Dual Approximate Dynamic Programming for Large Scale Hydro Valleys

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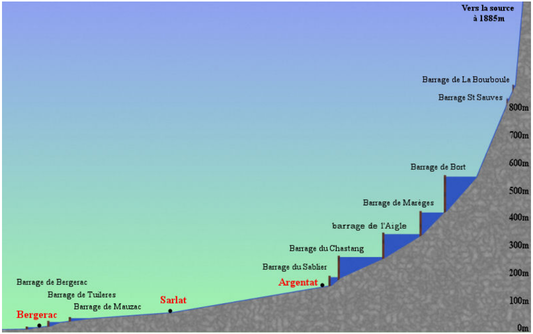
[1] Joint work with J.-C. Alais and F. Pacaud,
supported by the FMJH Program Gaspard Monge for Optimization.

Motivation

Electricity production management for hydro valleys



- *1 year time horizon*: compute each month the "values of water" (Bellman functions)
- *stochastic framework*: rain, market prices
- *large-scale valley*: 5 dams and more

We wish to remain within the scope of Dynamic Programming.

How to push the curse of dimensionality limits?

Aggregation methods

- fast to run method
- require some homogeneity between units

Stochastic Dual Dynamic Programming (SDDP)

- efficient method for this kind of problems
- strong assumptions (convexity, linearity)

Dual Approximate Dynamic Programming (DADP)

- spatial decomposition method
- complexity almost linear in the number of dams
- approximation methods in the stochastic framework

This talk: present numerical results for large-scale hydro valleys using DADP, and comparison with DP and SDDP.

Lecture outline

1. Dams management problem
   - Hydro valley modeling
   - Optimization problem
2. DADP in a nutshell
   - Spatial decomposition
   - Constraint weakening
3. Numerical experiments
   - Academic examples
   - More realistic examples

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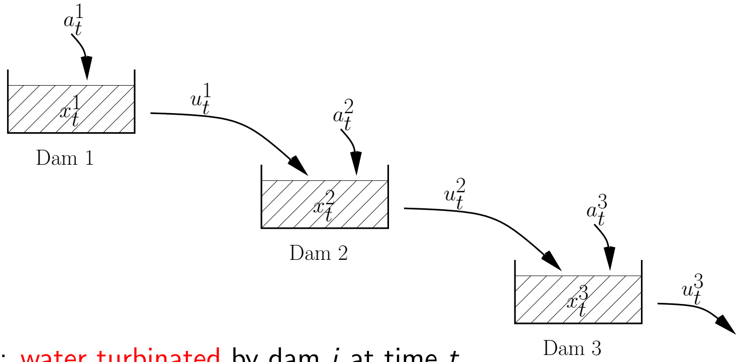
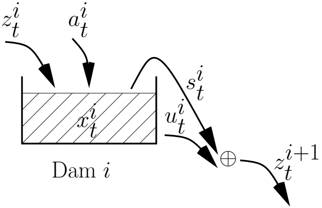
Operating scheme



u_t^i : water turbinated by dam i at time t,
x_t^i : water volume of dam i at time t,
a_t^i : water inflow at dam i at time t,
p_t^i : water price at dam i at time t,

Randomness: $w_t^i = (a_t^i, p_t^i)$, $w_t = (w_t^1, \ldots, w_t^N)$.

Dynamics and cost functions



Dam dynamics:

$$x_{t+1}^i = f_t^i(x_t^i, u_t^i, w_t^i, z_t^i) \ ,$$
$$= x_t^i - u_t^i + a_t^i + z_t^i - s_t^i \ ,$$

z_t^{i+1} being the outflow of dam i:

$$z_t^{i+1} = g_t^i(x_t^i, u_t^i, w_t^i, z_t^i) \ ,$$
$$= u_t^i + \underbrace{\max\left\{0, x_t^i - u_t^i + a_t^i + z_t^i - \overline{x}^i\right\}}_{s_t^i} \ .$$

We assume the Hazard-Decision information structure (u_t^i is chosen once w_t^i is observed), so that $\underline{u}^i \leq u_t^i \leq \min\left\{\overline{u}^i, x_t^i + a_t^i + z_t^i - \underline{x}^i\right\}$.

Gain at time $t < T$: $\quad L_t^i(x_t^i, u_t^i, w_t^i, z_t^i) = p_t^i u_t^i - \epsilon (u_t^i)^2$.

Final gain at time T: $\quad K^i(x_T^i) = -a^i \min\{0, x_T^i - \widehat{x}^i\}^2$.

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Stochastic optimization problem

The global optimization problem reads:

$$\max_{(\boldsymbol{X},\boldsymbol{U},\boldsymbol{Z})} \mathbb{E}\left(\sum_{i=1}^{N} \left(\sum_{t=0}^{T-1} L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i) + K^i(\boldsymbol{X}_T^i) \right) \right),$$

subject to:

$$\boldsymbol{X}_{t+1}^i = f_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i) \ , \ \ \forall i \ , \ \ \forall t \ ,$$

$$\sigma\big(\boldsymbol{U}_t^i\big) \subset \sigma\big(\boldsymbol{W}_0, \ldots, \boldsymbol{W}_t\big) \ , \qquad \forall i \ , \ \ \forall t \ ,$$

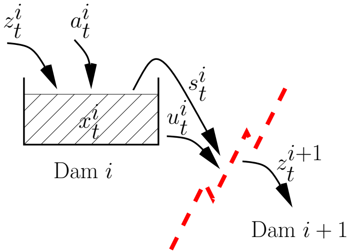
$$\boldsymbol{Z}_t^{i+1} = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i) \ , \ \ \forall i \ , \ \ \forall t \ .$$

Assumption. Noises $\boldsymbol{W}_0, \ldots, \boldsymbol{W}_{T-1}$ are *independent over time*.

Price decomposition

- Dualize the coupling constraints $\boldsymbol{Z}_t^{i+1} = g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i)$.
  Note that the associated multiplier $\boldsymbol{\Lambda}_t^{i+1}$ is a random variable.

- Minimize the dual problem (using a gradient-like algorithm).



- At iteration k, the duality term:

  $$\boldsymbol{\Lambda}_t^{i+1,(k)} \cdot \left(\boldsymbol{Z}_t^{i+1} - g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i)\right) ,$$

  is the difference of two terms:

  - $\boldsymbol{\Lambda}_t^{i+1,(k)} \cdot \boldsymbol{Z}_t^{i+1} \rightsquigarrow$ dam $i+1$,
  - $\boldsymbol{\Lambda}_t^{i+1,(k)} \cdot g_t^i(\cdots) \rightsquigarrow$ dam i.

- Dam by dam decomposition for the maximization in $(\boldsymbol{X}, \boldsymbol{U}, \boldsymbol{Z})$ at $\boldsymbol{\Lambda}_t^{i+1,(k)}$ fixed.

DADP core idea

The i-th subproblem writes:

$$\max_{\boldsymbol{U}^i,\boldsymbol{Z}^i,\boldsymbol{X}^i} \mathbb{E}\Bigg(\sum_{t=0}^{T-1} \Big(L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i) + \boldsymbol{\Lambda}_t^{i,(k)} \cdot \boldsymbol{Z}_t^i - \boldsymbol{\Lambda}_t^{i+1,(k)} \cdot g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i) \Big) + K^i(\boldsymbol{X}_T^i) \Bigg) ,$$

but $\boldsymbol{\Lambda}_t^{i,(k)}$ depends on the whole past of noises $(\boldsymbol{W}_0, \ldots, \boldsymbol{W}_t)$...

The core idea of DADP is

- to replace the constraint $\boldsymbol{Z}_t^{i+1} - g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i) = 0$ by its conditional expectation with respect to $\boldsymbol{Y}_t^i$:

$$\mathbb{E}\big(\boldsymbol{Z}_t^{i+1} - g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i) \mid \boldsymbol{Y}_t^i\big) = 0 ,$$

- where $(\boldsymbol{Y}_0^i, \ldots, \boldsymbol{Y}_{T-1}^i)$ is a "well-chosen" information process.

Subproblems in DADP

DADP thus consists of a constraint relaxation.

This constraint relaxation is equivalent to replace the original multiplier $\boldsymbol{\Lambda}_t^{i,(k)}$ by its conditional expectation $\mathbb{E}\big(\boldsymbol{\Lambda}_t^{i,(k)} \mid \boldsymbol{Y}_t^{i-1}\big)$.

The expression of the i-th subproblem becomes:

$$\max_{\boldsymbol{U}^i,\boldsymbol{Z}^i,\boldsymbol{X}^i} \mathbb{E}\Bigg(\sum_{t=0}^{T-1} \Big(L_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i) + \mathbb{E}\big(\boldsymbol{\Lambda}_t^{i,(k)} \mid \boldsymbol{Y}_t^{i-1}\big) \cdot \boldsymbol{Z}_t^i$$
$$- \mathbb{E}\big(\boldsymbol{\Lambda}_t^{i+1,(k)} \mid \boldsymbol{Y}_t^{i}\big) \cdot g_t^i(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i)\Big)$$
$$+ K^i(\boldsymbol{X}_T^i)\Bigg) .$$

If each process $\boldsymbol{Y}^i$ follows a dynamical equation, DP applies.

A crude relaxation: $\boldsymbol{Y}_t^i \equiv \text{cste}$

1. The multipliers $\boldsymbol{\Lambda}_t^{i,(k)}$ appear only in the subproblems by means of their expectations $\mathbb{E}\big(\boldsymbol{\Lambda}_t^{i,(k)}\big)$, so that each subproblem involves a 1-dimensional state variable.
2. For the gradient algorithm, the coordination task reduces to:

$$\mathbb{E}\big(\boldsymbol{\Lambda}_t^{i,(k+1)}\big) = \mathbb{E}\big(\boldsymbol{\Lambda}_t^{i,(k)}\big) + \rho_t \mathbb{E}\Big(\boldsymbol{Z}_t^{i+1,(k)} - g_t^i\big(\boldsymbol{X}_t^{i,(k)}, \boldsymbol{U}_t^{i,(k)}, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^{i,(k)}\big)\Big) .$$

3. The constraints taken into account by DADP are in fact:

$$\mathbb{E}\Big(\boldsymbol{Z}_t^{i+1} - g_t^i\big(\boldsymbol{X}_t^i, \boldsymbol{U}_t^i, \boldsymbol{W}_t^i, \boldsymbol{Z}_t^i\big)\Big) = 0 .$$

The DADP solutions do not satisfy the initial constraints: need to use an heuristic method to regain admissibility.

How to regain admissible policies?

We have computed N local Bellman functions V_t^i at each time t, each depending on a single state variable x^i, whereas we need one global Bellman function V_t depending on the global state $(x^1, \dots, x^N)$ in order to design the decisions at time t.

Heuristic procedure: form the following global Bellman function:

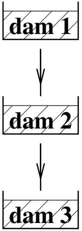
$$\widehat{V}_t(x^1, \dots, x^N) = \sum_{i=1}^{N} V_t^i(x^i) \ ,$$

and solve at each time t the one-step DP problem:

$$\max_{u,z} \ \sum_{i=1}^{N} L_t^i(x^i, u^i, w_t^i, z^i) + \widehat{V}_{t+1}(x_{t+1}^1, \dots, x_{t+1}^N) \ ,$$

with $\quad x_{t+1}^i = f_t^i(x^i, u^i, w_t^i, z^i) \quad$ and $\quad z^{i+1} = g_t^i(x^i, u^i, w_t^i, z^i) \ .$

Four case studies



Discretization

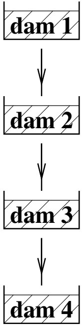
$T \rightsquigarrow 12$

$\boldsymbol{X} \rightsquigarrow 41$

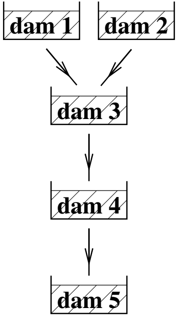
$\boldsymbol{U} \rightsquigarrow 6$

$\boldsymbol{W} \rightsquigarrow 10$

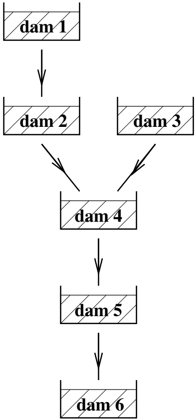
3-Dams



4-Dams



5-Dams



6-Dams

Results

Valley	3-Dams	4-Dams	5-Dams	6-Dams
DP CPU time	*5'*	*1630'*	*461200'*	N.A.
DP value	2482.0	3742.7	4681.6	N.A.
$\mathrm{SDDP_d}$ value	2474.2	3736.4	4672.2	7014.8
$\mathrm{SDDP_d}$ CPU time	*0.3'*	*2'*	*16'*	*320'*
$\mathrm{SDDP_c}$ value	2481.6	3742.2	4680.6	7029.0
$\mathrm{SDDP_c}$ CPU time	*3'*	*4'*	*5'*	*8'*

Table: Results obtained by DP, $\mathrm{SDDP_d}$ and $\mathrm{SDDP_c}$

Valley	3-Dams	4-Dams	5-Dams	6-Dams
DADP CPU time	*3'*	*6'*	*5'*	*13'*
DADP value	2401.3	3667.0	4633.7	6816.5
Gap with DP	$-\mathbf{3.2\%}$	$-\mathbf{2.0\%}$	$-\mathbf{1.0\%}$	$-\mathbf{3.0\%}$

Table: Results obtained by DADP "Expectation"

Results obtained using a 4 cores – 8 threads Intel®Core i7 based computer.

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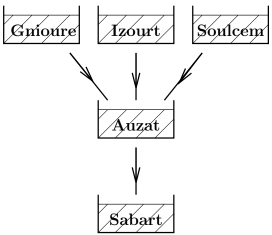
Two “true” valleys

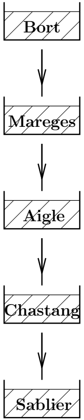


Discretization

$T \rightsquigarrow 12$, $W \rightsquigarrow 10$

realistic grids for U and X

Vicdessos



Dordogne

Results

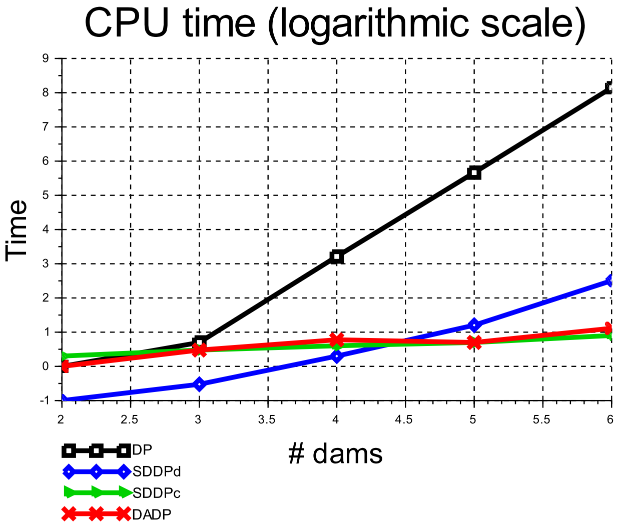
Valley	**Vicdessos**	**Dordogne**
$\mathrm{SDDP_d}$ CPU time	*90*'	*86000*'
$\mathrm{SDDP_d}$ value	2232.1	21904.5
$\mathrm{SDDP_c}$ CPU time	*10*'	*17*'
$\mathrm{SDDP_c}$ value	2244.3	22136.1

Table: Results obtained by $\mathrm{SDDP_d}$ and $\mathrm{SDDP_c}$

Valley	**Vicdessos**	**Dordogne**
DADP CPU time	*10*'	*155*'
DADP value	2237.4	21499.8
Gap with $\mathrm{SDDP_d}$	$-$**0.3%**	$-$**2.8%**

Table: Results obtained by DADP “Expectation”

CPU time comparison



Conclusions and perspectives

Conclusions for DADP

- Fast numerical convergence of the method.
- Near-optimal results even when using a “crude” relaxation.
- Method that can be used for very large valleys

General perspectives

- Apply to more complex topologies (smart grids).
- Connection with other decomposition methods.
- Theoretical study.

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