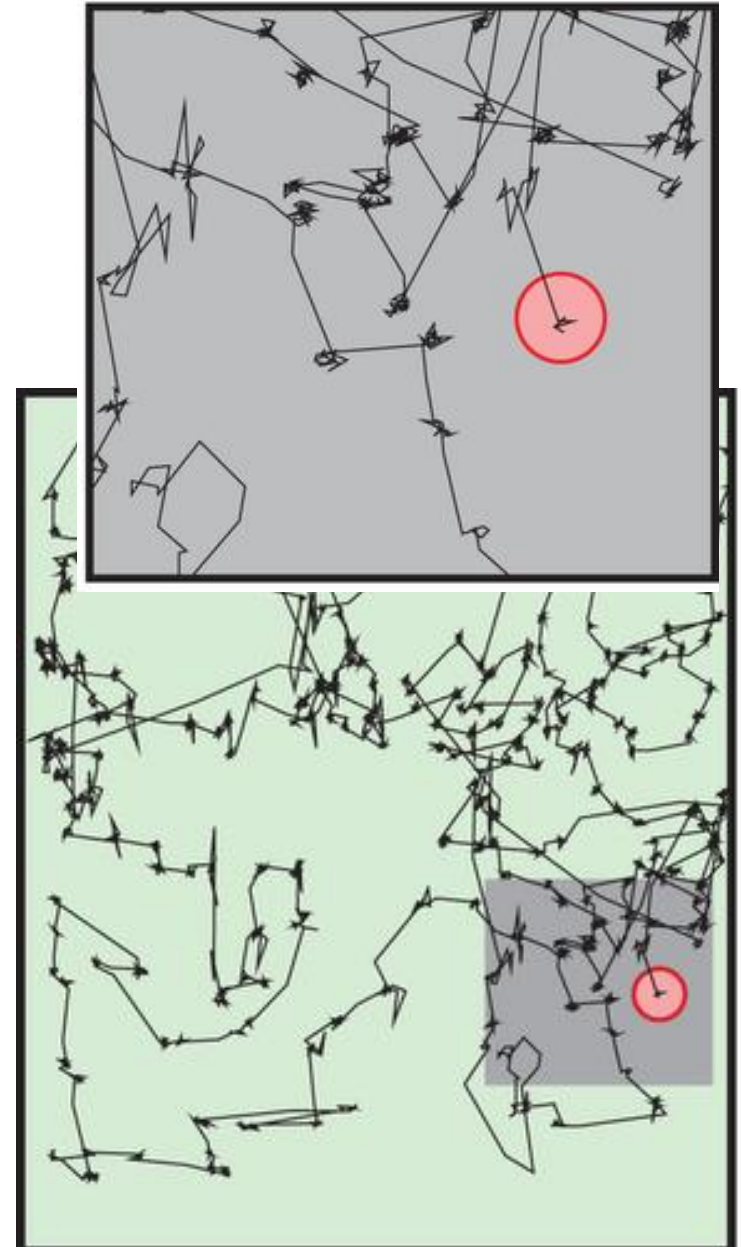
SACCADES, MICRO-SACCADES AND SUPER-RESOLUTION

Additionally to voluntary saccades, there are *involuntary micro-movements* of low amplitude and high frequency, that play a fundamental role in the *improvement of visual acuity*.

See *super-resolution* techniques.



[Chen, Luo, Hu 2007]



[Credidio et al 2012]

COURSE #1: CONCLUSIONS

Motion perception plays a major role in biological vision, for the interpretation of objects and the reconstruction of scenes geometry.

Biological vision systems, whether human or animal, use a wide variety of visual cues to perceive depth:

- Static or Dynamic
- Binocular or Monocular
- Geometry, Shade/Occlusion, Blur,...

Computer vision may also exploit different cues, as well as optical, mechanical or electronical devices, to improve motion estimation and 3d perception (see course #2 on Co-design).

REFERENCES

[Giese 2014] Martin A. Giese, *Biological and body motion perception*, in Oxford Handbook of Perceptual Organization, J. Wagemans, ed., Oxford University Press, 2014.

[Giese 2006] Martin A. Giese, *Visual perception*, lecture material, University of Tübingen, 2006.

[Reichardt 1956] W. Reichardt, *Autocorrelation, a principle for evaluation of sensory information by the central nervous system*, W.A. Rosenblith (Ed.), Principles of Sensory Communications, Wiley, New York (1961), pp. 303–317

[Ewert 1993] S. Wachowitz, J.-P. Ewert, *A key by which the toad's visual system gets access to the domain of prey*, Physiology & Behavior, 60-3, Sep. 1996, Pages 877–887.

[Tautz 2008] Jürgen Tautz, *The Buzz about Bees: Biology of a Superorganism*, Springer, 2008.

[Krapp et Wicklein 2008] H.G. Krapp, M. Wicklein *Central processing of visual information in insects*. In: The Senses: A Comprehensive Reference, ed. by A. Basbaum, A. Kaneko, G.M. Shepherd, G. Westheimer, Academic Press, 2008, p. 131–204.

[Pinard 2018] C. Pinard, L. Chevalley, A. Manzanera, D. Filliat *Learning Structure-from-Motion from motion*. Workshop “Geometry Meets Deep Learning”, ECCV, 2018.

REFERENCES

[Johansson 1950] Gunnar Johansson *Configurations in Visual Event Perception*. Uppsala: Almqvist & Wiskell (1950).

[Mc Dermott 2001] J. McDermott, Y. Weiss, E.H. Adelson, *Beyond Junctions: Nonlocal Form Constraints on Motion Interpretation*, *Perception* 2001, 30-8, pages 905-923

[Hoffman 2008] D.M. Hoffman, A.R. Girshick, Kurt Akeley, M.S. Banks, *Vergence–accommodation conflicts hinder visual performance and cause visual fatigue*, *Journal of Vision*, 2008 vol. 8 no. 3 article 33.

[Steele 2014] Kenneth M. Steele, *Psychology of Perception*, Lecture material, Appalachian State University, Fall 2014 <http://www1.appstate.edu/~kms/>

[Yarbus 1967] Alfred L. Yarbus, *Eye movements and vision*, Plenum Press, New York, 1967.

[Chen 2007] Y. Chen, Y. Luo, D. Hu, *Unified regularization framework for blind image super-resolution*, *Optical Engineering* Volume 46, Issue 12, December 2007.

[Credidio 2012] H. F. Credidio, E. N. Teixeira, S. D. S. Reis, A. A. Moreira, J. S. Andrade Jr, *Statistical patterns of visual search for hidden objects*, *Scientific reports*, Nature publishing group.